

Scattering is an **elastic process** in which a photon interacts with a **free** electron. After the interaction, the emerging photon is somehow different from the initial one. On the basis of the outcome, there are three main scattering processes in astrophysics.

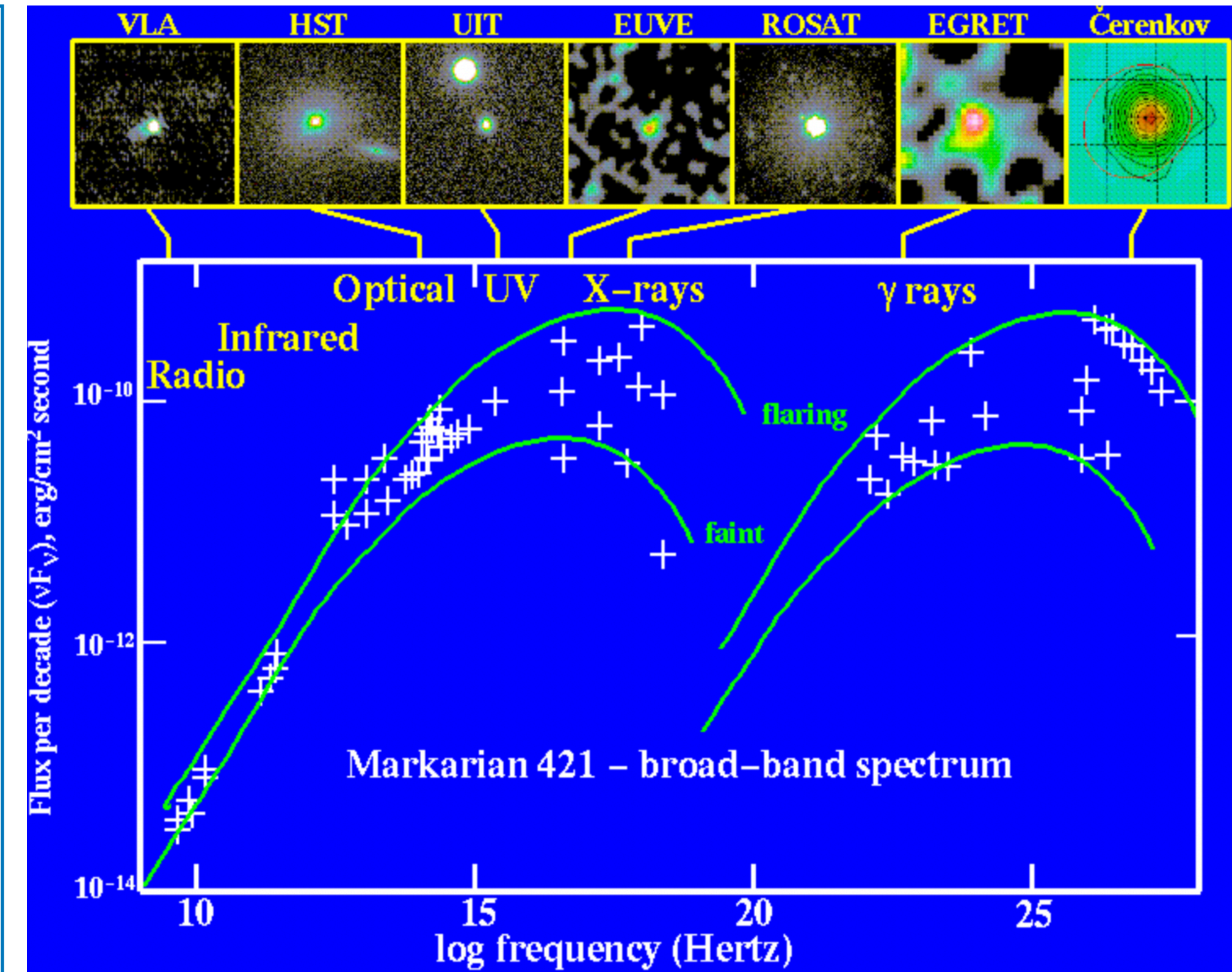
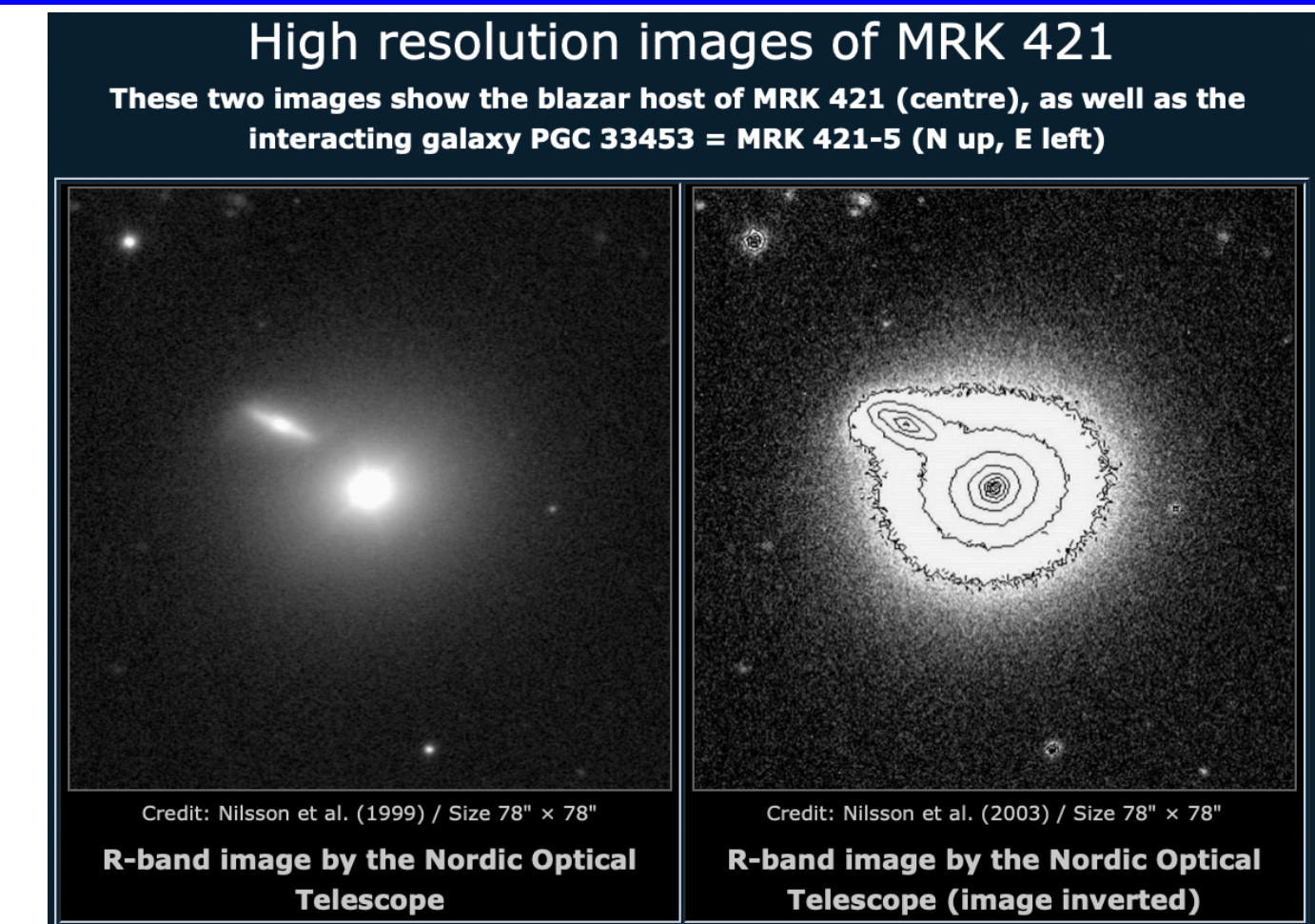
Thomson

Compton

Inverse Compton

... on textbooks (and lecture notes)

- G. Ghisellini: "Radiative Processes in High Energy Astrophysics" §5
- M. Longair: "High Energy Astrophysics" § 9
- G. B. Rybicki & A. P. Lightman: "Radiative Processes in Astrophysics" § 3.4 §7
- C. Fanti & R. Fanti: "Lezioni di radioastronomia" § 5
- T. Padmanabhan: "Theoretical Astrophysics" §6.4 & 6.6 & 6.7



