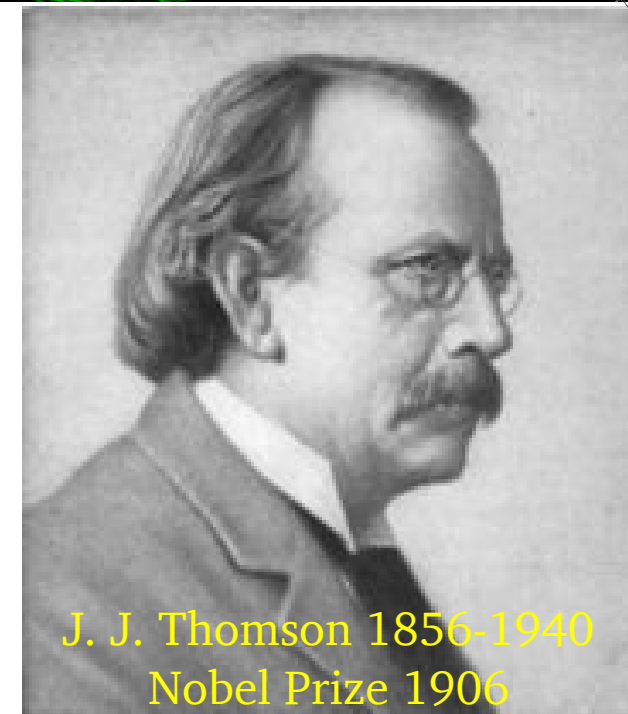


"It is the scattering of electromagnetic radiation by a free non-relativistic charged particle."

The electric and magnetic components of the incident wave accelerate the particle. As it accelerates, it, in turn, emits radiation and thus, the wave is scattered. Thomson scattering is an important phenomenon in plasma physics and was first explained by the physicist



The main cause of the acceleration of the particle will be due to the electric field component of the incident wave. The particle will move in the direction of the oscillating electric field, resulting in electromagnetic dipole radiation. The moving particle radiates most strongly in a direction perpendicular to its motion and that radiation will be polarized along the direction of its motion. Therefore, depending on where an observer is located, the light scattered from a small volume element may appear to be more or less polarized."



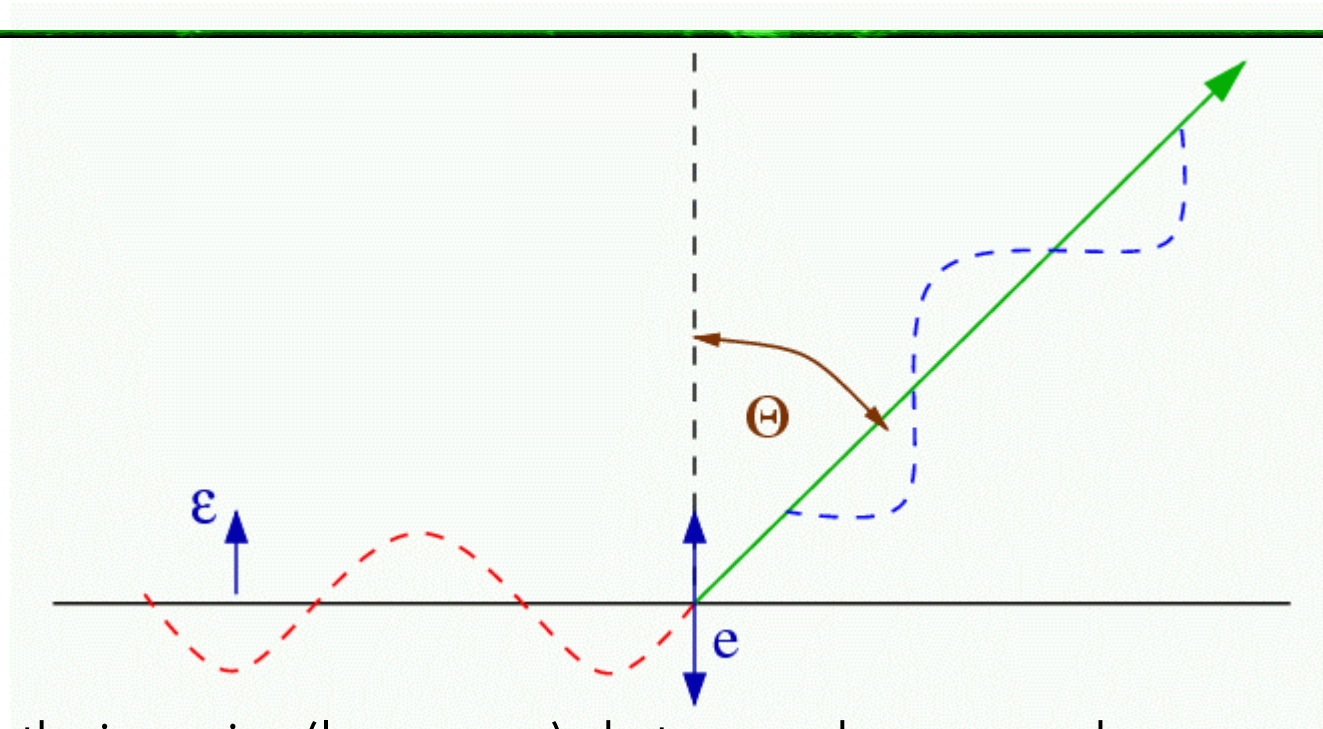
Thompson, Compton & IC scattering

on textbooks

- G. Ghisellini: "Radiative Processes in High Energy Astrophysics" § 5
- M. Longair: "High Energy Astrophysics" § 9
- G. B. Rybicky & A.P. Lightman: "Radiative Processes in Astrophysics" § 3.4 § 7
- C. Fanti & R. Fanti: "Lezioni di radioastronomia" § 5
- T. Padmanabhan: "Theoretical Astrophysics" § 6.4 & 6.6 & 6.7

Photon-electron interaction -1-

$$h\nu \ll m_e c^2$$



- for a non relativistic particle, the incoming (low-energy) photons can be expressed as a continuous e-m wave and the magnetic field ($H_0 = E_0$) can be "neglected" (i.e. studied as a consequence of the E field behaviour).
- the average incoming Poynting flux is $\langle |S| \rangle = cE_0^2/8\pi$
- the charge feels a force due to a linearly polarized incoming wave:
- the energy of the scattered radiation is the same as the incoming wave



Thomson scattering (2)

Force from a linearly polarised wave $\vec{F} = e\hat{\epsilon}E_0 \sin \omega_0 t$

Dipole moment

$$\vec{d} = e\vec{r} \quad \rightarrow \quad \ddot{\vec{d}} = e\ddot{\vec{r}} = \frac{e^2 E_0}{m} \hat{\epsilon} \sin \omega_0 t$$

$$\vec{d} = - \left(\frac{e^2 E_0}{m \omega_0^2} \right) \hat{\epsilon} \sin \omega_0 t = \vec{a}(t)$$

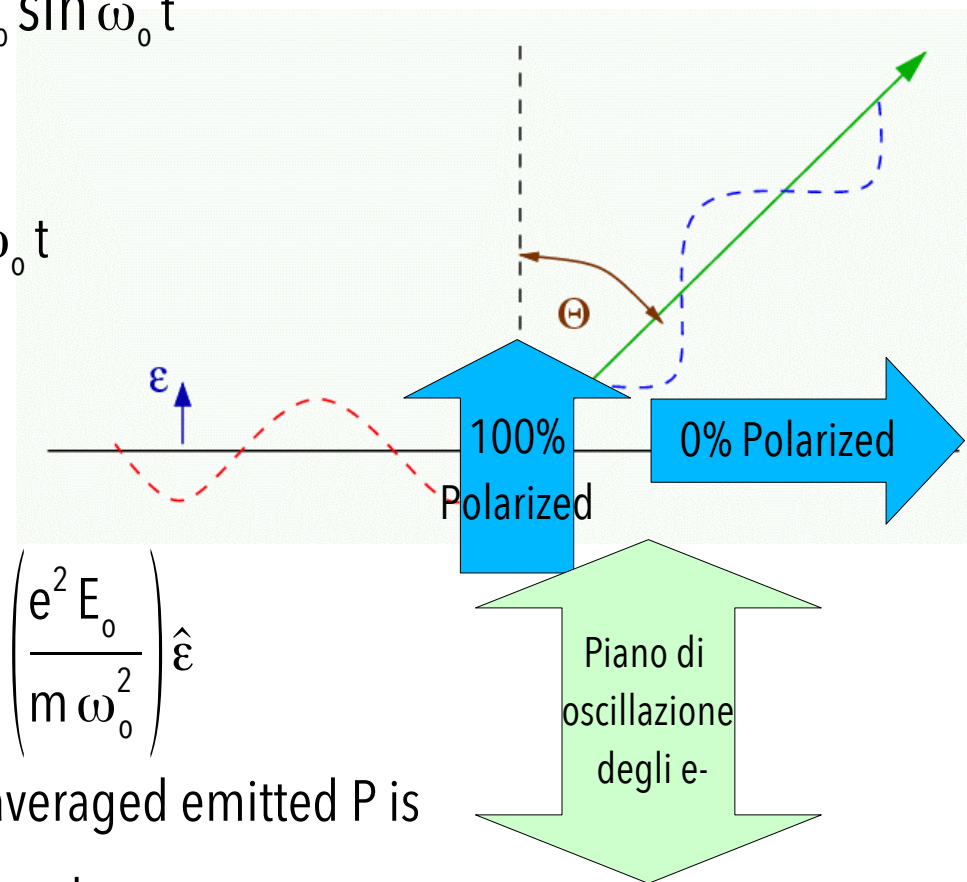
describes a dipole with amplitude $\vec{d}_0 = - \left(\frac{e^2 E_0}{m \omega_0^2} \right) \hat{\epsilon}$

taking into account the Larmor's formula, the time averaged emitted P is

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \left\langle \frac{2}{3} \frac{\ddot{\vec{d}}^2}{c^3} \right\rangle = \left\langle \frac{e^4 E_0^2}{8\pi m^2 c^3} \sin^2 \Theta \right\rangle = \langle S \rangle \frac{d\sigma}{d\Omega}$$

where $\frac{d\sigma}{d\Omega} = \frac{e^4}{m^2 c^4} \sin^2 \Theta = r_0^2 \sin^2 \Theta$

Differential cross section for the scattering





An electron oscillates when interacting with the E field of incoming radiation and

$$x(t) = \frac{e E_0}{m_e \omega^2} \cos \omega t$$

the consequent acceleration is $a(t) = \frac{e E_0}{m_e} \cos \omega t$

the emitted power by the scattering particle is $-\left\langle \frac{d\varepsilon}{dt} \right\rangle = \frac{2}{3} \frac{e^2}{c^3} a^2 = \frac{2}{3} \frac{e^2}{c^3} \left(\frac{e E_0}{m_e} \right)^2 \frac{1}{2} = \frac{1}{3} \frac{e^4 E_0^2}{m_e^2 c^3}$

However, remember the module of the Poynting vector of the incoming radiation $\langle |S| \rangle = c E_0^2 / 8\pi$

the **Thomson cross section** of the electron can be defined:

$$\sigma_T = \frac{\left\langle \frac{d\varepsilon}{dt} \right\rangle}{\langle |S| \rangle} = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.65 \times 10^{-25} \text{ cm}^2$$

where $r_o = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13} \text{ cm}$ electron classical radius

N.B.: the cross section is independent of frequency (at least apparently)

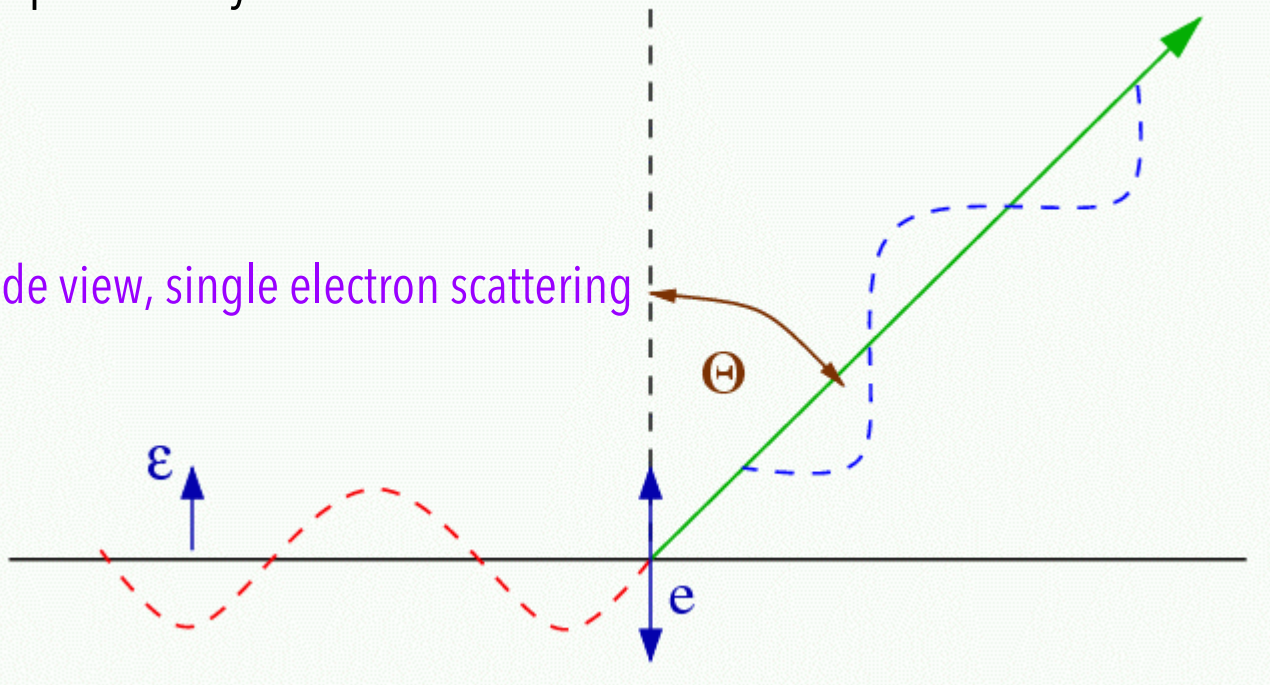
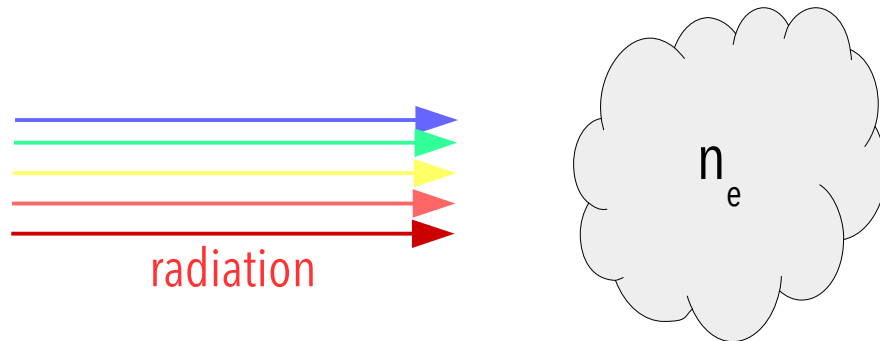
This breaks down at high energies, when classical physics needs quantum mechanics



How polarization is created

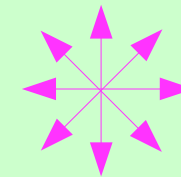
<https://www.youtube.com/watch?v=fSAzPQtSAPQ> from 1:40" to 3:30"

Side view, single electron scattering



All the incoming photons follow about the same trajectory (e.g. single point source)

Front view: unpolarized radiation makes the electrons oscillate in all directions

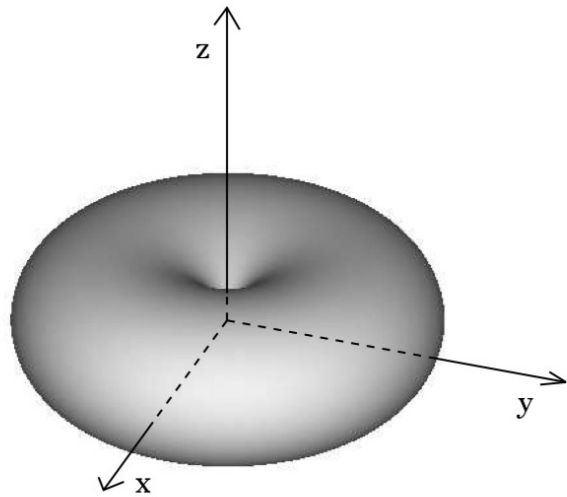
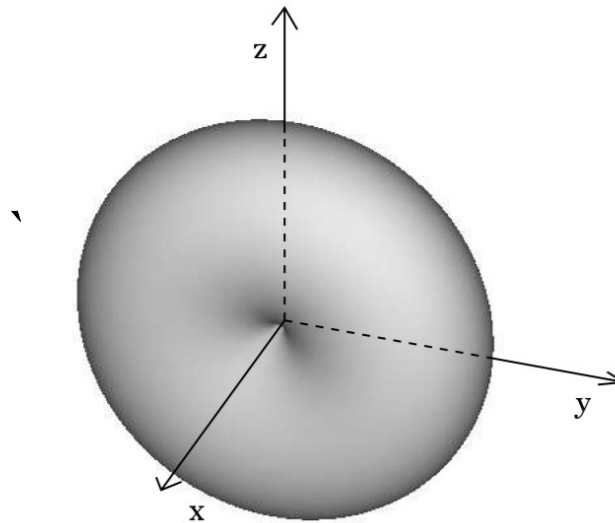




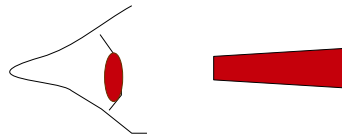
How polarization is created

The side (red) observer DO NOT receive any radiation from the "blue" oscillations