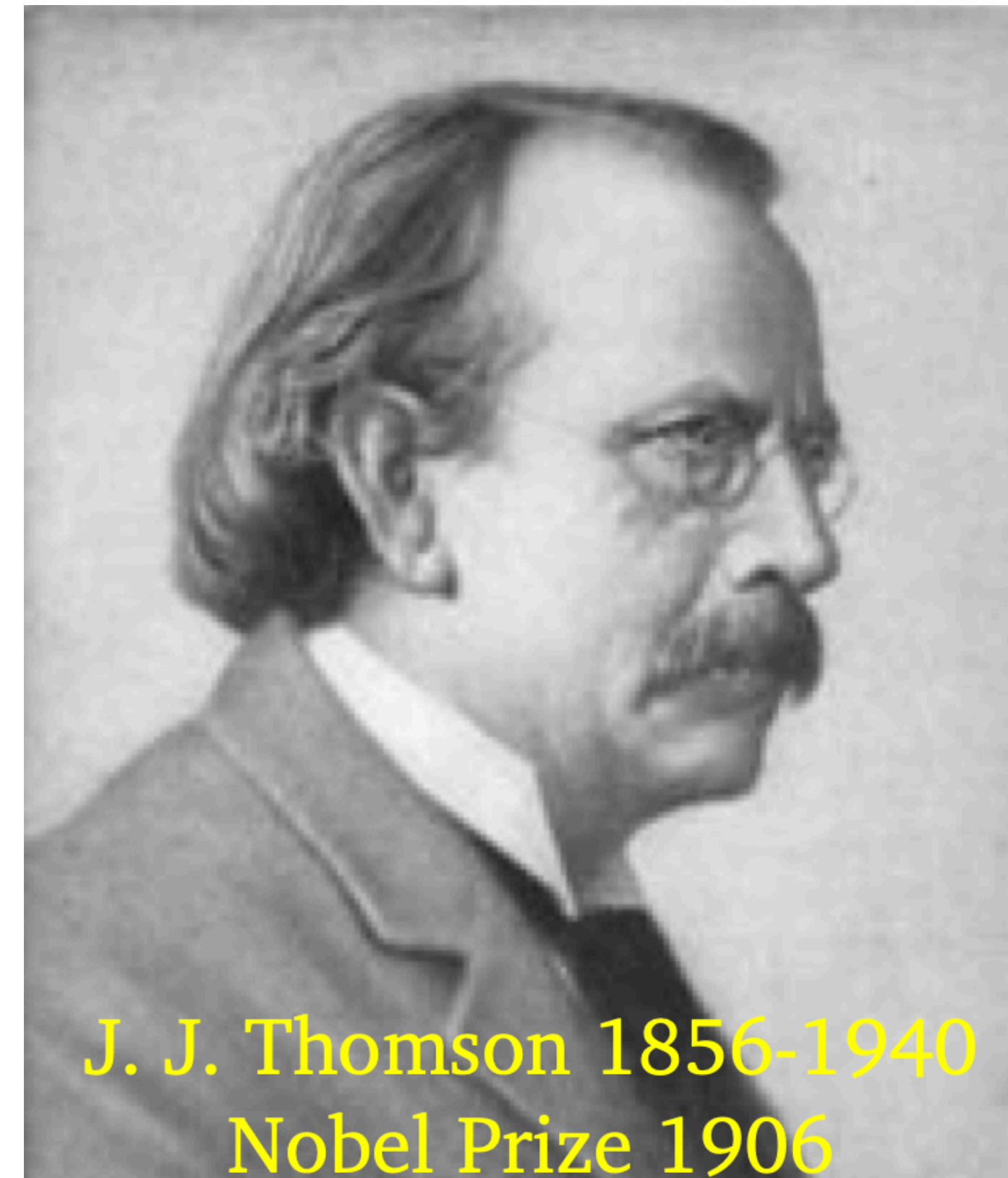
Thomson scattering takes place when **a low energy photon** ($h\nu \ll m_e c^2$) is **absorbed by a free electron**, initially at rest.

The electric and magnetic components of the incident wave accelerate the electron, which, in turn, emits radiation. Thus, the wave is scattered, and this process is an important phenomenon in plasma physics. It was first explained by the physicist J.J Thomson

The main cause of the acceleration of the particle will be due to the electric field component of the incident wave. The particle will move in the direction of the oscillating electric field, resulting in electromagnetic dipole radiation.

The moving particle radiates most strongly in a direction perpendicular to its motion and that radiation can be **polarized** (based on the geometry of the scattering).

Therefore, depending on where an observer is located, the light scattered from a small volume element may appear to be more or less polarized.



Scattering processes: Thomson

For a non-relativistic electron (best, at rest!), the incoming (low-energy) photons can be expressed as a continuous electro-magnetic wave and the magnetic field can be "neglected", since the magnetic force is $\frac{e}{c} \vec{v} \times \vec{B}$

The charge feels a force due to a linearly polarised incoming wave: $\vec{F} = e\hat{\epsilon}E_0 \sin \omega_0 t$, producing a dipole oscillation $\vec{d} = \frac{eE_0}{m_e\omega_0^2} \hat{\epsilon} \sin \omega_0 t$

The corresponding second derivative is $\ddot{\vec{d}} = \frac{eE_0}{m_e} \hat{\epsilon} \sin \omega_0 t$

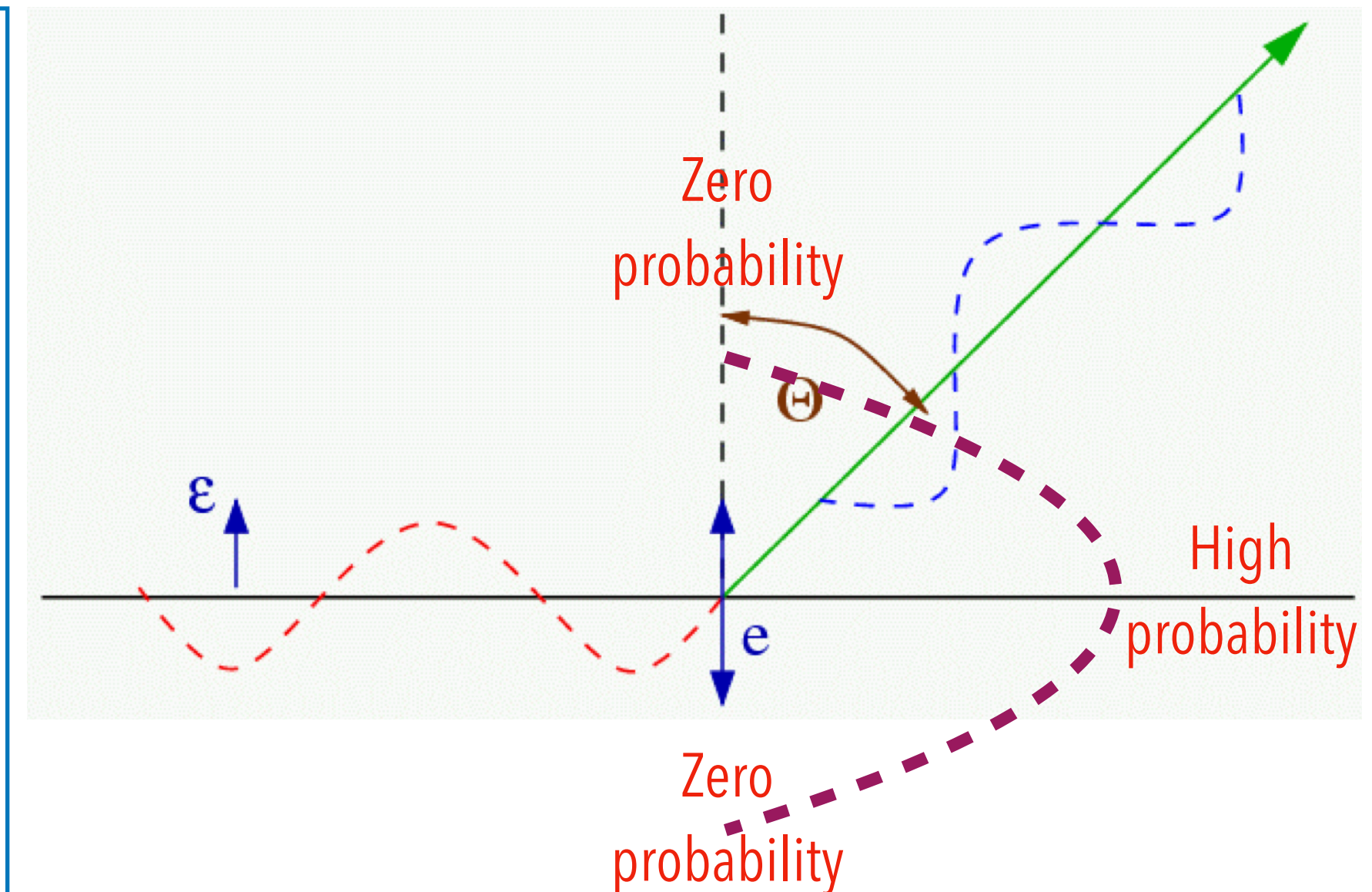
Its squared time average is $\langle \ddot{\vec{d}}^2 \rangle = \frac{e^2 E_0^2}{2m_e}$

then the Larmor formula providing the average radiated energy $-\langle \frac{d\epsilon}{dt} \rangle = \frac{1}{3} \frac{e^4 E_0^2}{m_e^2 c^3}$