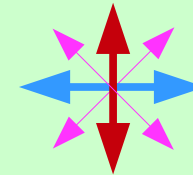
The top (blue) observer DO NOT receive any radiation from the "red" oscillations



Both observers measure 100% polarized radiation



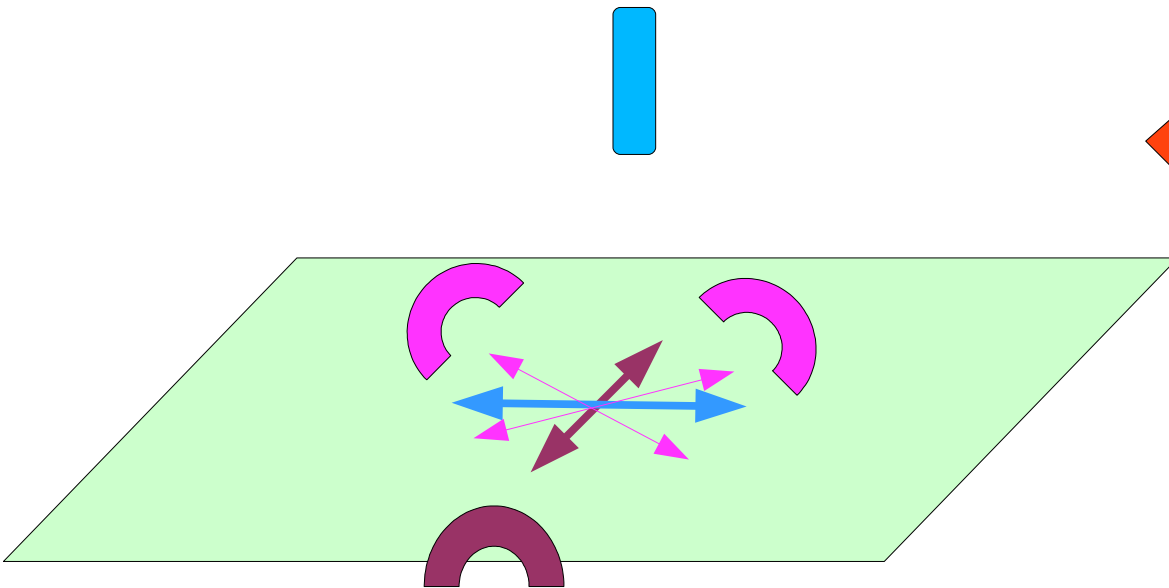
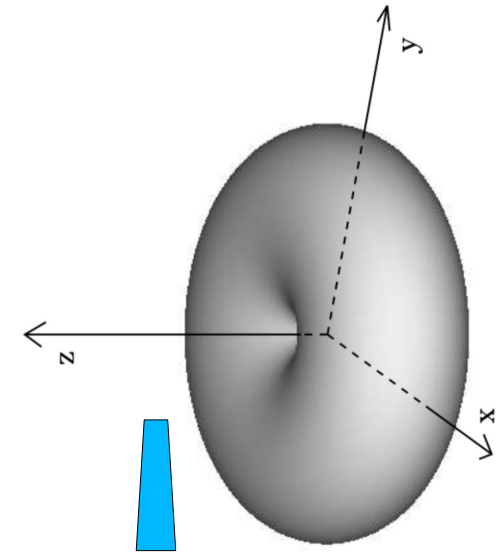
Front view: unpolarized radiation
makes the e^- 's oscillate in all directions



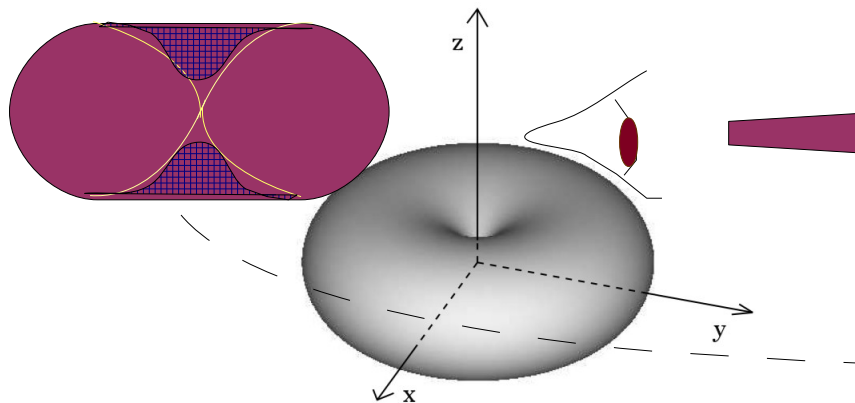


How polarization is created

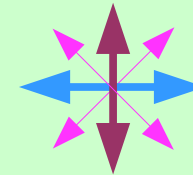
The orange observer receive partially polarized radiation



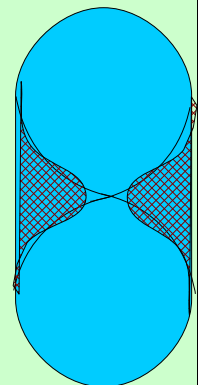
Emission pattern of the
bordaux double arrow



Front view: unpolarized radiation
makes the e-'s oscillate in all directions



Emission pattern of the
cyan double arrow





1. Scattering symmetric wrt the scattering angle (backward=forward)

2. The amount of scattered radiation depends solely on the incident radiation

$$-\frac{dW}{dt} = \sigma_T c U_{\text{rad}}$$

3. Even if the incident radiation is unpolarized, TS can polarize scattered radiation depending on the scattering angle

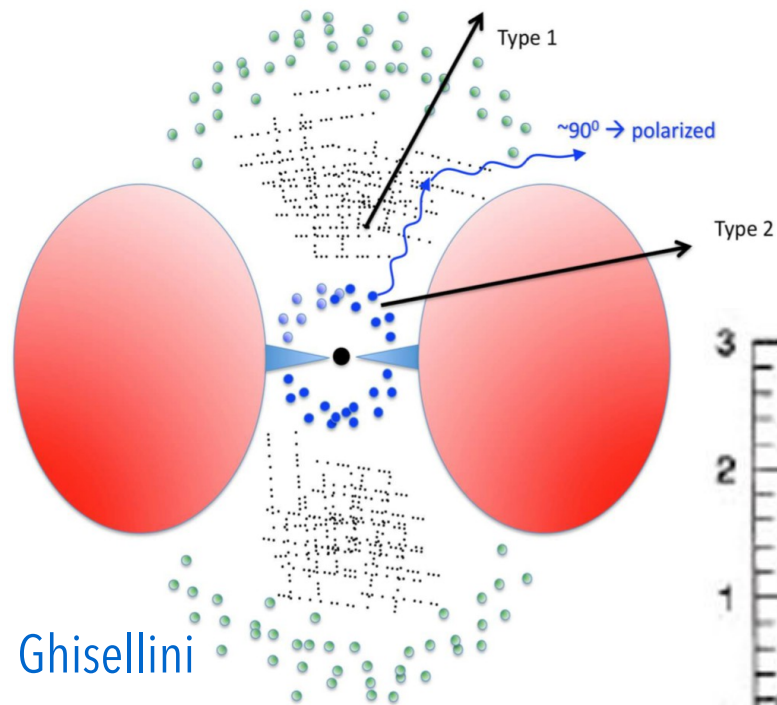
4. TS is one of the main processes capable of trapping photons in a given region.

Optical depth for TS in a medium is $\tau_T = \int \sigma_T n_e dl$

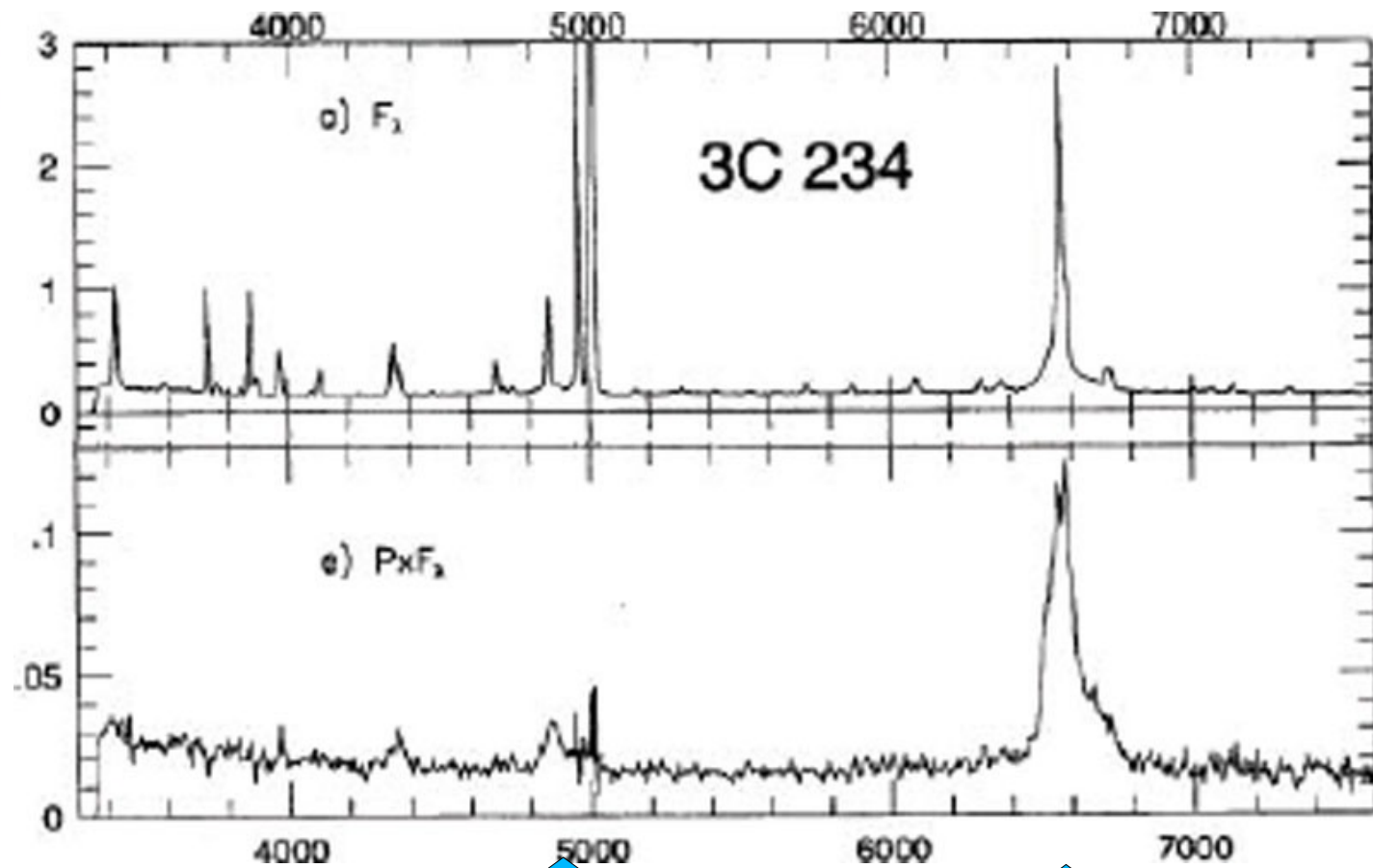
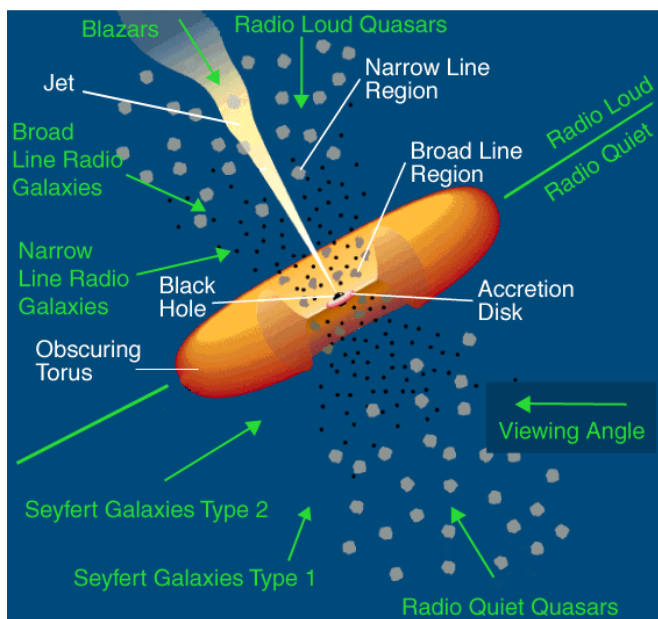
Photons are scattered in random directions, i.e. random walk in which each step is the

mean free path $\lambda_T = \frac{1}{\sigma_T n_e}$

Optical spectrum of a "type II" AGN in total intensity and in polarized light:



"broad line emission" is visible in polarized light
(when unpolarized emission is dropped)





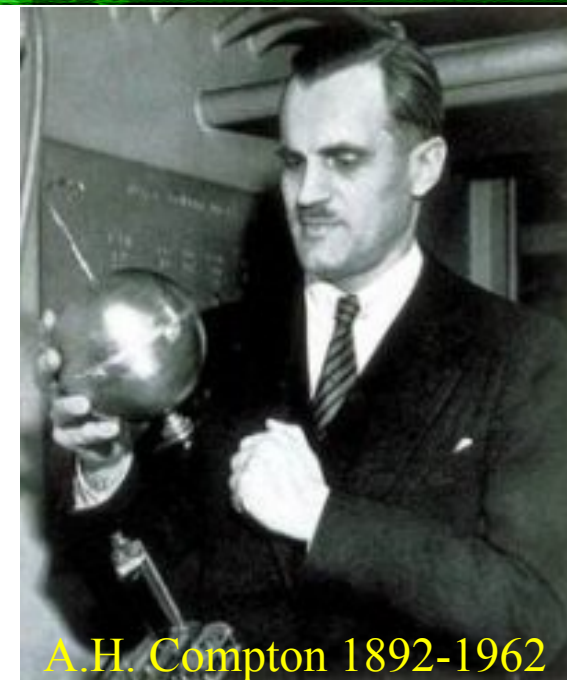
Compton scattering

"Compton scattering or the Compton effect, is the decrease in energy (increase in λ) of a high energy (X – or γ | ray) photon, as it interacts with a free electron.

More important for astrophysics, Inverse Compton scattering also exists, where the photon gains energy (decreasing in wavelength) upon interaction with a high energy electron.

The amount the wavelength increases by is called the Compton shift.

Although nuclear Compton scattering also exists, *what is meant by Compton scattering usually is the interaction involving free electrons only.*

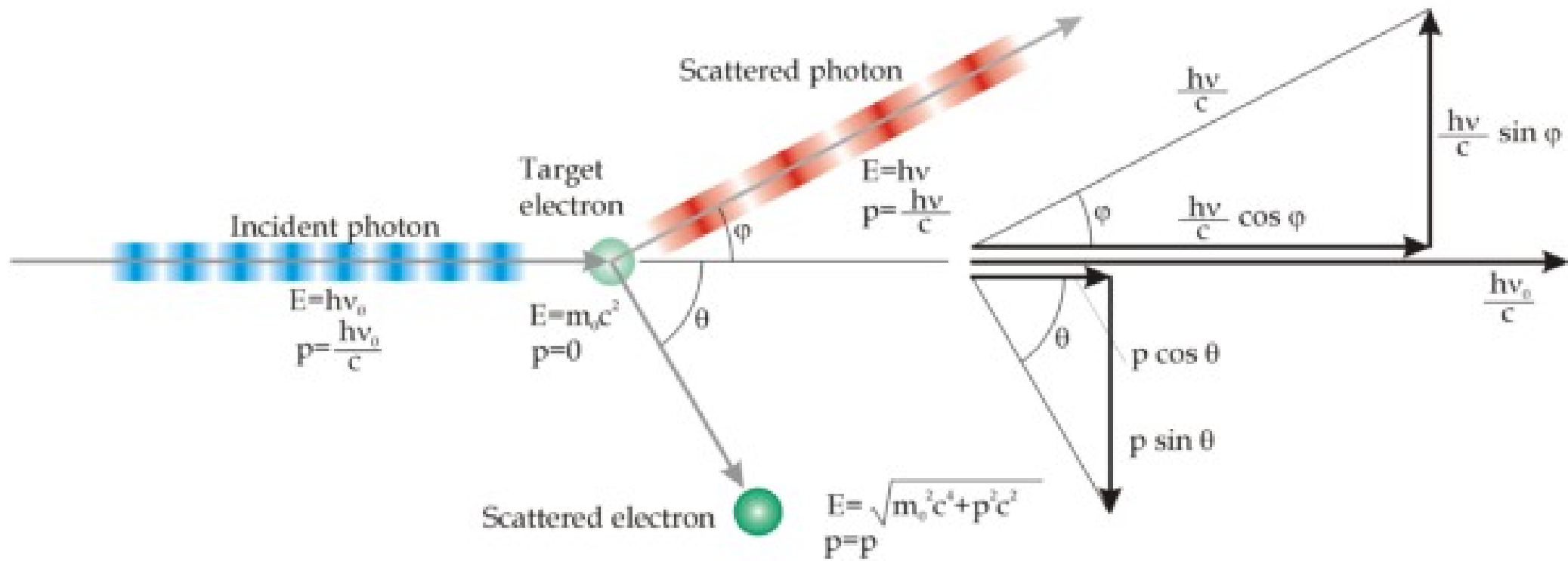


Compton effect was observed by Arthur Holly Compton in 1923, for which he was awarded the Nobel Prize in Physics in 1927..

The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon.

Thomson scattering, the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength.

Light must behave as if it consists of particles in order to explain the Compton scattering."