Comparing to the incoming Poynting flux $\langle |S| \rangle = \frac{cE_0^2}{8\pi}$ we can define the **Thomson cross**

section σ_T as $\sigma_T = \frac{\langle d\epsilon/dt \rangle}{\langle |S| \rangle} = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_0^2 = 6.65 \times 10^{-25} \text{ [cm}^{-2}\text{]}$

– the energy of the scattered radiation is the same as the incoming wave



Note that $r_0 = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13} \text{ [cm]}$ is the electron classical radius.

Note that the cross section of the scattering does not depend on the frequency (...apparently, at least)

All this breaks down at high photon frequencies, when classical physics needs significant corrections.

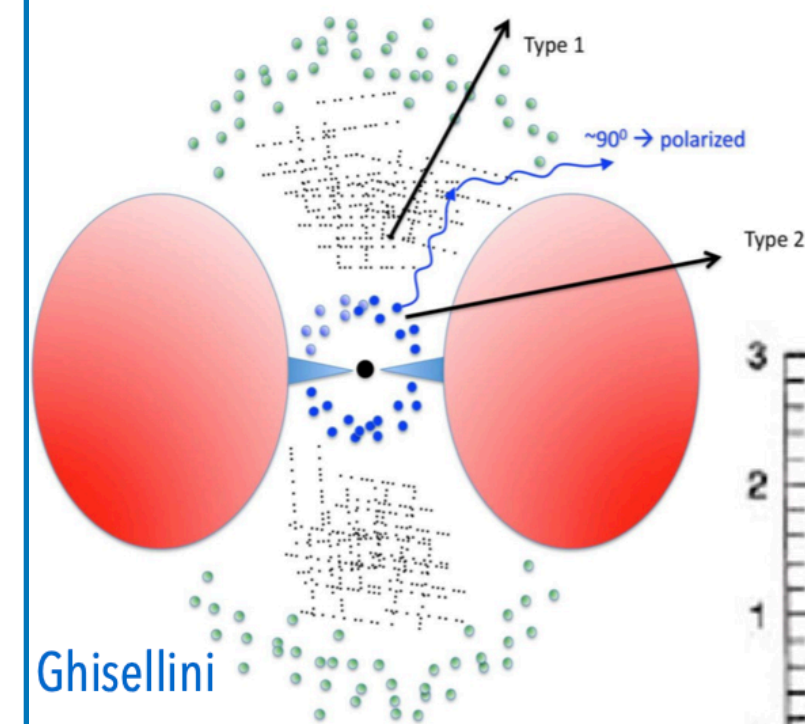
See <https://www.youtube.com/watch?v=fSAzPQtSAPQ> in particular from 1:40" to 3:30" – not available at the moment

Scattering processes: Thomson

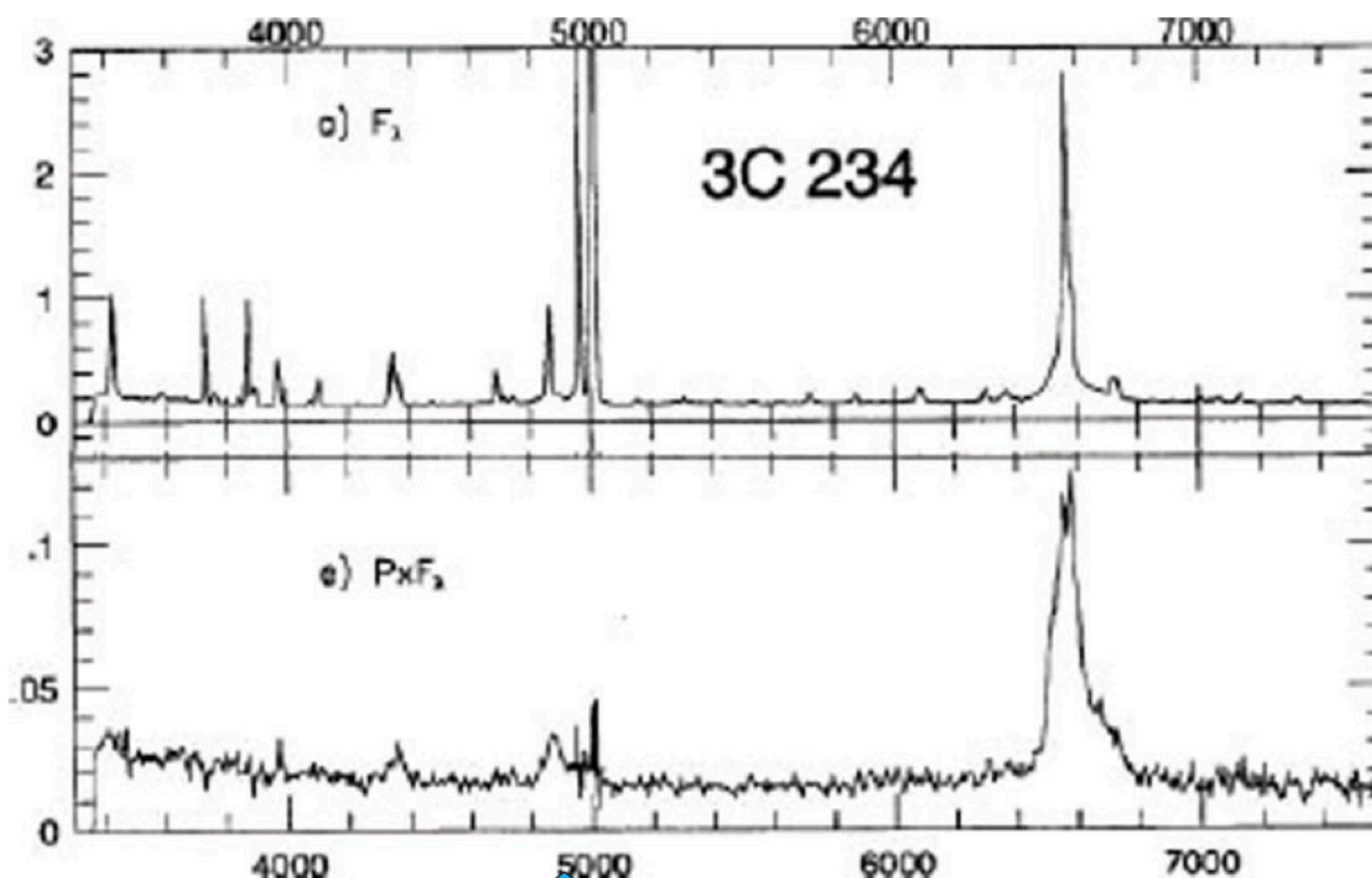
The geometry of the scattering may produce polarised radiation.

Observer 1 and 2 are on a plane perpendicular to the direction of the incoming radiation.

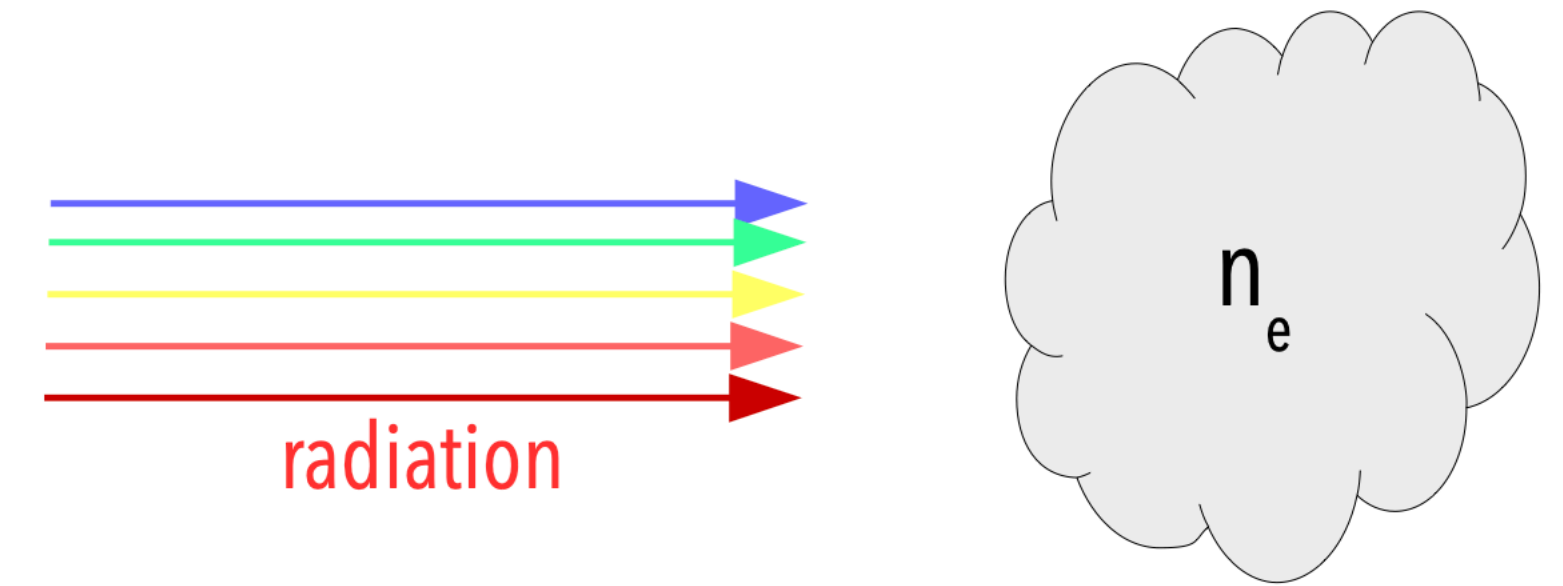
In this case, they observe fully linearly polarised emission (the original radiation is not polarised at all!). At intermediate angles with respect the initial direction of the photon flux, the fractional polarisation ranges from 0 (aligned to the initial direction) to 100% at 90 deg



"broad line emission" is visible in polarized light
(when unpolarized emission is dropped)



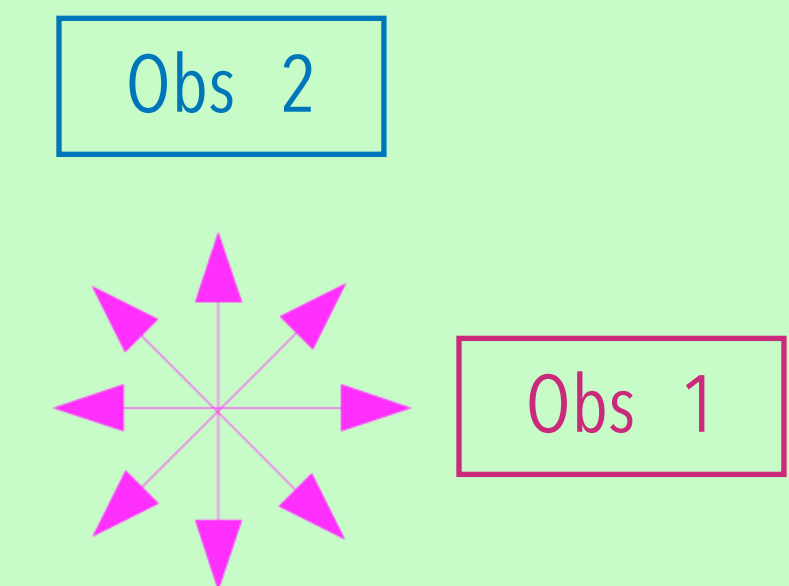
Top: total intensity
Bottom: polarised intensity
Note the different scales!



All the incoming photons follow about the same trajectory (e.g. single point source)

Top: side view
Bottom: front view

Front view: unpolarized radiation makes the electrons oscillate in all directions



Summary

1. Scattering is symmetric with respect to the scattering angle (backward=forward)
2. The amount of scattered radiation depends solely on the incident radiation. $-\frac{d\varepsilon}{dt} = \sigma_T c U_{\text{rad}}$
3. Even if the incident radiation is unpolarized, Thomson scattering can polarize scattered radiation depending on the scattering angle.
4. Thomson scattering is one of the main processes capable of trapping photons in a given region.

The optical depth for Thomson Scattering in a medium is $\tau_T = \int \sigma_T n_e dl$

Photons are scattered in random directions (following the dipole emission pattern as a probability distribution) and the random walk in which each step defines the photon mean free path: $\lambda_T = \frac{1}{\sigma_T n_e}$

Thomson scattering, i.e. the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength.

Compton scattering (or the Compton effect), is the **decrease in frequency** (increase in λ) **of a high energy** (X – or γ – ray) **photon, as it interacts with a free electron.**

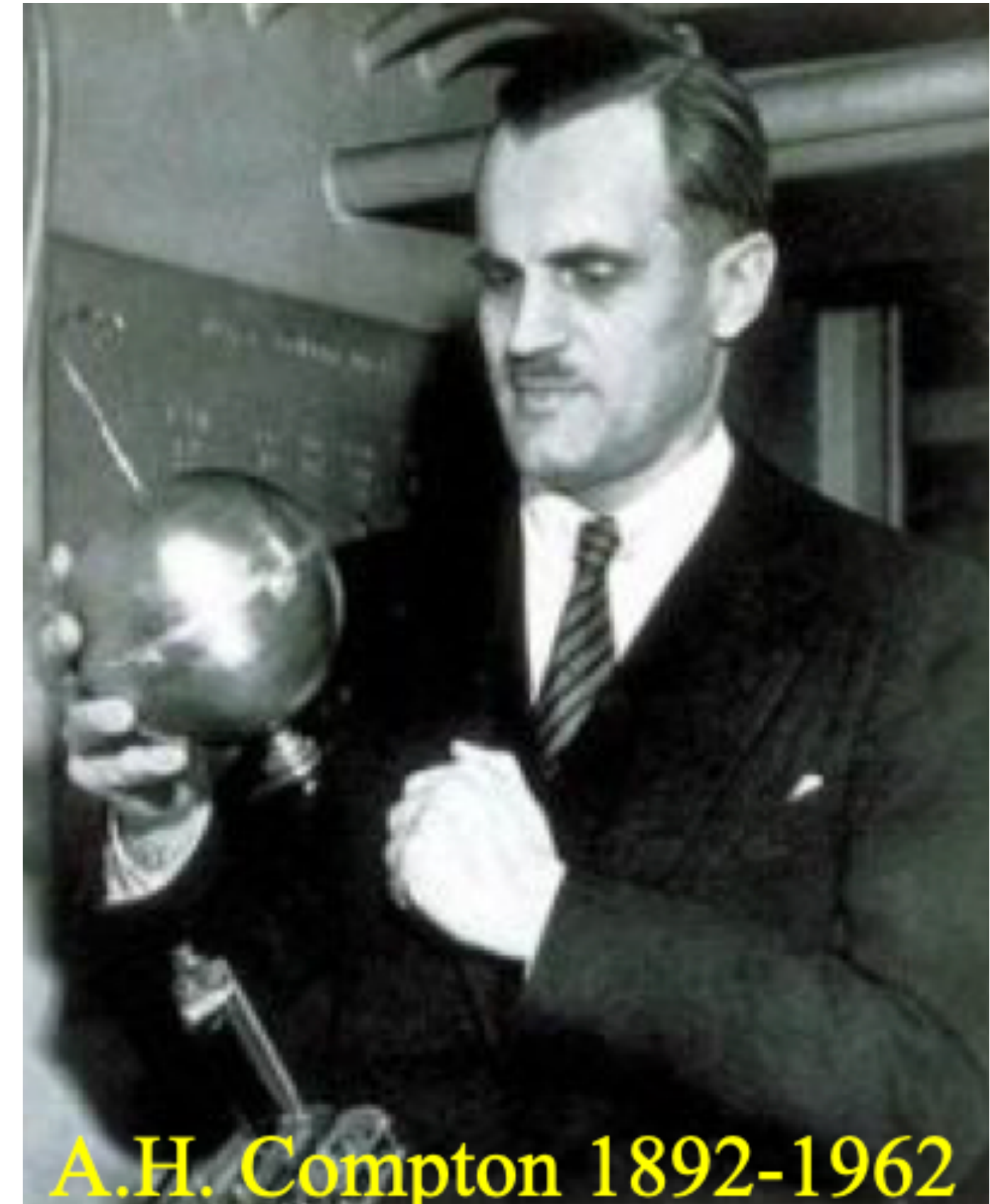
The amount the wavelength increase is called the **Compton shift**.