Energy conservation:

$$h\nu_i + m_e c^2 = h\nu_f + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

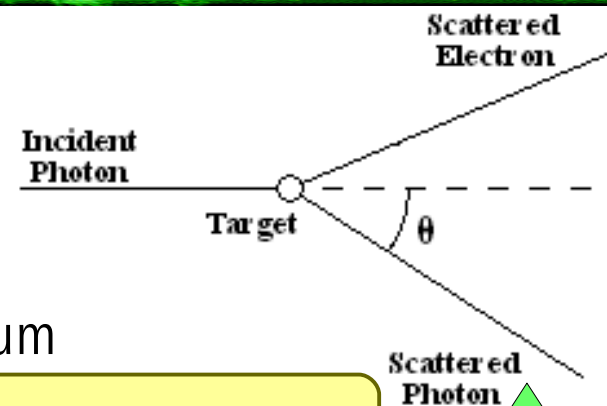
momentum conservation:

$$\vec{p}_i = \vec{p}_f + \vec{p}_e$$

$$\rightarrow p = E/c = h\nu/c \rightarrow \text{photon momentum}$$

$$\vec{p}_e = \vec{p}_i - \vec{p}_f$$

$$\rightarrow p_e^2 = (\vec{p}_i - \vec{p}_f) \cdot (\vec{p}_i - \vec{p}_f) = p_i^2 + p_f^2 - 2p_i p_f \cos \theta$$

FYI,
Computations
never asked

$$p_e^2 = h^2 \nu_i^2 + h^2 \nu_f^2 - 2h^2 \nu_i \nu_f \cos \theta \quad \leftarrow \text{multiply by } c^2. \text{ Then take the square of energy conservation}$$

$$p_e^2 c^2 + m_e^2 c^4 = h^2 \nu_i^2 + h^2 \nu_f^2 + m_e^2 c^4 + 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2 - 2h^2 \nu_i \nu_f$$

$$\text{Compare the two highlighting } p_e^2 c^2 \text{ we get } \rightarrow -2h^2 \nu_i \nu_f \cos \theta = 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2 - 2h^2 \nu_i \nu_f$$

$$2h^2 \nu_i \nu_f (1 - \cos \theta) = 2h m_e c^2 (\nu_i - \nu_f) \rightarrow \frac{1 - \cos \theta}{m_e c^2} = \frac{\nu_i - \nu_f}{h \nu_i \nu_f} = \frac{1}{h} \left(\frac{1}{\nu_f} - \frac{1}{\nu_i} \right)$$

$$\frac{1}{\nu_f} - \frac{1}{\nu_i} = \frac{\lambda_f}{c} - \frac{\lambda_i}{c} = \frac{h}{m_e c^2} (1 - \cos \theta)$$

$$\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \theta) = \lambda_0 (1 - \cos \theta)$$

Compton wavelength for electrons

$$\lambda_0 = \frac{h}{m_e c} = 0.02426 \text{ \AA}$$

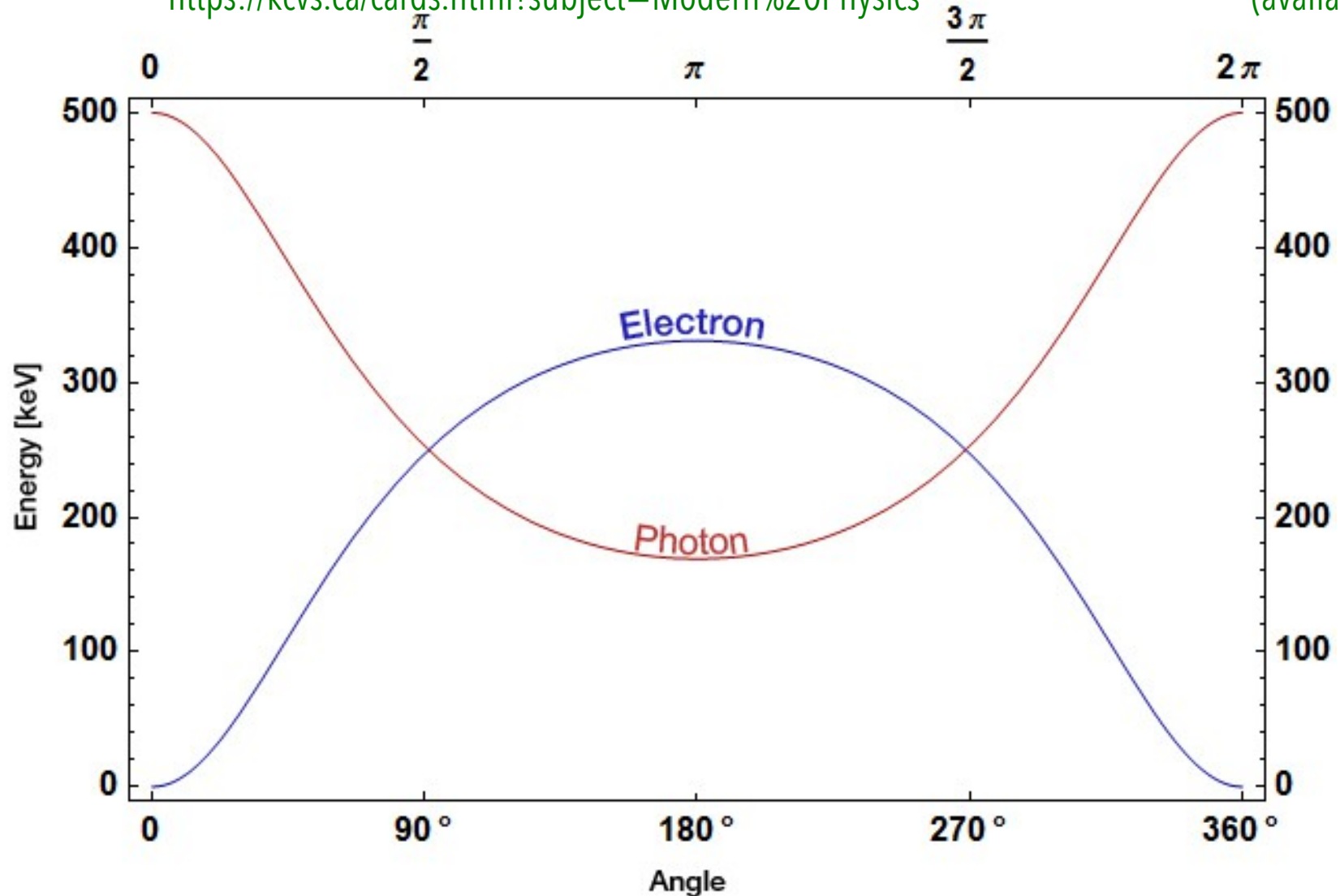


Compton scattering

a Java applet showing the geometry of the scattering can be found at

http://www.student.nada.kth.se/~f93-jhu/phys_sim/compton/Compton.htm (currently unavailable)

<https://kcvs.ca/cards.html?subject=Modern%20Physics> (available?)





Scattered photon energy

$$h\nu_e = \frac{h\nu_i}{1 + h\nu_i(1 - \cos\theta)/m_e c^2}$$

$$\lambda_e - \lambda_i = \frac{h}{m_e c} \cdot (1 - \cos\theta) = \lambda_c (1 - \cos\theta) = 0.02426 \cdot (1 - \cos\theta) \text{ [Å]}$$

Final energy of the scattered photon is a function of:

- initial energy of the photon
- scattering angle
- the photon always loses energy, unless $\theta = 0$
- the scattering is closely elastic when $\lambda \approx \lambda_c$ (i.e. $h\nu \approx m_e c^2$)

When the photons involved in the collision have large energies, the scattering becomes less efficient and quantum electrodynamics effects reduce the cross section.

The Thomson cross section becomes the **Klein-Nishina cross section**



In case $h\nu \sim m_e c^2$, the probability of interaction decreases, also as a function of the scattering angle; additional constraints come from relativistic quantum mechanics.

If we define $x \equiv \frac{h\nu}{m_e c^2}$

FYI,
Computations
never asked

$$\sigma = \sigma_T \frac{3}{4} \left[\frac{1+x}{x^3} \left(\frac{2x(1+x)}{1+2x} - \ln(1+2x) \right) + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right]$$

known as the Klein - Nishina cross section:

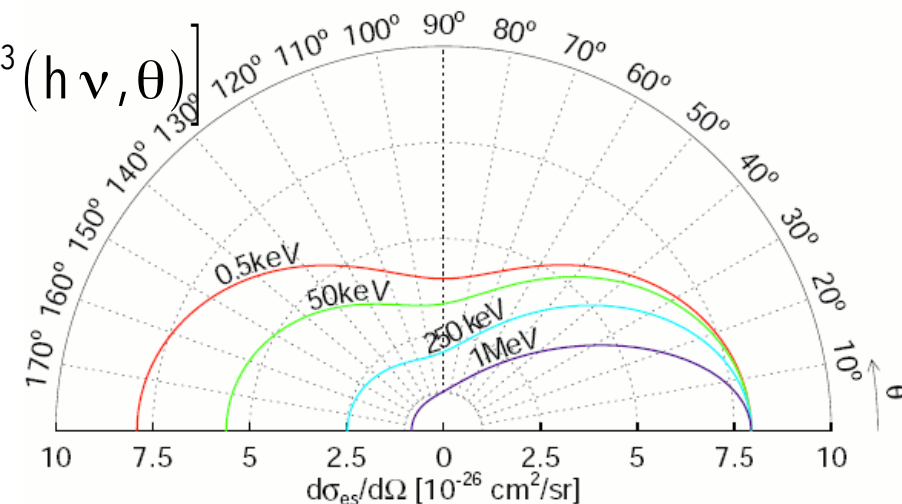
$$\text{if } x \gg 1 \rightarrow \sigma = \sigma_T \frac{3}{8} \frac{1}{x} \left(\ln 2x + \frac{1}{2} \right)$$

It originates from a differential cross section based on the scattering angle

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left[P(h\nu, \theta) - P^2(h\nu, \theta) \sin^2 \theta + P^3(h\nu, \theta) \right]$$

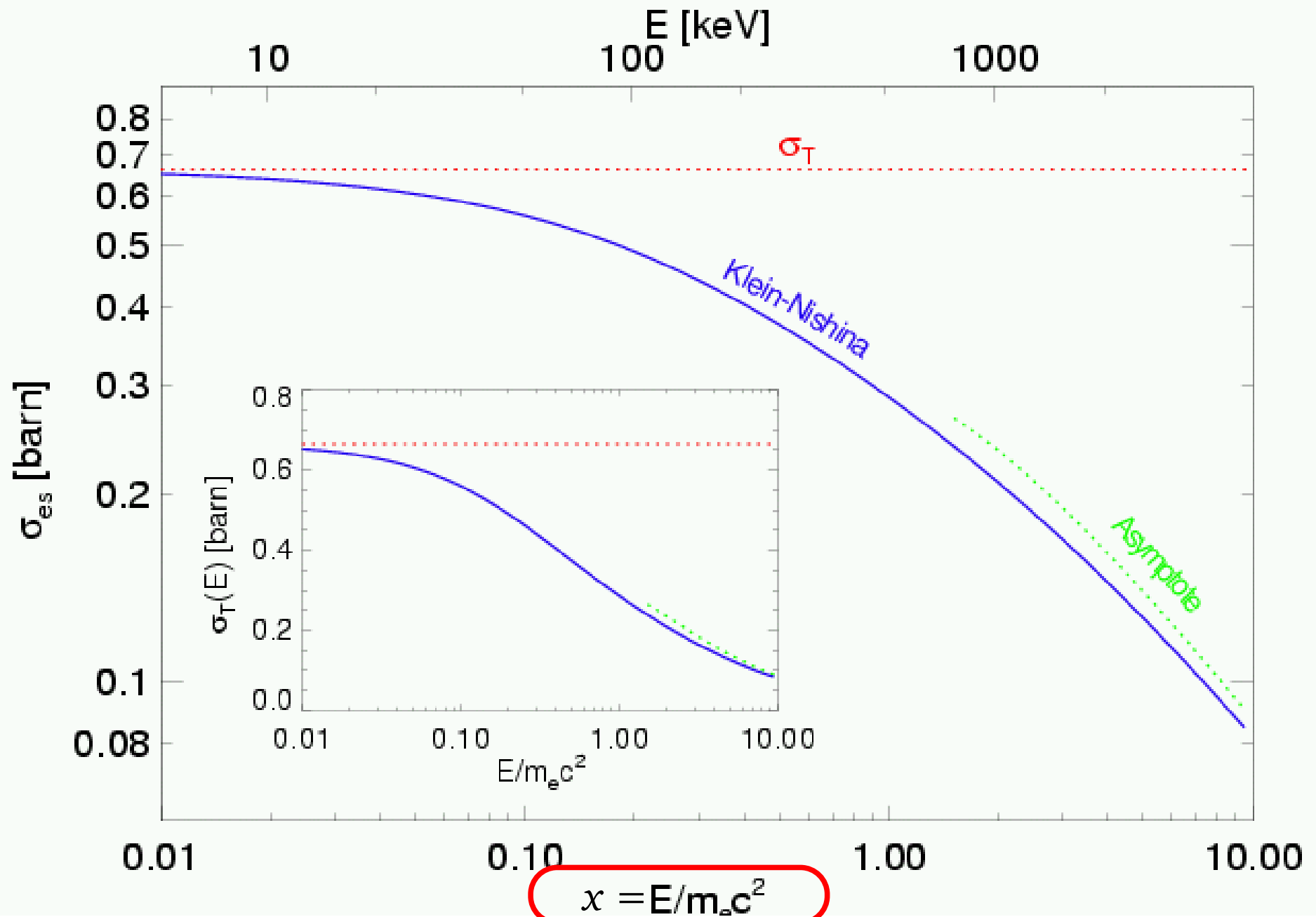
where the probability is defined as

$$P(h\nu, \theta) = \frac{1}{1 + \frac{h\nu}{m_e c^2} (1 - \cos \theta)}$$



and the Klein - Nishina cross section becomes the Thomson cross section at low photon energies

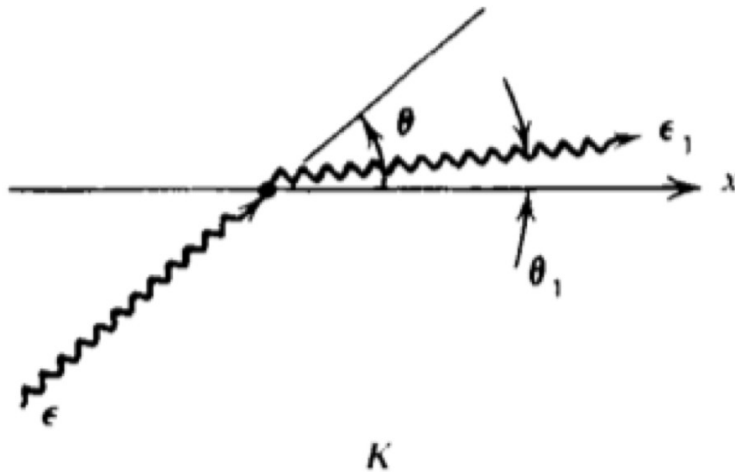
The **Klein - Nishina** cross section is relevant at high photon energies (Hard X-Rays and beyond)



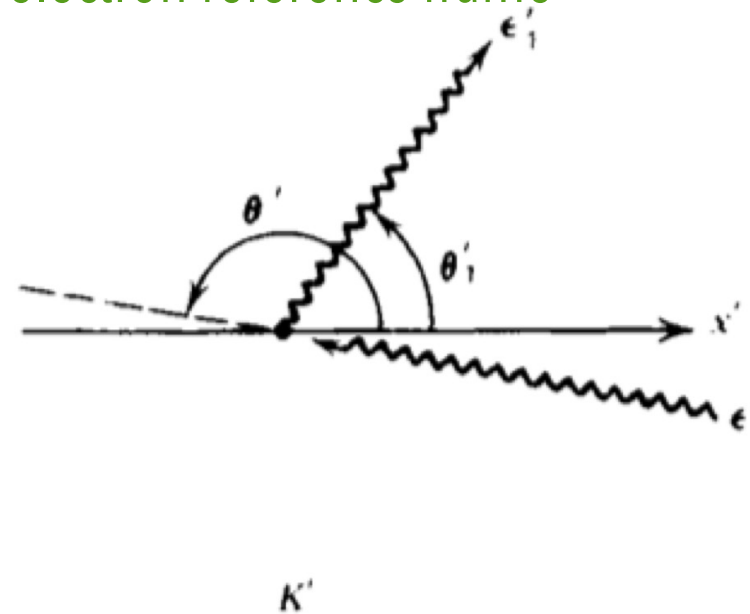
Moving, energetic (relativistic) electrons “cool down” by increasing the energy of low energy seed photons.

Remember that relativistic electrons come along their $1/\gamma$ beaming cone

Lab reference frame



electron reference frame



Moving, energetic (relativistic) electrons “cool down” by increasing the energy of low energy seed photons. To make computations simpler, it is possible to consider the centre of momentum frame where the photon energy is much less than $\sim m_e c^2$ and then $\sim \sigma_T$ can be used.

⇒ In the electron rest frame we have Thomson scattering

1. Lab system ⇒ electron rest frame:

$$h\nu_e = h\nu_L \gamma (1 - \beta \cos \theta)$$

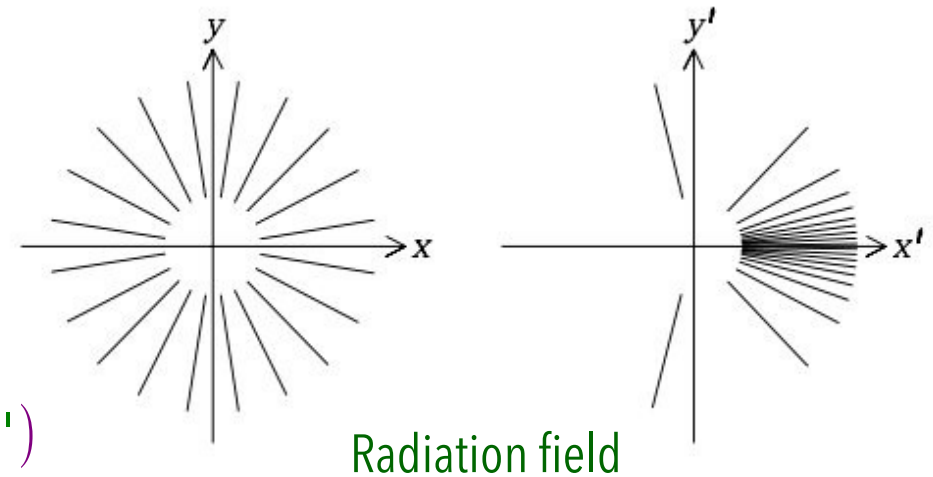
2. Thomson scattering in electron rest frame

$$h\nu_e' = h\nu_e$$

3. Electron rest frame ⇒ Lab system

$$h\nu_L' = h\nu_e' \gamma (1 + \beta \cos \theta_1')$$

$$h\nu_L' \approx \gamma^2 h\nu_L$$



i.e. ⇒ Very efficient energy transfer from electron to photon!

Angle measured in the RF of the electron



Inverse Compton scattering

The scattering angle is important for the energy of the outgoing photon:

1 — maximum energy gained by the photon if $\theta = \pi$ and $\theta_1' = 0$

☐ the photon is blue – shifted (face on collision)

2 – minimum energy for the scattered photon if $\theta = 0$ and $\theta_1' = \pi$

☐ the photon is red – shifted (end on collision)

The maximum energy that can be gained is (averaged over all angles):

$$\epsilon' \approx \frac{4}{3} \gamma^2 \epsilon$$

or, in terms of frequency

$$\nu' \approx \frac{4}{3} \gamma^2 \nu$$

In case it is possible to measure both the initial and final photon energy/frequency, then γ (or γ^2) can be deduced

$$\gamma \approx \sqrt{\frac{3}{4} \frac{\nu'}{\nu}}$$

It is a Lorentz invariant, i.e. the same in the laboratory and electron rest frame

$$E' = h\nu' = h\nu\gamma(1 + \beta\cos\theta) = E\gamma(1 + \beta\cos\theta)$$

Let's consider a region where there is a plasma of relativistic electrons and a radiation field:
in the electron framework, the photon energy is

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon'}{dt'} = c\sigma_T \frac{\langle E_{\text{rad}}'^2 \rangle}{8\pi} = c\sigma_T \gamma^2 \langle (1 + \beta\cos\theta)^2 \rangle \langle E_{\text{rad}}^2 \rangle$$

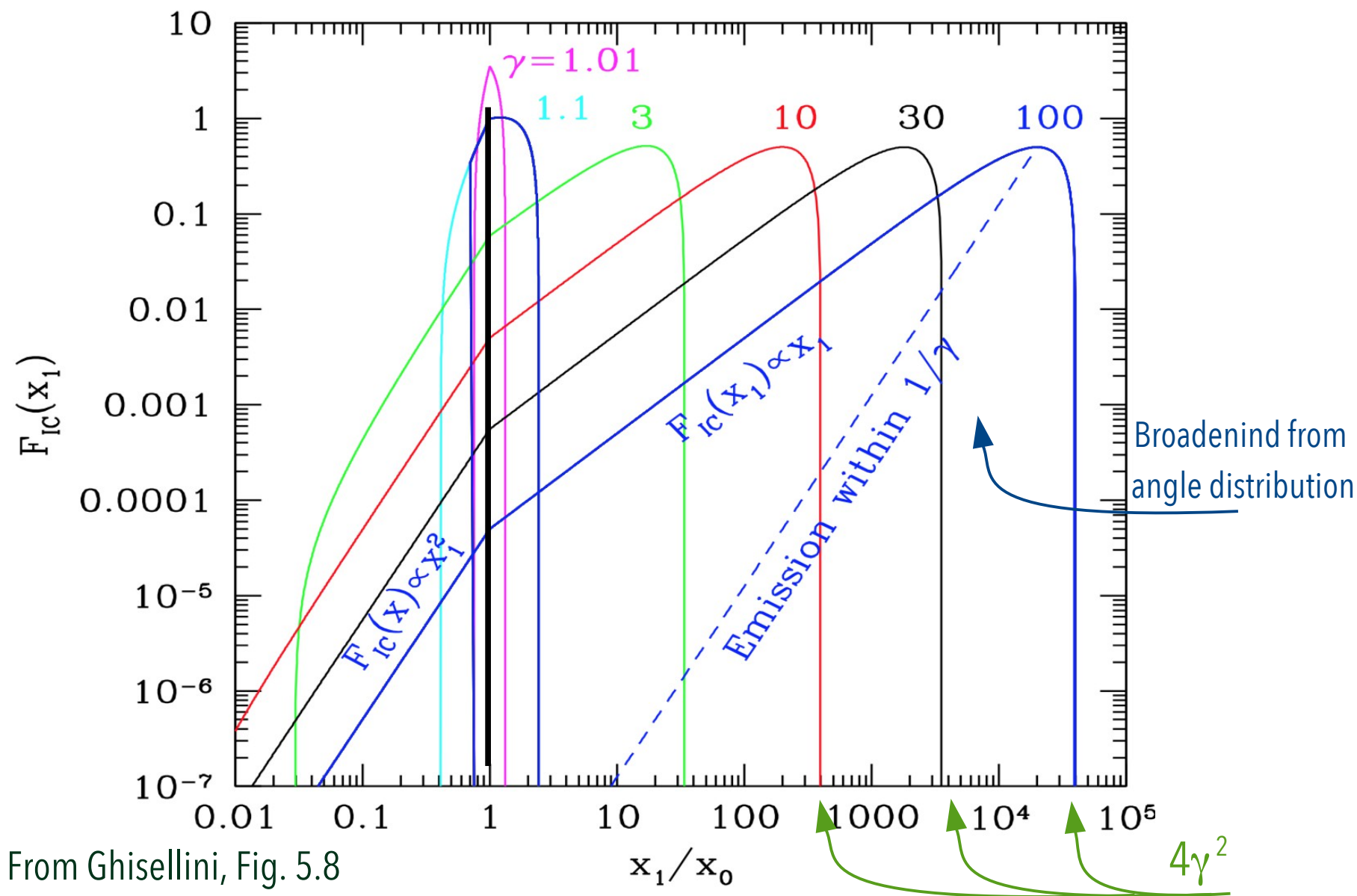
since the emitted energy is a Lorentz invariant. It is then possible to write

$$\langle E_{\text{rad}}^2 \rangle = U_{\text{ph}} \quad \langle (1 + \beta\cos\theta)^2 \rangle = 1 + \frac{1}{3}\beta^2$$

Finally, the energy coming out the scattering region is

$$\frac{d\varepsilon_{\text{out}}}{dt} = c\sigma_T U_{\text{ph}} \gamma^2 \left(1 + \frac{1}{3}\beta^2 \right)$$

Example: single energy ($x_0 = h\nu_0$) photons scattered to energies x_1 by electrons with different γ and distribution of scattering angles (N.B. All spectra are normalized, just for comparison of shift in energy)



From Ghisellini, Fig. 5.8



Inverse Compton emission .vs. Synchrotron emission

The energy associated with the seed photons, prior of the scattering is

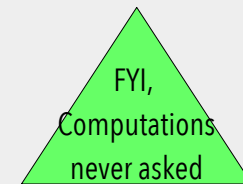
$$\frac{d\varepsilon_{\text{in}}}{dt} = c \sigma_T U_{\text{ph}}$$

Then the net effect of the Inverse Compton scattering is

$$\frac{d\varepsilon_{\text{out}}}{dt} - \frac{d\varepsilon_{\text{in}}}{dt} = \left(\frac{d\varepsilon}{dt} \right)_{\text{ic}} = c \sigma_T U_{\text{ph}} \left[\gamma^2 \left(1 + \frac{1}{3} \beta^2 \right) - 1 \right]$$

$$\gamma^2 \left(1 + \frac{1}{3} \beta^2 \right) - 1 = \gamma^2 + \frac{1}{3} \gamma^2 \beta^2 - 1 = \gamma^2 - 1 + \frac{1}{3} \gamma^2 \beta^2 \rightarrow$$

but $\gamma^2 - 1 = \gamma^2 \beta^2$ then $\rightarrow = \gamma^2 \beta^2 + \frac{1}{3} \gamma^2 \beta^2 = \frac{4}{3} \gamma^2 \beta^2$



Finally we get the amount of power provided by the relativistic electrons to the seed photons

$$\left(\frac{d\varepsilon}{dt} \right)_{\text{ic}} = \frac{4}{3} c \sigma_T \gamma^2 \beta^2 U_{\text{ph}} \quad (**)$$



Let's recall the synchrotron emission $\left(\frac{d\varepsilon}{dt} \right)_{\text{syn}} = \frac{4}{3} c \sigma_T \gamma^2 \beta^2 U_H$

The same relativistic electrons radiate via synchrotron and inverse Compton

Let's compare the two losses:

$$\frac{\left(\frac{d\varepsilon}{dt} \right)_{\text{syn}}}{\left(\frac{d\varepsilon}{dt} \right)_{\text{ic}}} = \frac{U_H}{U_{\text{ph}}}$$

Their contributions add up:

$$-\left(\frac{d\varepsilon}{dt} \right)_{\text{sy+ic}} = C_s \varepsilon^2 \left[H^2 + 8\pi U_{\text{rad}} \right]$$

The increased cooling rate implies that the **electron radiative lifetime is consequently reduced**

$$t_{\text{sy+ic}}^* \approx \frac{1}{\varepsilon^*} \frac{1}{H^2 + 8\pi U_{\text{rad}}} \approx \frac{3 \cdot 10^8}{(H^2 / 8\pi + U_{\text{rad}}) [\text{eV cm}^{-3}]} \frac{1}{\varepsilon^* [\text{GeV}]} \text{ yr}$$

Therefore.... **hard times for relativistic electrons!**

(Wait a little while for an example: radiative lifetime for a relativistic electrons in a radio lobe.)

$H_{\text{eq}}, U_{\text{rad}}$, have strong dependence on z !, i.e. **when**

They have strong dependence on the location in the Universe, i.e. **where**

How to evaluate U_{rad} ?

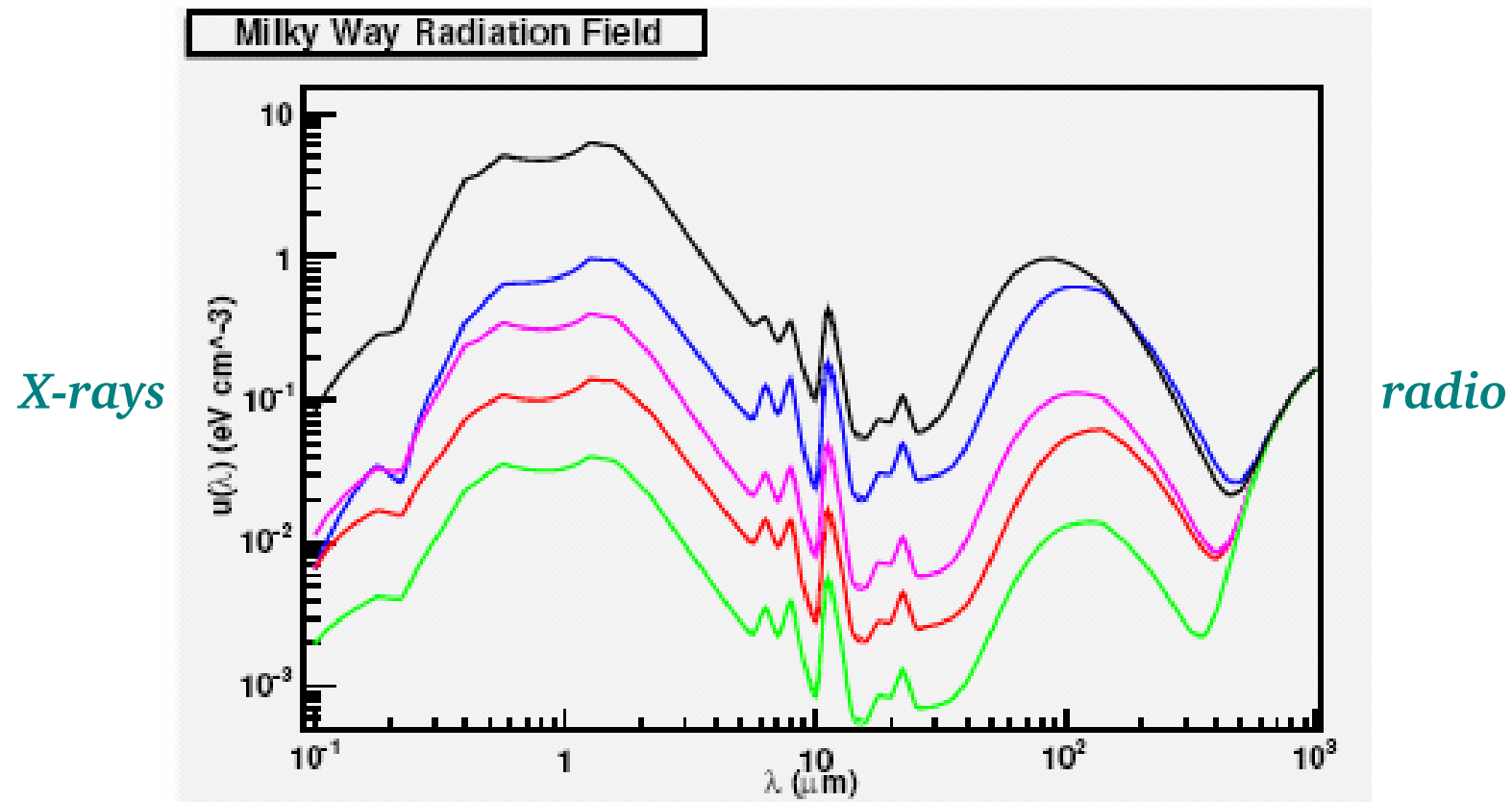
CMB is a radiation which can be found in every corner of the Universe

$$H_{\text{CMB}} \sim 3.28(1+z)^2 \mu\text{G}$$

H_{CMB} is the equivalent to the H field strength necessary to have the same losses for Synchrotron and IC with CMB photons

The "Local Radiation Field"

Evaluation of U_{rad} : as taken in a number of selected locations in/around the Milky Way.



Spatial variation of the total radiation field throughout the Galaxy.

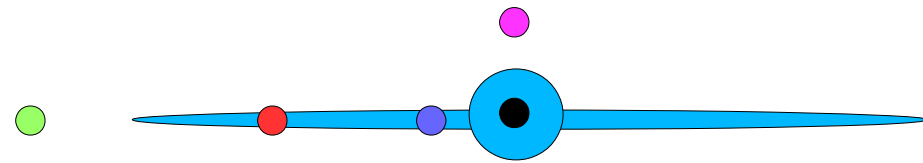
Black line : total radiation field for GC.

Magenta line : total radiation field for $R=0$, $z=5$ kpc,

Blue line : total radiation field for $R=4$, $z=0$ kpc,

Red line : total radiation field for $R=12$, $z=0$ kpc,

Green line : total radiation field for $R=20$, $z=0$ kpc.



This diagram will be discussed a bit further when considering the ISM

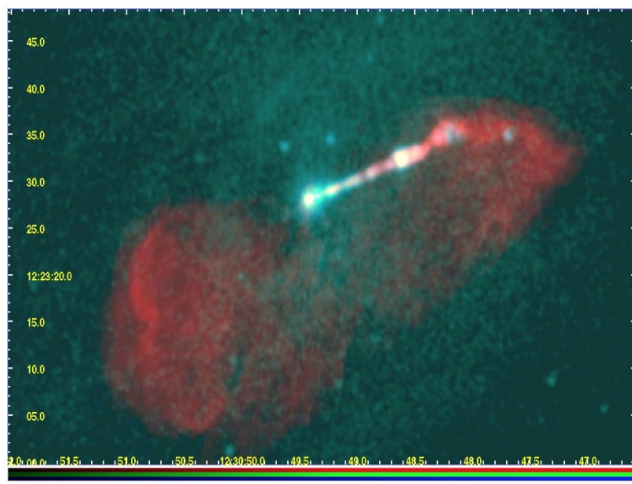
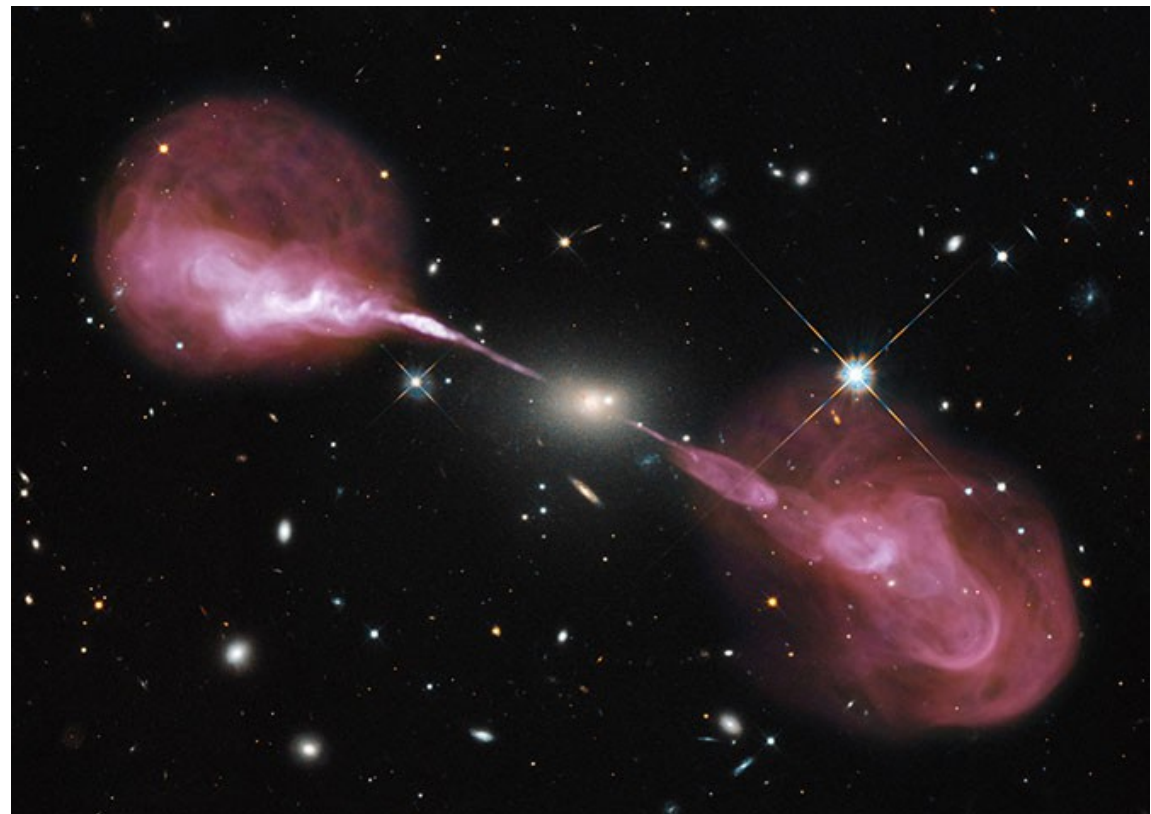
If U_{rad} can be determined, then the magnetic field can be measured!