Although nuclear Compton scattering also exists, what is meant by Compton scattering usually is the interaction involving free electrons only.

Compton effect was observed by Arthur Holly Compton in 1923, for which he was awarded the Nobel Prize in Physics in 1927.

The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon, namely **light must behave as if it consists of particles in order to explain the Compton scattering.**"

More important for astrophysics, Inverse Compton scattering also exists, where the photon gains energy (decreasing in wavelength) upon interaction with a high energy electron.



The physical/mathematical description of an elastic collision:

Energy conservation: $h\nu_i + m_e c^2 = h\nu_f + \sqrt{p_e^2 c^2 + m_e^2 c^4}$

Momentum conservation: for photons, $p = E/c = h\nu/c$ then, after the collision the momentum of the electron is:

$$\vec{p}_e = \vec{p}_i - \vec{p}_f \rightarrow p_e^2 = (p_i - p_f) \cdot (p_i - p_f) = p_i^2 + p_f^2 - 2p_i p_f \cos \varphi$$

$$p_e^2 = \frac{h^2 \nu_i^2}{c^2} + \frac{h^2 \nu_f^2}{c^2} - \frac{2h^2 \nu_i \nu_f}{c^2} \cos \varphi \quad [\leftarrow \text{multiply by } c^2]$$

Then take the square of the energy conservation

$$p_e^2 c^2 + m_e^2 c^4 = h^2 \nu_i^2 + h^2 \nu_f^2 + m_e^2 c^4 + 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2 - 2h^2 \nu_i \nu_f$$

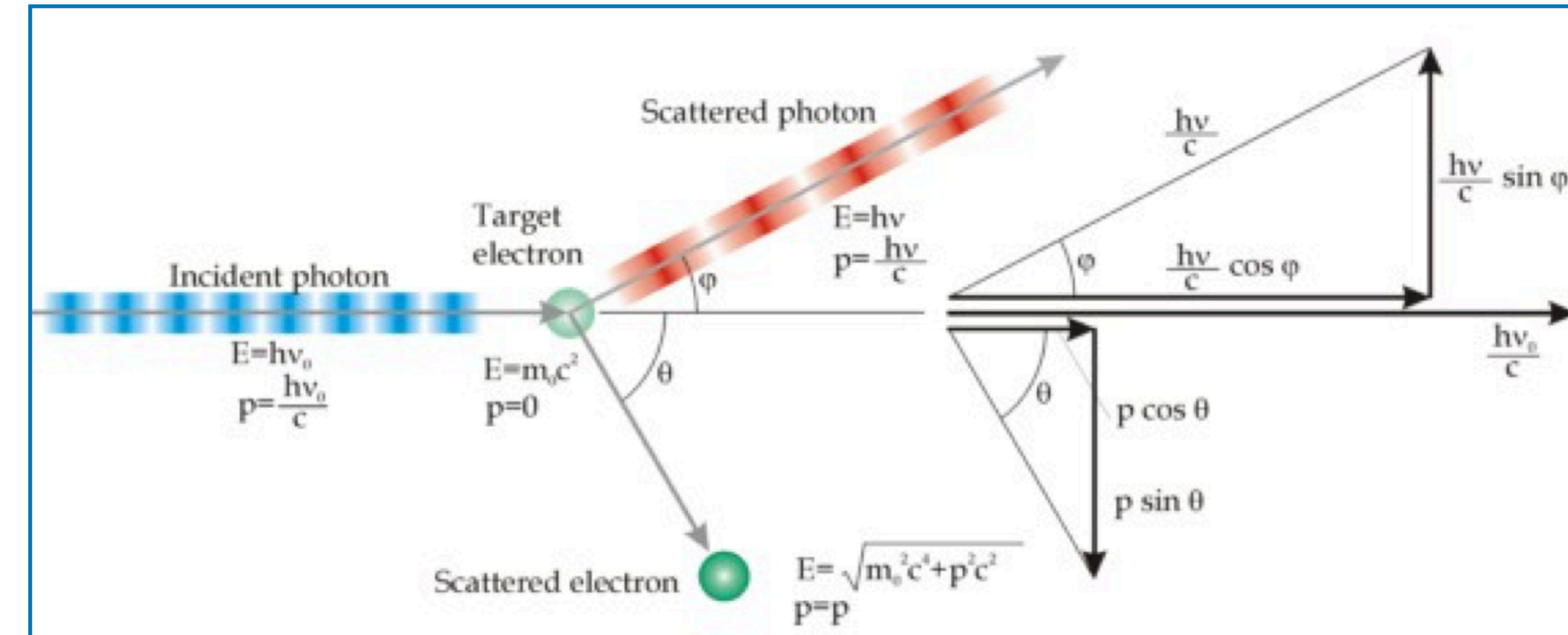
Let's compare the two, extracting $p_e^2 c^2$ we get:

$$-2h^2 \nu_i \nu_f \cos \varphi = 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2 - 2h^2 \nu_i \nu_f$$

$$2h^2 \nu_i \nu_f (1 - \cos \varphi) = 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2$$

getting rid of common factors

$$h\nu_i \nu_f (1 - \cos \varphi) = (\nu_i - \nu_f) m_e c^2$$



$$\frac{(1 - \cos \varphi)}{m_e c^2} = \frac{\nu_i - \nu_f}{h\nu_i \nu_f} = \frac{1}{h} \left(\frac{1}{\nu_f} - \frac{1}{\nu_i} \right)$$

$$\frac{1}{\nu_f} - \frac{1}{\nu_i} = \frac{\lambda_f}{c} - \frac{\lambda_i}{c} = \frac{h}{m_e c^2} (1 - \cos \varphi)$$

$$\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \varphi) = \lambda_c (1 - \cos \varphi)$$

Where $\lambda_c = \frac{h}{m_e c} = 0.02426 \text{ \AA}$

is the **Compton wavelength** for electrons

The scattered photon energy is then
$$h\nu_f = \frac{h\nu_i}{1 + h\nu_i(1 - \cos \varphi)/m_e c^2}$$

The final energy of the scattered photon is then dependent on:

- Initial energy of the photon
- Scattering angle

Some graphical explanation of the influence of the scattering angle to the energy transfer:

<https://www.youtube.com/watch?v=zTzJJ9IGPOU>

The incoming photon always loses energy, unless $\varphi = 0$

The scattering angle is not fully isotropic and anisotropies becomes more and more evident with increasing photon energies (e.g. $h\nu \sim m_e c^2$ in the diagram the right)

When the photons involved in the collision have large energies, the scattering becomes less efficient (smaller probability of interaction).

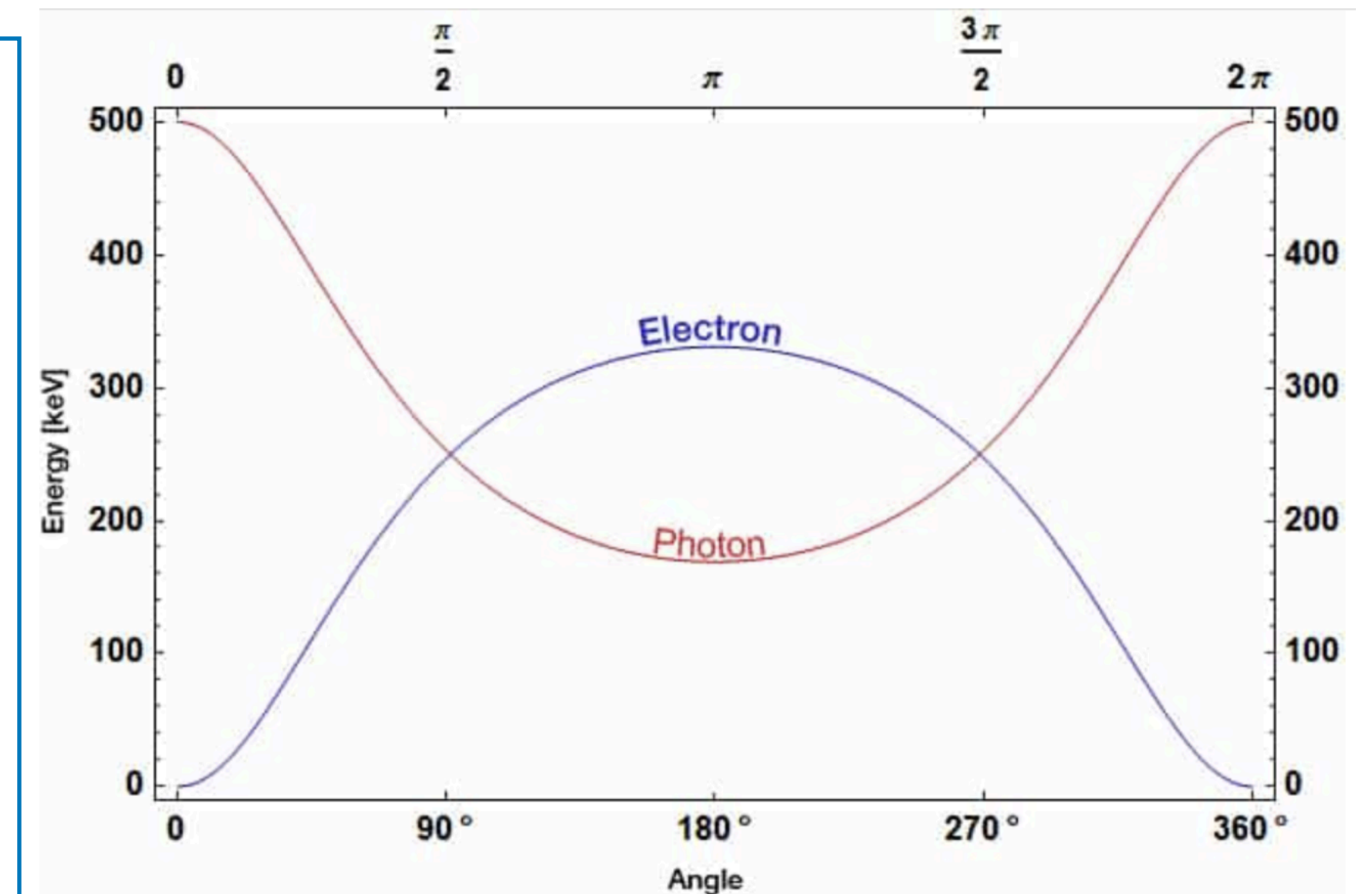
Quantum electrodynamic effects reduce the Thomson cross section to the Klein–Nishina expression



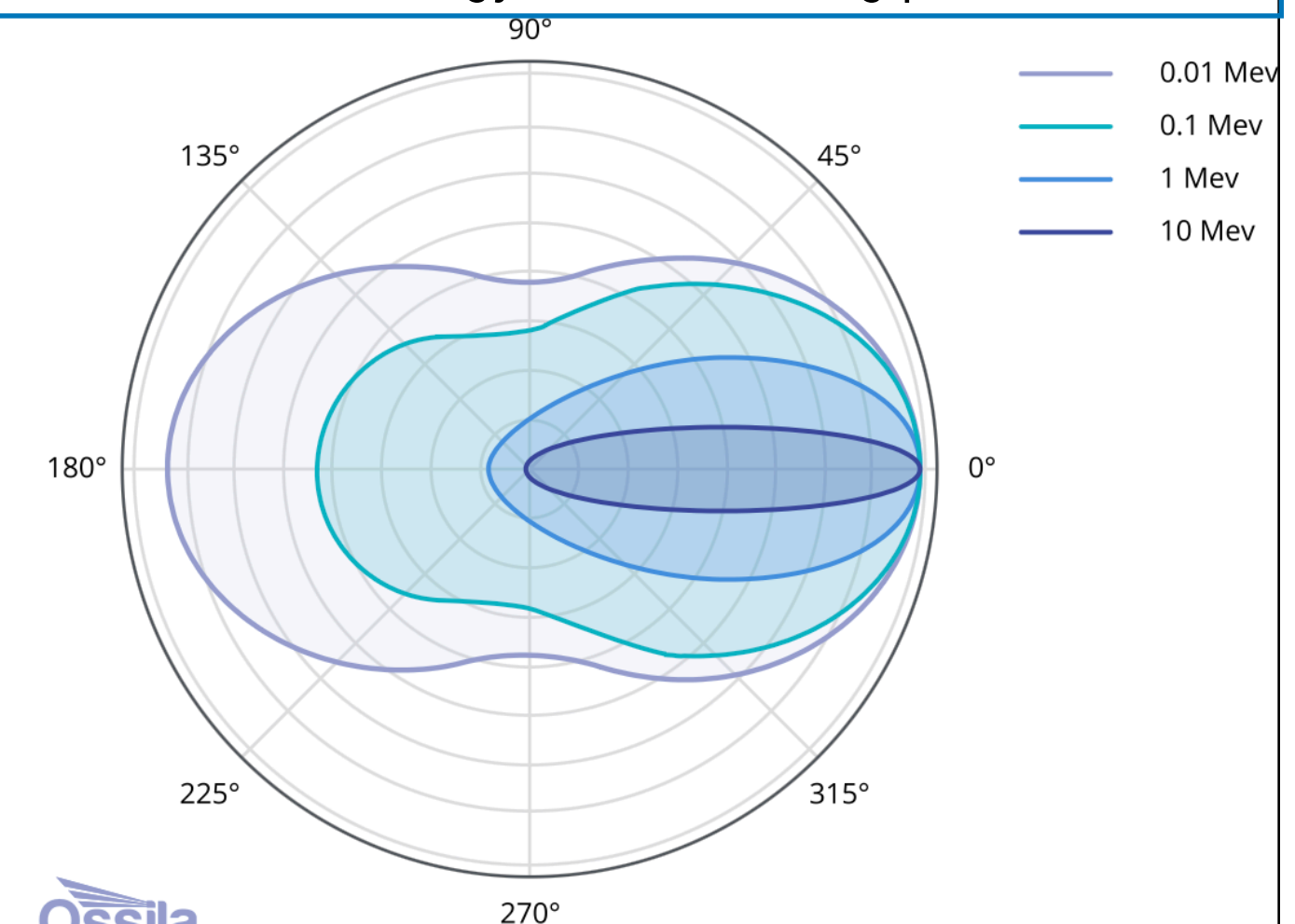
Oskar Klein (1894 -1977)



Yoshio Nishina (1890–1951)



Top: Energy share between an incoming 500 keV photon and an electron according to the scattering angle. The max energy transfer occurs when the scattered photon is reflected back, and the electron is sent on the opposite side of the scattered electron. **Bottom:** probability of scattering angle distribution based on the energy of the incoming photon.



If we define $x = \frac{h\nu_i}{m_e c^2}$ the **Klein–Nishina cross section** can be expressed as:

$$\sigma_{\text{K-N}} = \sigma_{\text{T}} \frac{3}{4} \left[\frac{1+x}{x^3} \left(\frac{2x(1+x)}{1+2x} - \ln(1+2x) \right) + \frac{1}{2x} \ln(1+2x) - \frac{1-3x}{(1-2x)^2} \right]$$

Whose limit for high photon energies ($x \gg 1$) is $\sigma = \sigma_{\text{T}} \frac{3}{4} \frac{1}{x} \left(\ln 2x + \frac{1}{2} \right)$