If $U_{\text{rad}} = U_{\text{CMB}}$ as taken in a number of selected locations given the typical γ of the electrons, in general, the ratio $L_{\text{radio}}/L_{\text{X}}$ is a measure of H^2/U_{CMB}^2 in the radio emitting region.

$$\frac{L_{\text{radio}}}{L_{\text{X}}} \approx \frac{U_{\text{H}}}{U_{\text{rad}}}$$

N.B. The energy density of the synchrotron radiation produced by the magnetized plasma is generally much lower than U_{CMB} .

The lobes of (giant) radio sources are suitable sites for this kind of studies



Let's consider a spherical cloud with radius R filled with magnetized relativistic plasma located at a distance D from the observer. The low-energy synchrotron photons may be up-scattered by the relativistic electrons via IC:

$$L_s = 4\pi D^2 \int S(\nu) d\nu \approx 4\pi D^2 S_{\max} \nu_c f(\alpha) \sim U_{\text{rad}} \frac{4\pi R^2 c}{3}$$

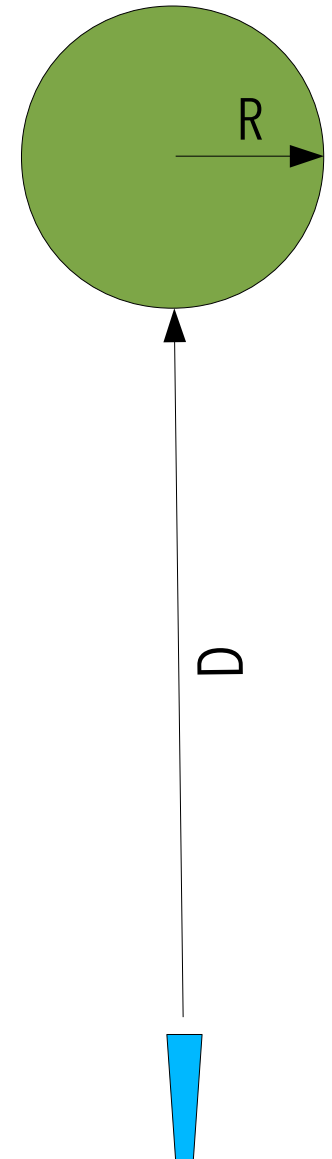
the energy density of the radiation field can then be derived

$$U_{\text{rad}} \approx \frac{3L_s}{4\pi R^2 c}$$

FYI,
Computations
never asked

finally the comparison between the IC and Synchrotron losses gives:

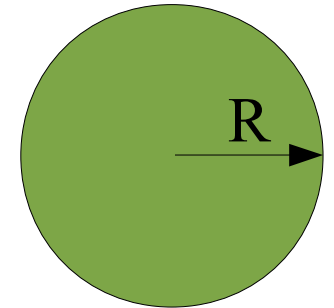
$$\frac{L_{\text{IC}}}{L_s} = \frac{U_{\text{rad}}}{H^2/8\pi} = \frac{6L_s}{R^2 H^2 c} \approx \frac{S_{\max} \nu_c}{\theta^2 H^2 c} f(\alpha)$$



In an optically thick medium we can use the R-J approximation of the Planck's function

$$B(\nu, T) \sim \frac{2kT_B}{\lambda^2} \quad \text{therefore we end up with}$$

$$\frac{L_{IC}}{L_s} = \left(\frac{T_{Bmax}}{10^{12} \text{ K}} \right)^5 \left(\frac{\nu_c}{\text{GHz}} \right) f(\alpha)$$



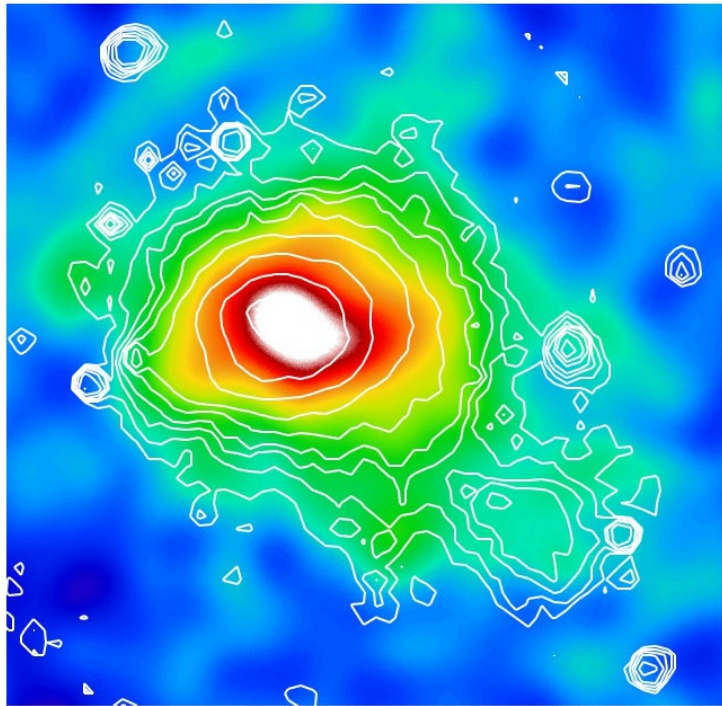
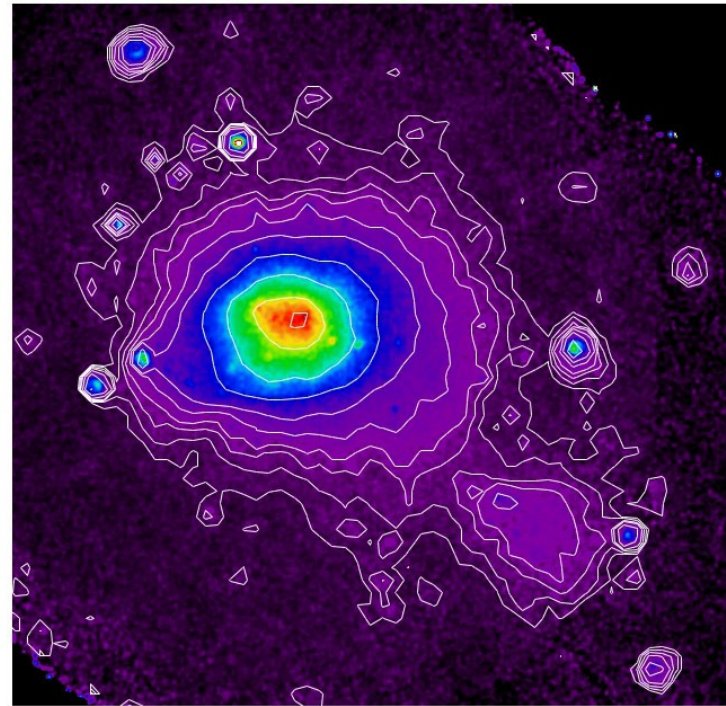
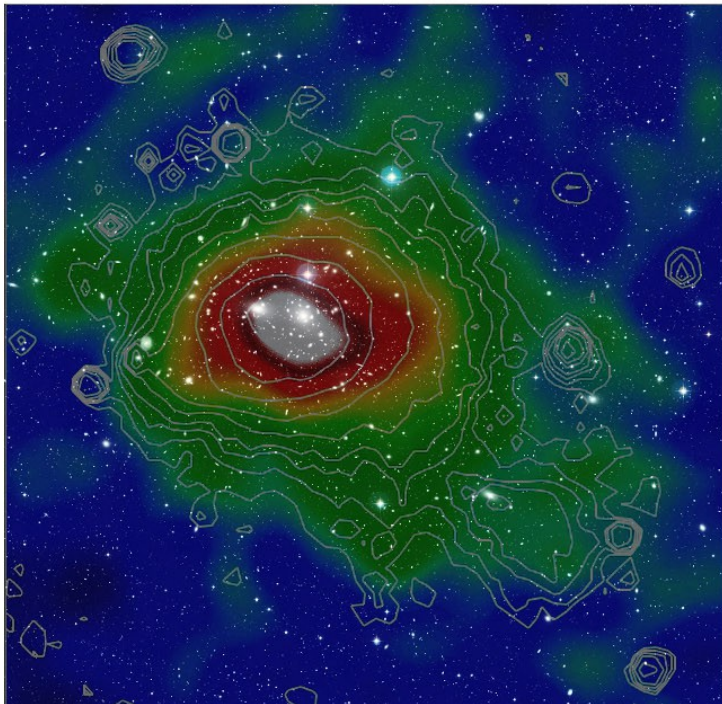
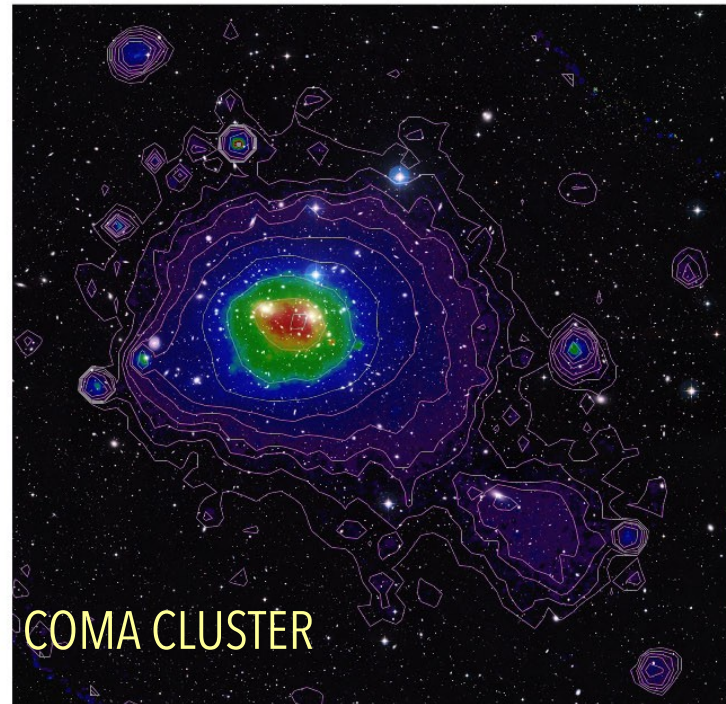
For brightness temperatures above 10^{12} K , the radiation field has undergone to a dramatic amplification; IC would become the most efficient and dominant cooling process (X-Rays) and a source would radiate its energy in a very short time. [Compton catastrophe]

Brightness temperature limit in radio source is $T_B \approx 10^{12} \text{ K}$

The SSC Catastrophe happens in bright and compact radio sources where relativistic beaming plays a role and enhances the energy density of the radiation field. (e.g. in Blazar sources)



A site with very hot (thermal) plasma (and a lot of seed photons!)



COMA CLUSTER



Comptonization - Multiple scattering

Compton & Compton⁻¹ are the two limiting cases where there is energy transfer photon $\leftrightarrow$ electron
There are astrophysical cases in which the two processes are mixed together

We want to evaluate if there is a substantial change in the spectrum of the seed photons
in case of multiple scattering with hot (but non-relativistic) electrons in thermal equilibrium.

Let's consider a large volume (uniformly) full of thermal plasma along with a sea of seed photons

⇒ Let's consider the process from the point of view of the photons

There are interactions where energy is transferred from photons to electrons (C scattering)

$$\Delta h\nu = h(\nu_i - \nu_f) = h\left(\frac{c}{\lambda_i} - \frac{c}{\lambda_f}\right) = hc\left(\frac{\lambda_f - \lambda_i}{\lambda_f \lambda_i}\right) = \frac{hc}{\lambda_f \lambda_i} \frac{h}{m_e c} (1 - \cos\theta) \approx \frac{h^2}{m_e \lambda_f \lambda_i} = \frac{h^2 \nu_f \nu_i}{m_e c^2}$$

$$\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{\text{phot}} \approx -\frac{h\nu}{m_e c^2}$$



Comptonization - Multiple scattering (2)

There are interactions where **energy is transferred from electrons to photons (IC scattering)** (positive since we are considering the process from the point of view of the photons)

From (**), for a non relativistic distribution ($\gamma \simeq 1$), also considering that $\frac{1}{2} m_e \langle v^2 \rangle = \frac{3}{2} kT \rightarrow v^2 \approx \frac{3kT}{m_e}$

$$\left\langle \frac{\Delta \epsilon}{\epsilon} \right\rangle_{\text{el}} \approx \frac{4}{3} \left(\frac{v}{c} \right)^2 = \frac{4}{3c^2} \frac{3kT}{m_e} = \frac{4kT}{m_e c^2}$$

The net energy transfer is:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{(4kT - h\nu)}{m_e c^2}$$

There is not a net energy transfer when $4kT = h\nu$

If $4kT \ll h\nu$ photons loose energy and electrons (gas) heats up

$4kT \gg h\nu$ photons gain energy and electrons (gas) cools down



Comptonization: -Ex. 1-

Let's consider a gas (plasma) region, D in diameter, where $4kT \gg h\nu$

$$\left\langle \frac{\Delta \epsilon}{\epsilon} \right\rangle \simeq \frac{4kT}{m_e c^2}$$

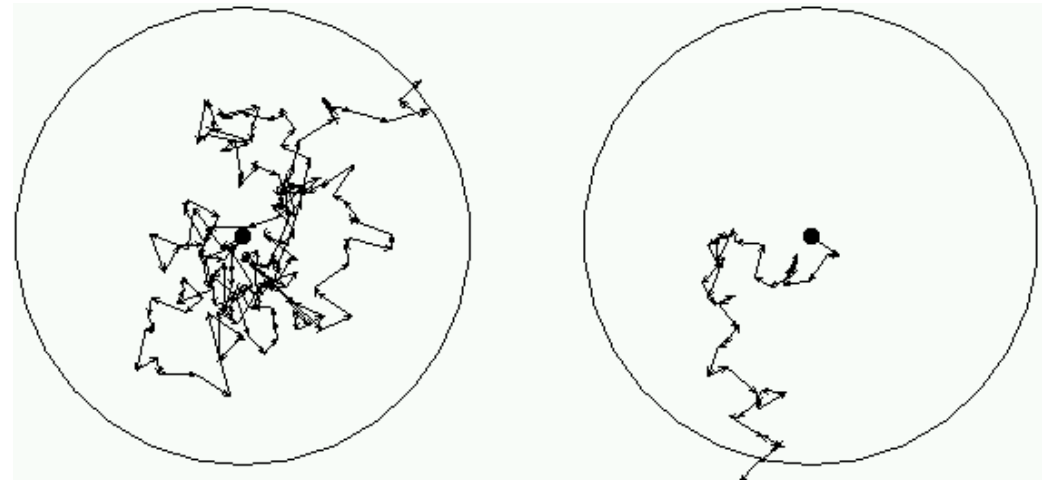
FYI,
Computations
never asked

The opacity to the scattering is $\tau_e = n_e \sigma_T D = \frac{D}{l_{\text{mfp}}}$

where the mean free path is $l_{\text{mfp}} = \frac{1}{n_e \sigma_T}$

The total number of collisions is: $N = \left(\frac{D}{l_{\text{mfp}}} \right)^2 = \tau_e^2$

The total energy gain is $\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{\text{tot}} \approx \frac{4kT}{m_e c^2} N = \frac{4kT}{m_e c^2} N dt$





Comptonization: -Ex. 2-

The total energy gain is $\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{\text{tot}} \approx \frac{4kT}{m_e c^2} N = \frac{4kT}{m_e c^2} N dt$

FYI,
Computations
never asked

After integration (in $d\nu, dt$) the mean photon energy as a function of time becomes

$$h\nu(t) = h\nu_0 e^{(4kT/m_e c^2)Nt} = h\nu_0 e^{(4kT/m_e c^2)N} = h\nu_0 e^{4y}$$

where y is known as "Compton y -parameter"

$$y = \frac{kT}{m_e c^2} N = \frac{kT n_e^2 \sigma_T^2 D^2}{m_e c^2}$$

when $\frac{h\nu_0}{4kT} e^{4y}$ is small $\rightarrow$ the energy of the photon increases exponentially

$\Rightarrow$ When Comptonisation is strong $\frac{h\nu_0}{4kT} e^{4y} \gg 1$ the energy of the scattered photons

saturate at $h\nu = 4kT$



Comptonization: -Ex. 3-

The spectrum of the radiation is substantially modified when $\nu_f = \nu(t) > \nu_o$

$$y \geq \frac{1}{4} \quad \text{i.e.} \quad \frac{4kT}{m_e c^2} N = \frac{4kT n_e^2 \sigma_T^2 D^2}{m_e c^2} > 1$$

The final equilibrium is achieved when

$$h \nu_f = 4kT \quad \text{i.e.} \quad y = \frac{1}{4} \ln \left(\frac{4kT}{h \nu_o} \right)$$

namely

$$\tau_e = \left[\ln \left(\frac{4kT}{h \nu_o} \right) \frac{m_e c^2}{4kT} \right]^{1/2}$$

If this condition is met (thermal equilibrium), the modified photon spectrum must follow the Bose-Einstein distribution

$$u(\nu) d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{(h\nu/kT) + \mu} - 1} d\nu$$

FYI,
Computations
never asked



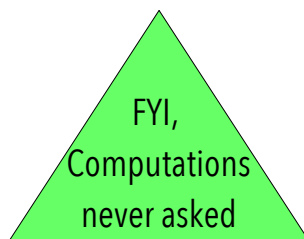
Comptonization -5- Modified Spectrum

At equilibrium, the photons will follow a Bose - Einstein distribution:

$$u(\nu) d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{(h\nu/kT)+\mu} - 1} d\nu$$

where μ is known as chemical potential (which is =0 for Planck's BB spectrum) and measures the rate at which photons are produced

in case $\frac{h\nu}{kT} + \mu \gg 1$



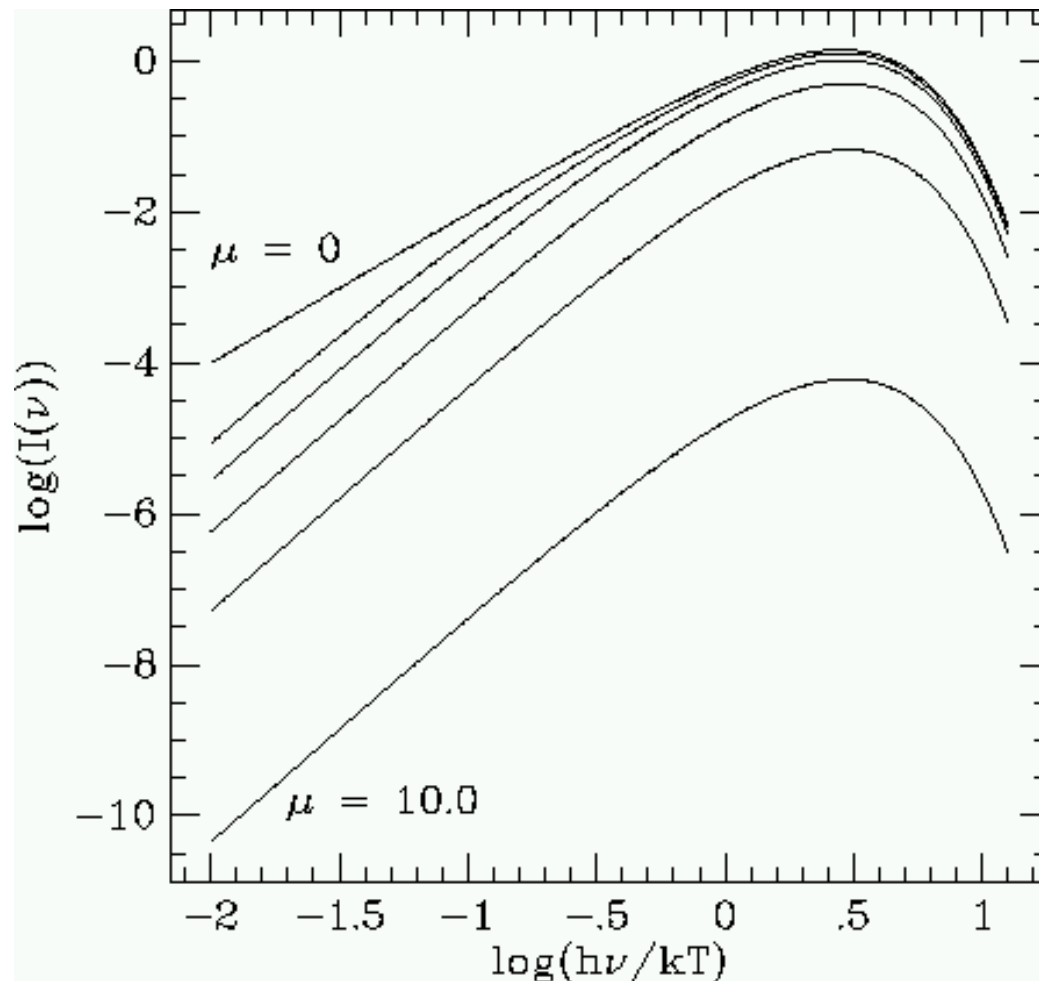
we have

$$u(\nu) d\nu = \frac{8\pi h \nu^3}{c^3} e^{-h\nu/kT} e^{-\mu}$$

i.e. the Wien's law modified by $e^{-\mu}$

The probability that an atom will emit a photon into a particular final state is increased by a factor of $(n+1)$ if there are already n photons in that state.

Photons are spin-1 bosons (Feynman+1965)





Kompaneets' equation

The exact description of the photon scattering (i.e. find the observed spectrum originated by various values of the y-parameter) is better obtained in the phase-space through the Kompaneets' equation:

$$\text{if } n = \frac{u(\nu)c^3}{8\pi h \nu^3} \quad \text{and} \quad x = \frac{h\nu}{kT}$$

FYI,
Computations
never asked

$$\frac{\partial n}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \underbrace{\left(n \right)}_{\text{Recoil effect}} + \underbrace{n^2}_{\text{Induced/stimulated emission}} + \underbrace{\frac{\partial n}{\partial x}}_{\text{Doppler motion}} \right]$$

Emission spectra come out as power-law with index $3+m$ where

$$m = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{1}{y}}$$

Examples: Accretion disks in XRBs (μQ), AGN,...S-Z effect

Sunyaev-Zeldovich effect



Rashid Sunyaev



Yakov Zeldovich
1914 – 1987

- **thermal**: high energy electrons interact with CMB photons via IC scattering
- **kinematic**: bulk motions modify the spectrum, second order effect, also known as Ostriker-Vishniac effect

Where does it take place?

For more information on the SZ effect: [Carlstrom + 2002 ARAA, vol 40, p.643-680](#)

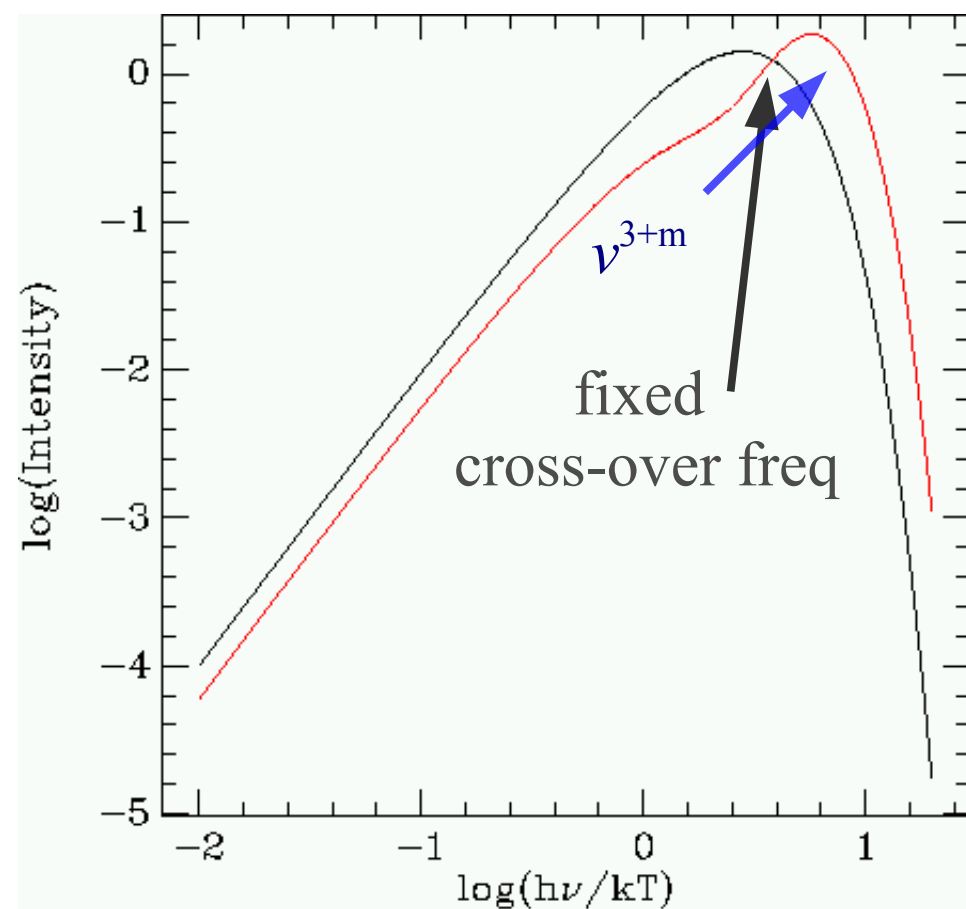
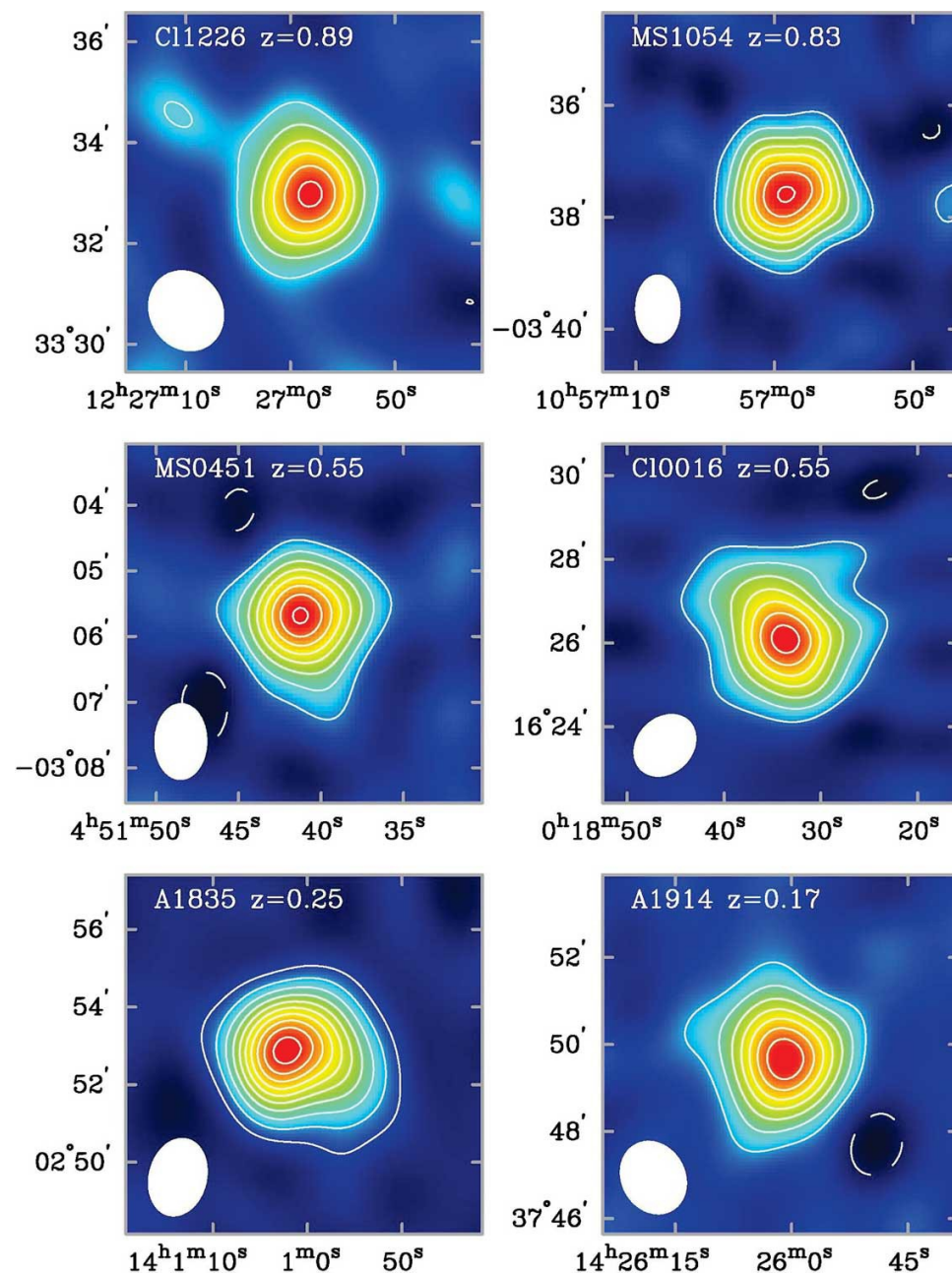
Sunyaev-Zeldovich effect (1)

Galaxy clusters have hot electrons capable of IC scattering of CMB photons. A CMB photon have a 1% probability of interacting with a high energy ICM electron.

$$\tau_{SZ} = n_e \sigma_T D$$

$$n_e \sim 10^{-3} \text{ cm}^{-3} \quad \sigma_T = 6.65 \cdot 10^{-25} \text{ cm}^2$$

$$D \sim 5 \text{ Mpc} \approx 4 \cdot 10^6 \cdot 3.08 \cdot 10^{18} \text{ cm} \sim 1.5 \cdot 10^{25} \text{ cm}$$





Sunyaev-Zeldovich effect (2)

From Carlstrom + 2002 ARAA, vol 40, p.643-680

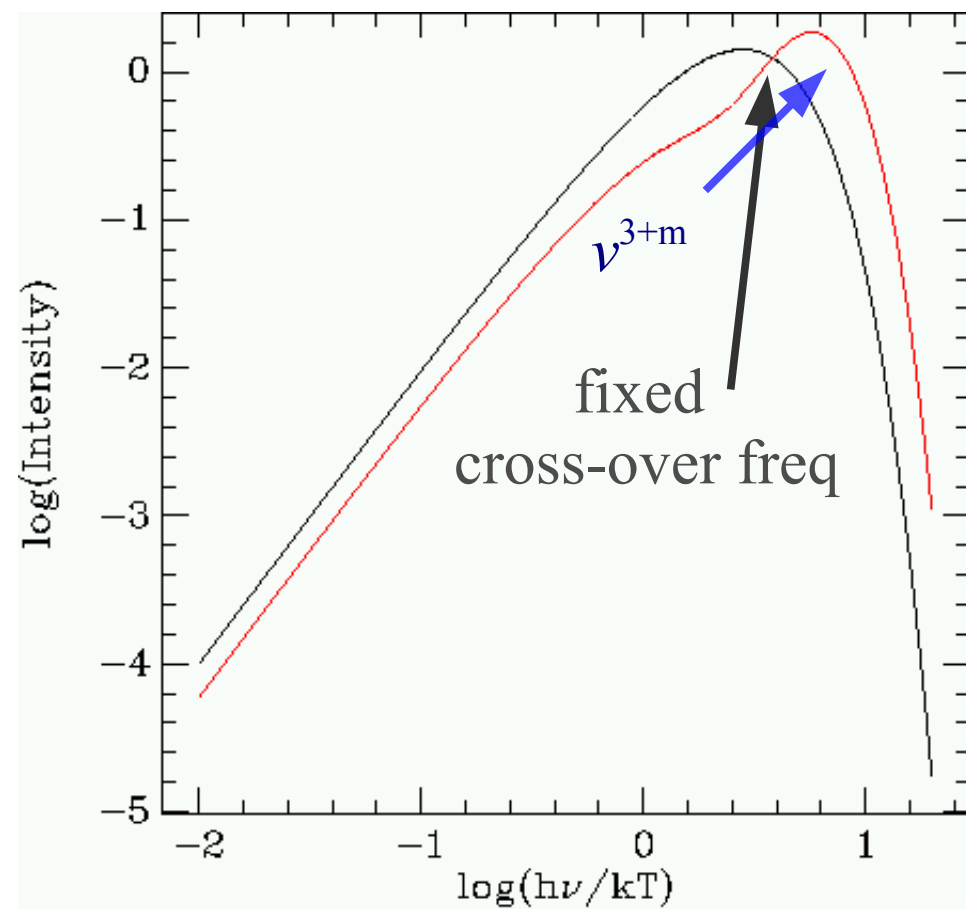
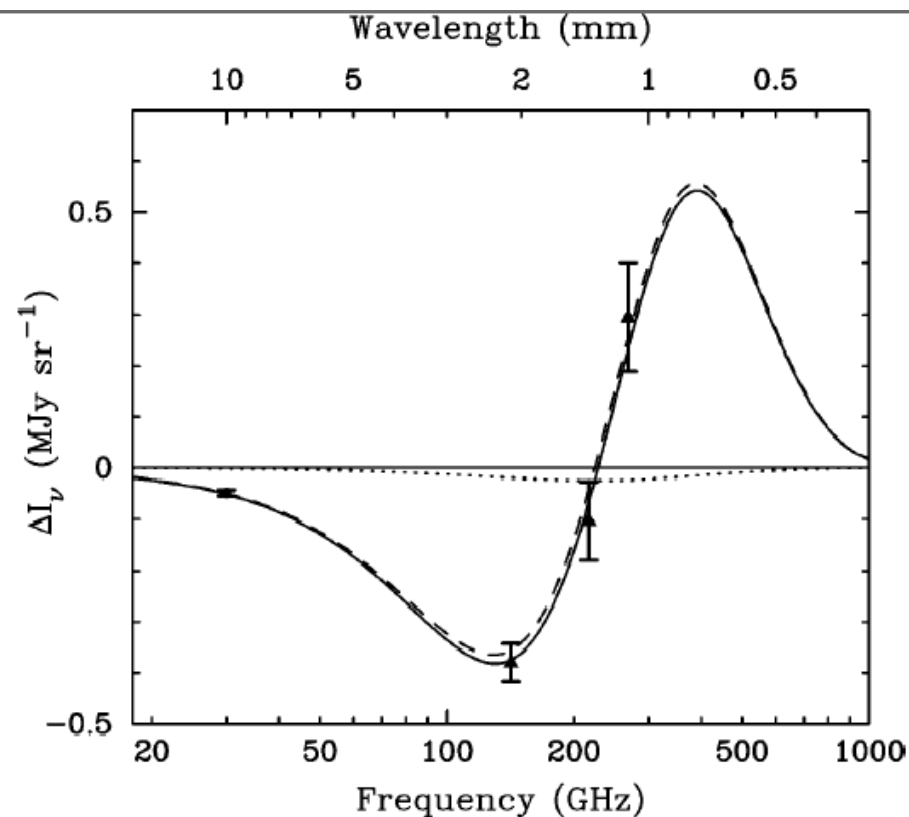


Figure 4 The measured SZE spectrum of Abell 2163. The data point at 30 GHz is from the Berkeley-Illinois-Maryland-Association (BIMA) array (LaRoque et al. 2002), at 140 GHz it is the weighted average of Diabolo and SuZIE measurements (Desert et al. 1998, Holzapfel et al. 1997) (*filled square*), and at 218 GHz and 270 GHz from SuZIE (Holzapfel et al. 1997) (*filled triangles*). The best fit thermal and kinetic SZE spectra are shown by the dashed and dotted lines, respectively, with the spectra of the combined effect shown by the solid line. The limits on the Compton y -parameter and the peculiar velocity are $y_0 = 3.56^{+0.41+0.27}_{-0.41-0.19} \times 10^{-4}$ and $v_p = 410^{+1030+460}_{-850-440}$ km s⁻¹, respectively, with statistical followed by systematic uncertainties at 68% confidence (Holzapfel et al. 1997, LaRoque et al. 2002).