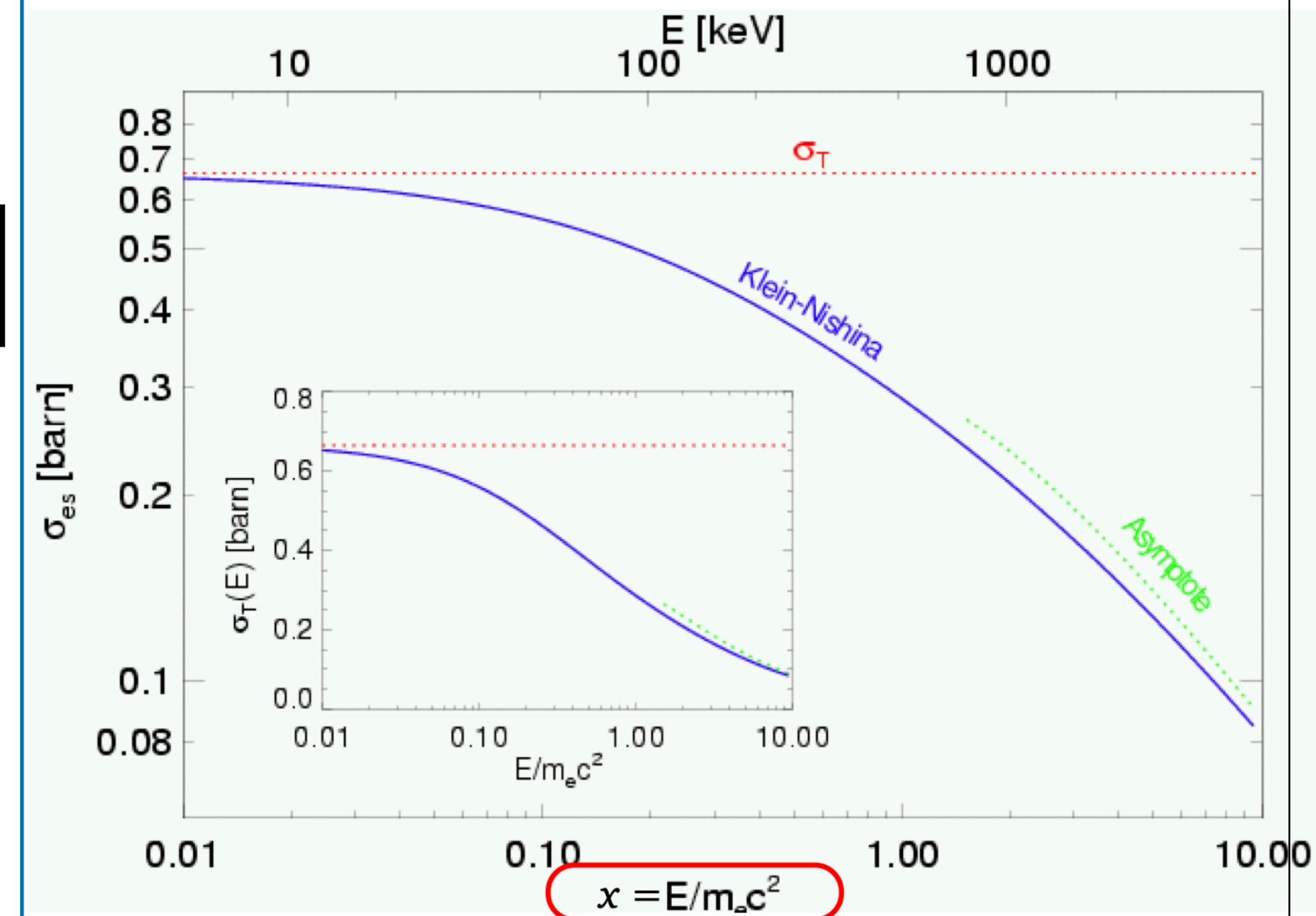
At low photon energies, the limit of the Klein–Nishina cross section is the Thomson

All this originates from a differential cross section based on the scattering angle

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e \left[P(h\nu, \varphi) - P^2(h\nu, \varphi) \sin^2 \varphi + P^3(h\nu, \varphi) \right]$$

Where $P(h\nu, \varphi) = \frac{1}{1 + \frac{h\nu}{m_e^2}(1 - \cos \varphi)}$ is the probability of scattering at an angle φ

- As mentioned earlier, the Compton scattering/effect is not particularly relevant in astrophysics since it generally turns a rare high energy photon into a common low energy photon



The **Thomson** and **Klein–Nishina** cross sections.
The inset is in Log–linear scale

Scattering processes: **INVERSE Compton**

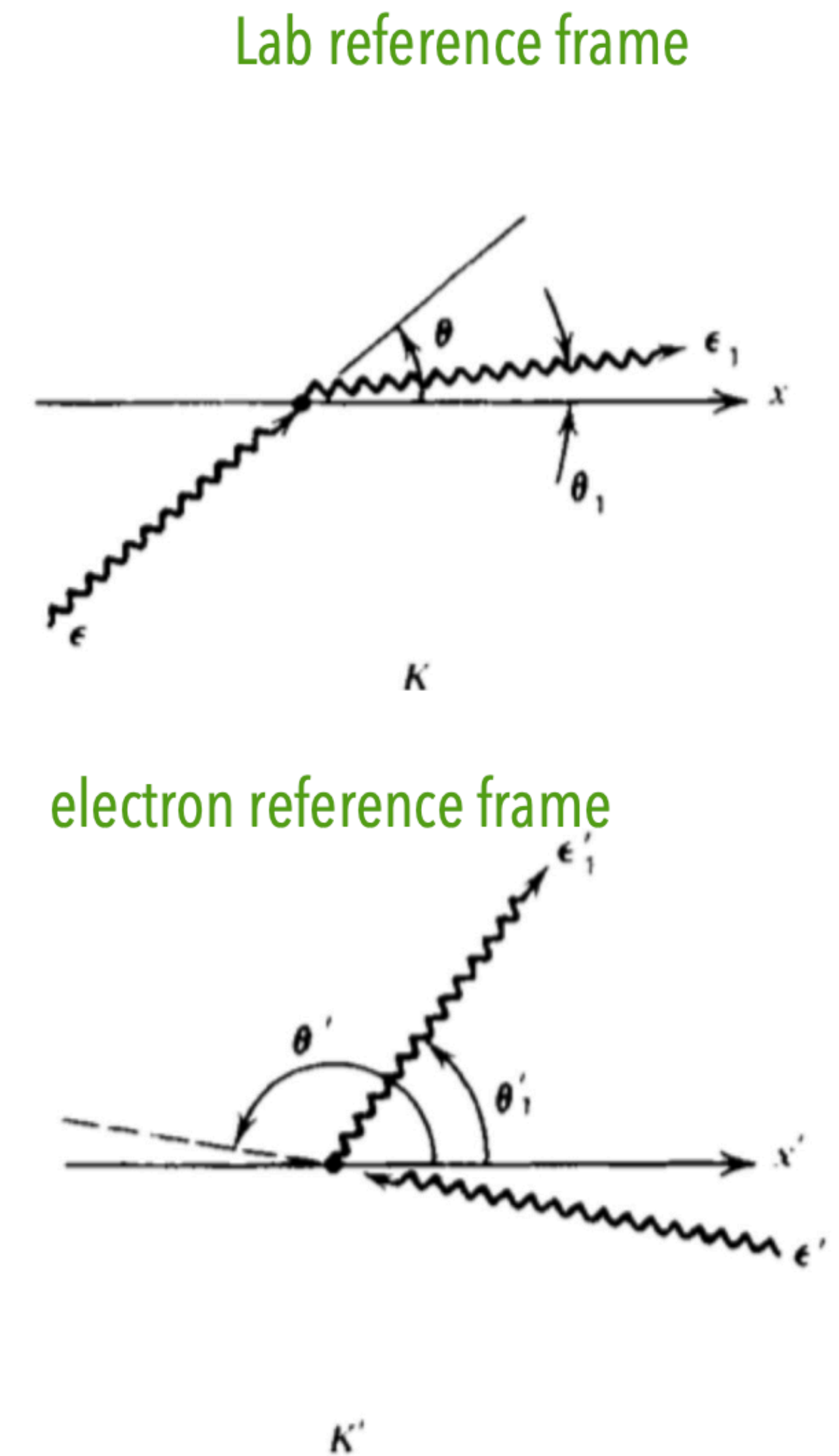
The inverse Compton scattering occurs when a high energy electron interacts with a low energy photon, transferring to it part of the kinetic energy, and creating a high(-er) energy photon. In a collective picture, energetic (e.g. relativistic) electrons "**cool down**" by increasing the energy of low frequency **seed** photons.

The process is described by considering the laboratory reference frame, then go to the electron reference frame and finally back into the laboratory reference frame. Prime quantities are referred to the electron RF.

If the electron is relativistic, the beaming effect ($1/\gamma$) is at work.