Sunyaev-Zeldovich effect: the Planck's view

The same galaxy cluster, A2319, as a function of frequency as seen by the Planck satellite. Red color means "hotter" than background (CMB); Blue is for "colder" than CMB.

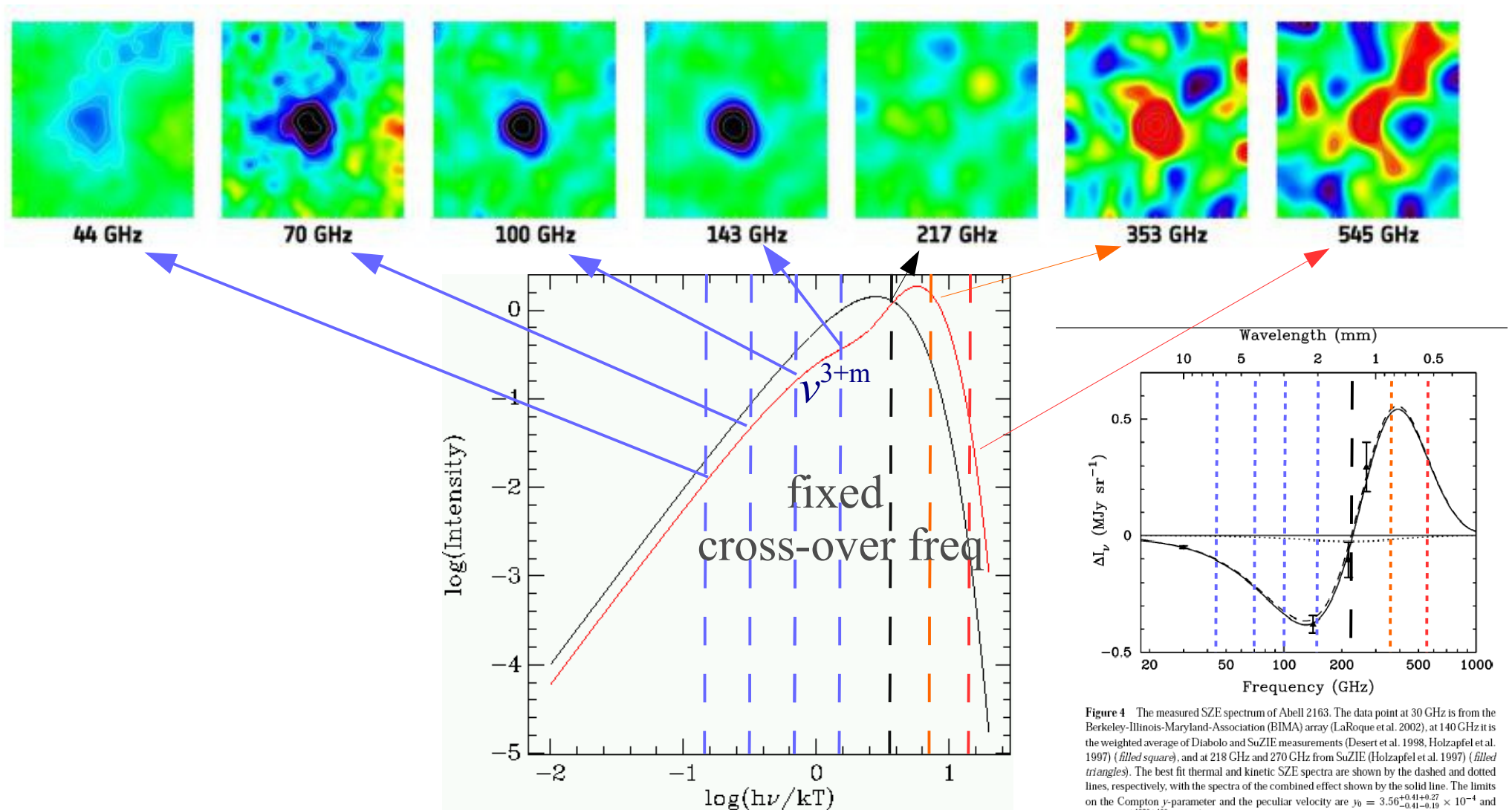
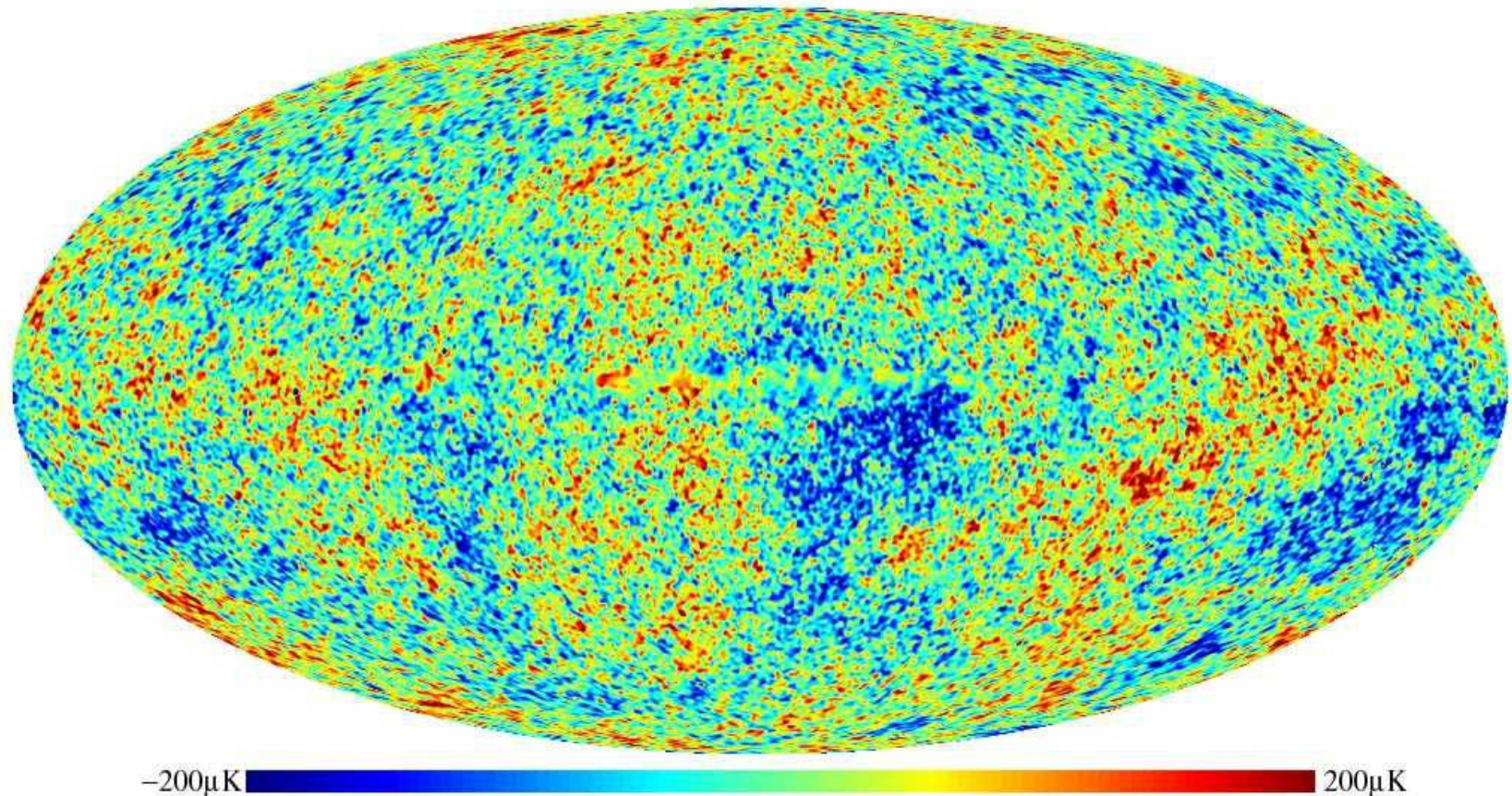


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The CMB sky-view

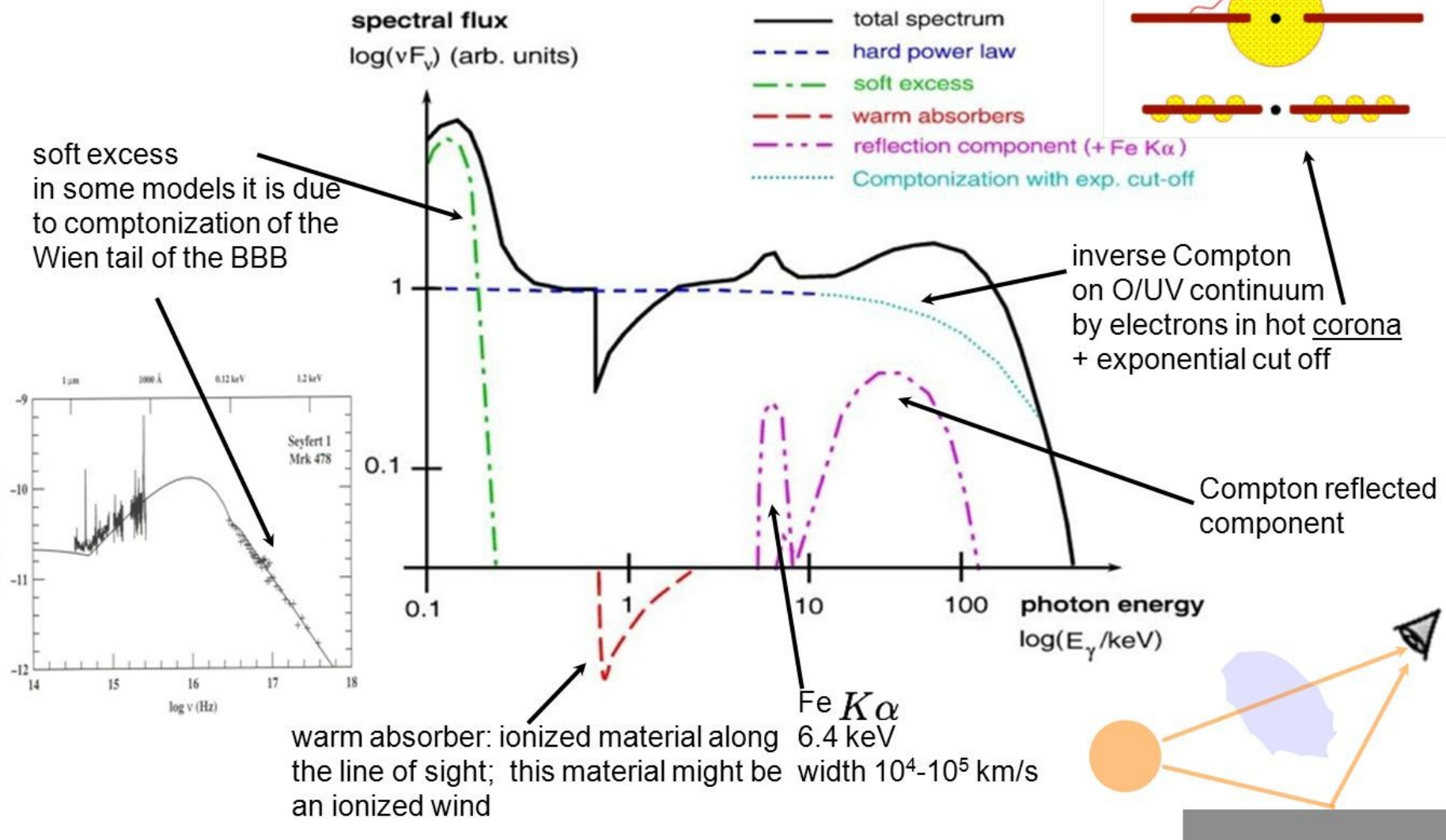
Cosmology is seeking for anisotropies in the CMB temperature, signature of inhomogeneities in the "primordial" mass distribution ("local" fluctuations of density).

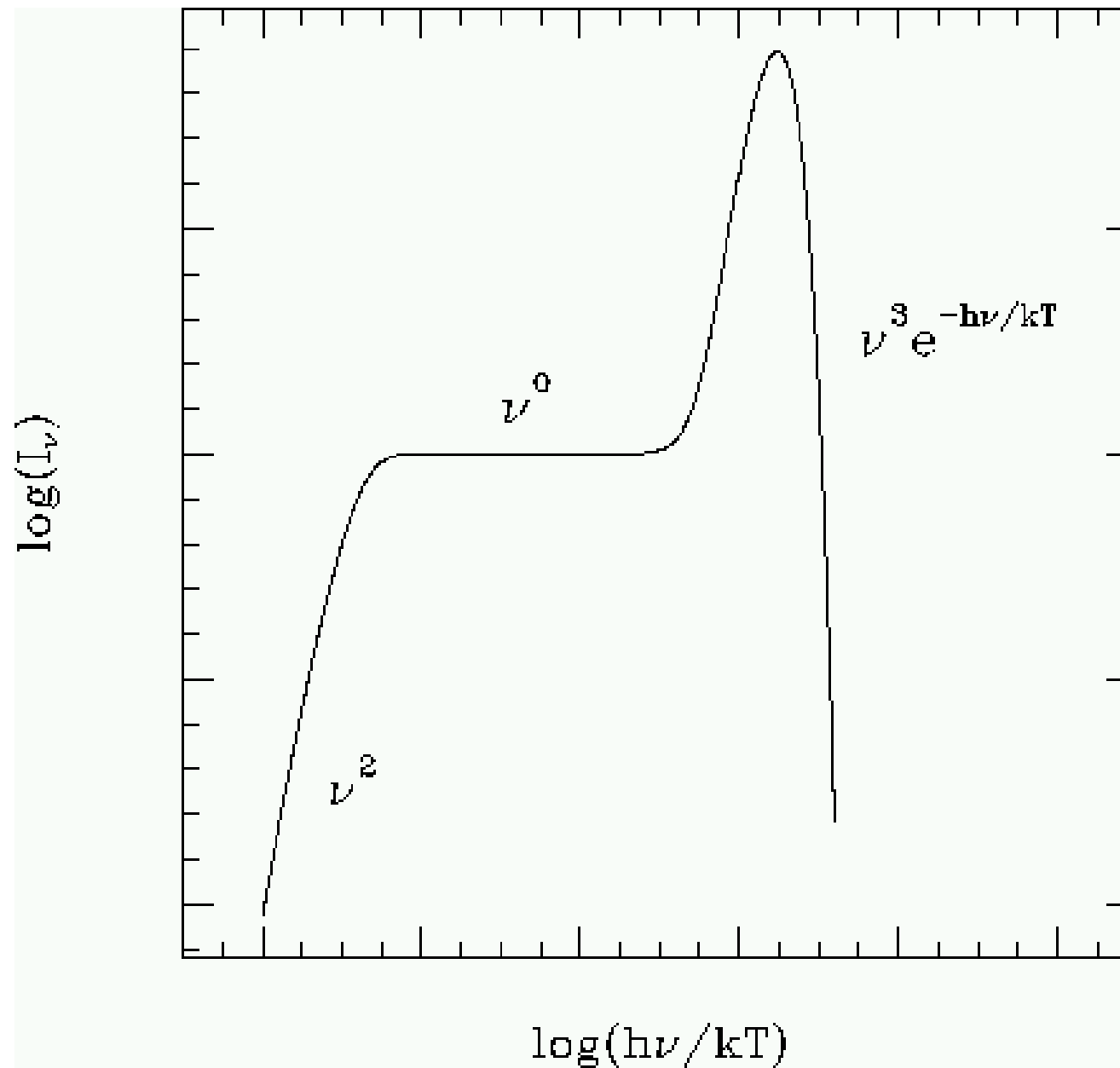


Local deviations from the CMB BB temperature of 2.72548 ± 0.00057 K

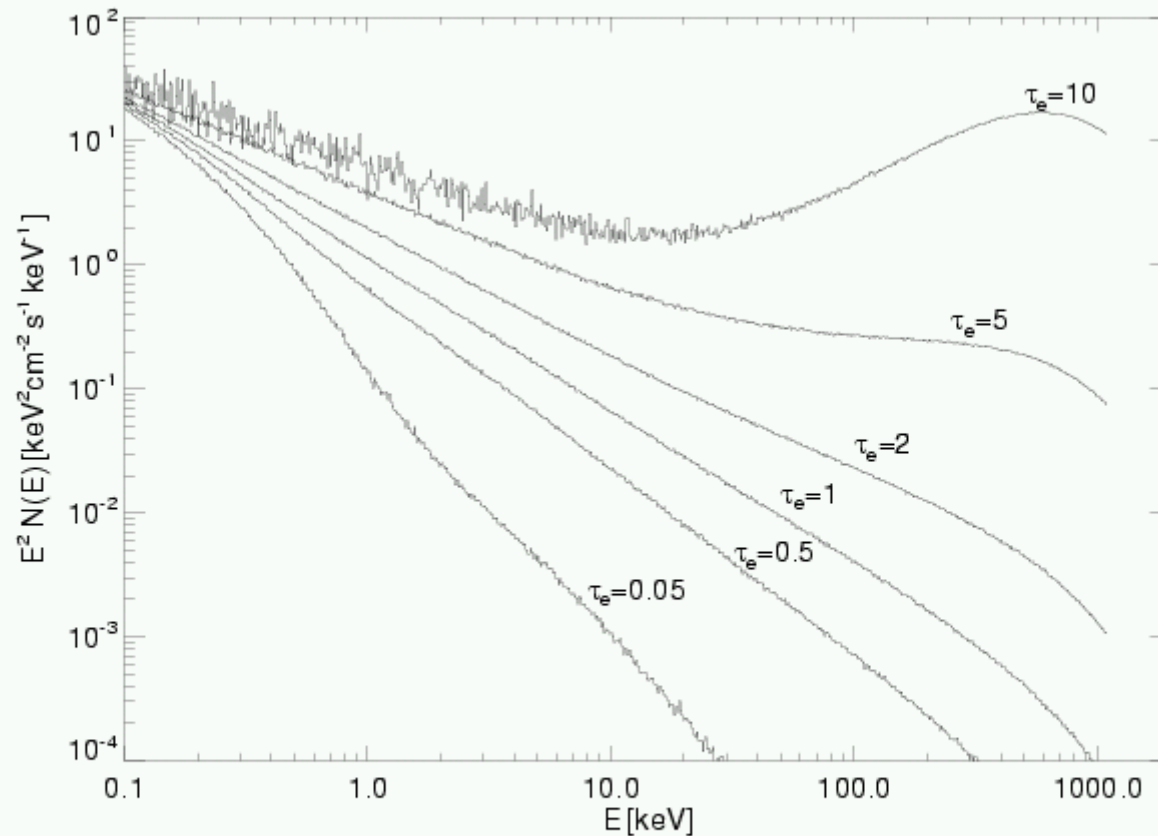
[Do not pay attention to details of the models]

X-ray spectral components





Comptonization: the spectrum is modified as a function of the opacity to this process

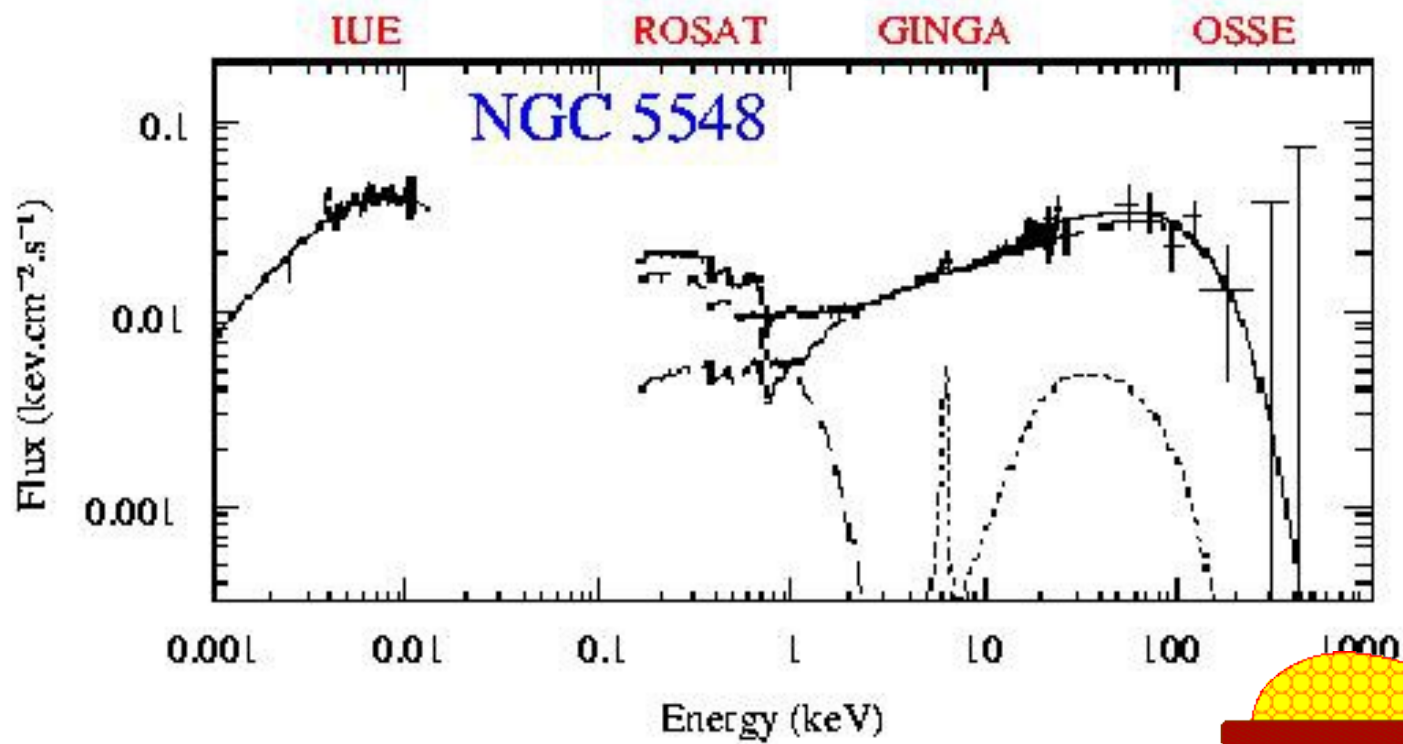
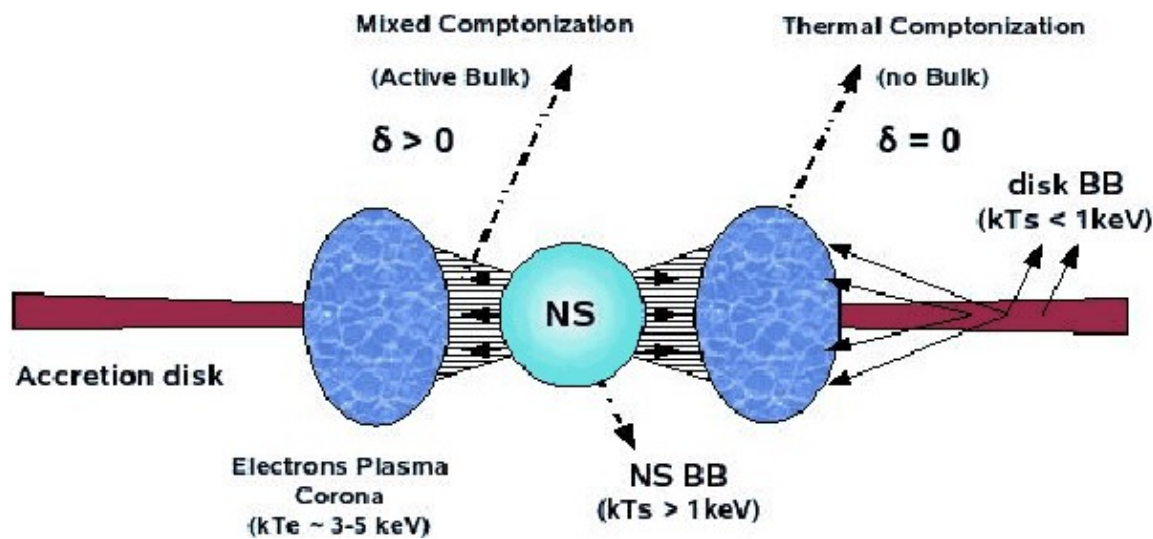
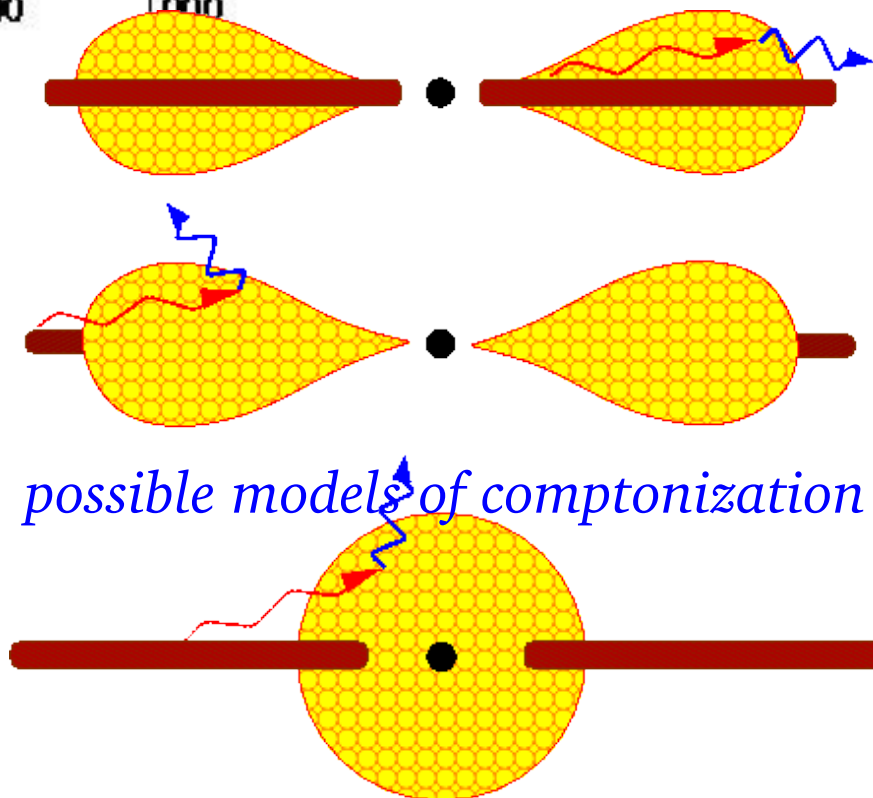


Sphere with $kT_e = 0.7m_e c^2$ (~ 360 keV), seed photons come from center of sphere.

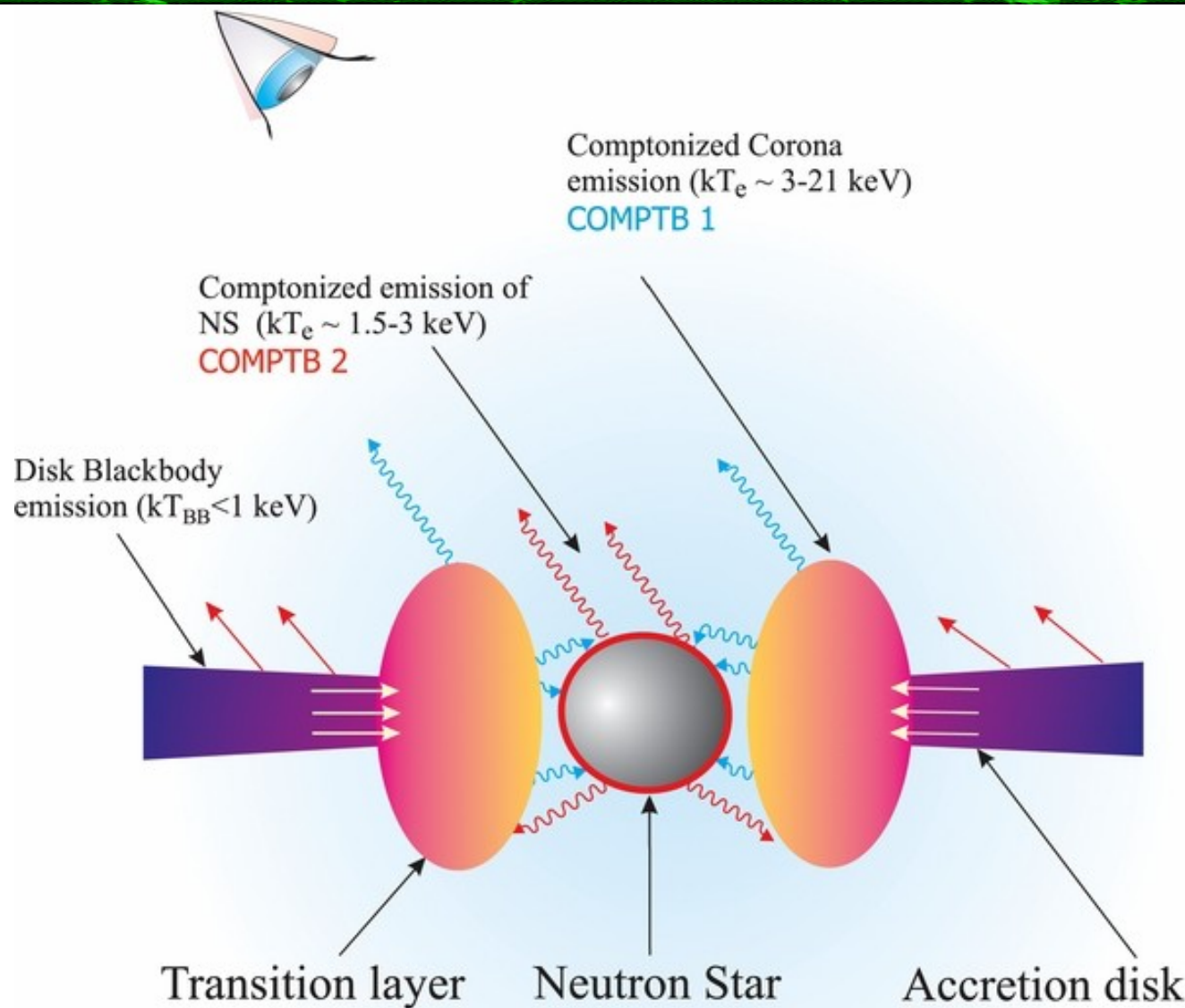
- $y \ll 1$: pure power-law spectrum.
- $y < 1$: power-law with exp. cut-off.
- $y \gg 1$: “Saturated Comptonization”.



Comptonization: Astrophysical examples



possible models of comptonization



A suggested geometry of GX 340+0. Disk and NS soft photons are upscattered off the hotter plasma of the TL located between the accretion disk and NS surface. Some fraction of these photons is seen directly by the Earth observer. Red and blue photon trajectories correspond to soft and hard (upscattered) photons, respectively. In our model, two Comptonization components are considered: the first one (*Comptb1*), with "seed" (disk) photon temperature $T_{s1} \sim 1$ keV and the CC electron temperature varies from 3 keV to 21 keV. The second Comptonized component (*Comptb2* with $T_{s2} \sim 1.5$ keV) and are related to the NS surface and its inner part of the TL (boundary) layer, respectively.



Comptonization summary

Effective in rarified plasma (no significant competing cooling mechanism)

Hot electrons ! (must transfer energy to low energy photons)

Total number of photons is conserved, i.e. possible BB photon distribution is modified

Equilibrium is achieved in "saturated sources"

(i.e. when the energy of the radiation is balanced with that of the electrons: $h\nu = 4kT$)



Summary of scattering processes

Free electrons & photons interact and may produce either Thomson, or Compton or Inverse Compton scattering depending on the relative energies

Thomson scattering does not change the photon energy, but may produce polarized radiation off the Scattering of unpolarized radiation

Compton scattering subtracts energy out of the photon transferring it to the electron

Inverse Compton scattering, depending on the geometry of the interaction, may produce high energy radiation from low energy photons.

Relativistic electrons are capable of producing high energy radiation in the X-rays and beyond

Comptonization is a process in which both direct and Inverse Compton scattering may take place. Depending on the balance of the energies of radiation and free electrons, the most energetic can transfer energy to the other component. Active in various astrophysical contexts.

2. I lobi della radiogalassia 3C 445, posta a 240 Mpc dalla Terra, sono caratterizzati da un campo magnetico di 0.1 microGauss. Che Lorentz factor, gamma, devono avere gli elettroni relativistici per emettere per sincrotrone a 200 GHz? I fotoni radio di frequenza 200 GHz vengono poi diffusi dagli stessi elettroni relativistici attraverso il Synchrotron-self Compton. A che lunghezza d'onda vengono a trovarsi dopo tale processo? La lunghezza d'onda risultante a che banda dello spettro elettromagnetico corrisponde?

10 Apr 2014

1. Sia data la radiosorgente Cen A, che si trova ad una distanza di circa 11 Mpc. Essa ha un flusso radio a 6 cm pari a 6.5 Jy con un indice spettrale pari a -0.8 [$S(\nu) = S_0 (\nu / \text{GHz})^{-0.8} \text{ Jy}$]. Calcolare **1.** la luminosita' monocromatica a 1GHz; **2.** la luminosita' radio totale tra 10 MHz e 100 GHz; **3.** supposto che il campo magnetico di equipartizione nei lobi sia di 5 μG , stimare la luminosita' X per Inverse Compton con i fotoni della CMB; **4.** quanto vale il fattore di Lorentz degli elettroni che emettono a 1 GHz?

04 Jul 2019

1. Sia data la radiosorgente Cyg A, che e' posta a $z=0.056$ (distanza di circa 235 Mpc), e che ha un flusso radio descrivibile dalla legge $S(\nu) = 2000 (\nu / \text{GHz})^{-0.8} \text{ Jy}$. Calcolare **1.** la luminosita' monocromatica a 1GHz; **2.** la luminosita' radio totale tra 10 MHz e 100 GHz; **3.** supposto che il campo magnetico di equipartizione nei lobi sia di 72.5 μG , stimare la luminosita' X per Inverse Compton con i fotoni della CMB.

04 Jun 2019