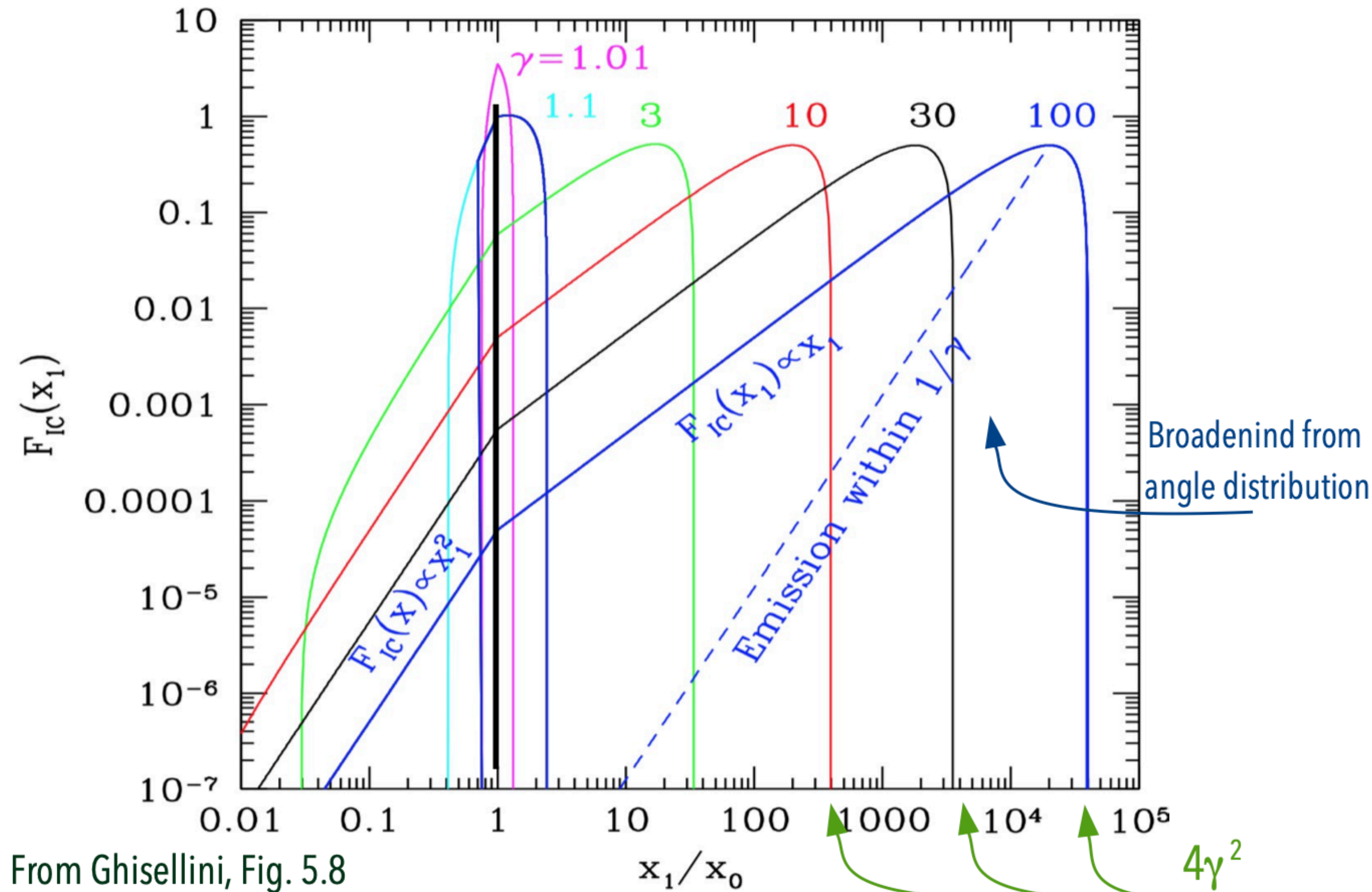
1. If the photon $h\nu_i$ enters the electron RF from the beaming angle, its energy is
$$h\nu_i' = h\nu_i \gamma (1 - \beta \cos \theta)$$
2. In case $h\nu_i' \leq m_e c^2$, then we can apply the Thomson scattering $h\nu_f' = h\nu_i'$
3. From the electron RF to the laboratory RF, in case the photon is within the beaming angle
$$h\nu_f = h\nu_f' \gamma (1 + \beta \cos \theta_1')$$

In summary, the net gain of the process is of the order of γ^2 , with the maximum amplification of $4\gamma^2$. Note **that not all the scattering imply an increase of the energy of the seed photon.**

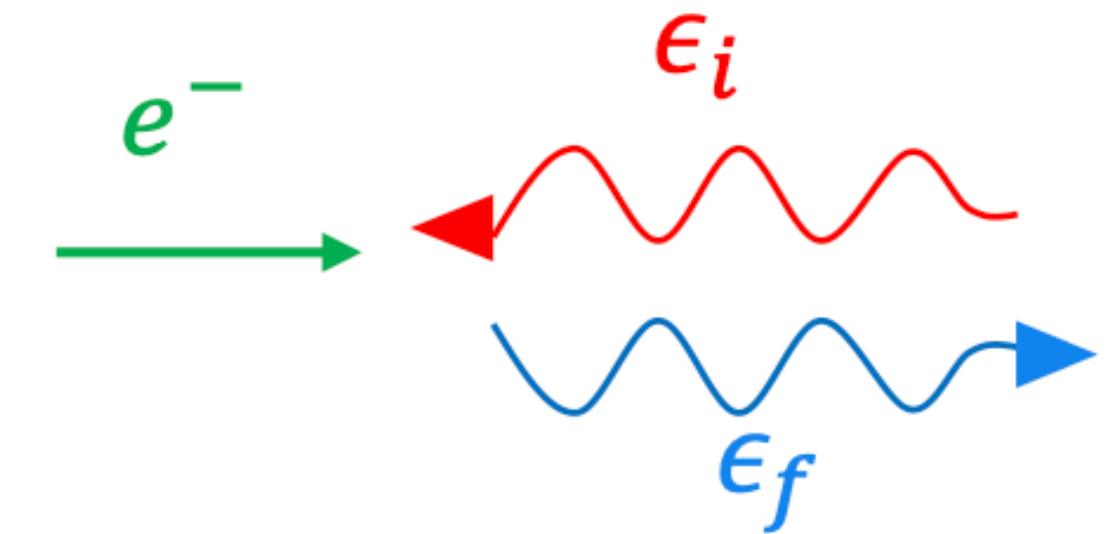


Scattering processes: **INVERSE Compton**

Example: single energy ($x_0 = h\nu_0$) photons scattered to energies x_1 by electrons with different γ and distribution of scattering angles (N.B. All spectra are normalized, just for comparison of shift in energy)



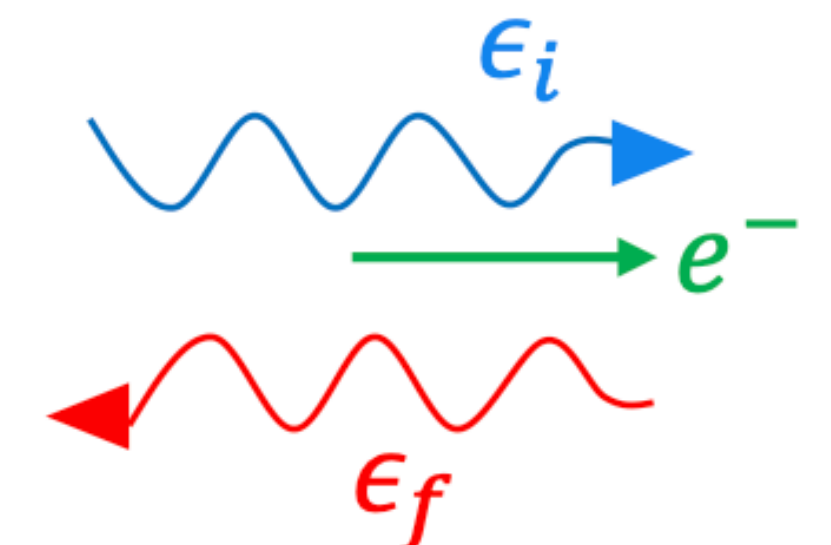
$$\epsilon_f = 4\gamma^2 \epsilon_i$$



Head on, $\theta_i = \pi$

Photon maximum energy **gain** (top)
and maximum energy **loss** (bottom)

$$\epsilon_f = \epsilon_i / 4\gamma^2$$



Tail on, $\theta_i = 0$

How the scattering angles determine the energy of the outgoing photon:

- maximum energy gained by the photon if $\theta = \pi$ and $\theta'_1 = 0$
 $\Rightarrow$ the photon is blue – shifted (head–on collision, and diffusion in the same direction of the motion of the electron)
- minimum energy for the scattered photon if $\theta = 0$ and $\theta'_1 = \pi$
 $\Rightarrow$ the photon is red – shifted (tail–on collision)

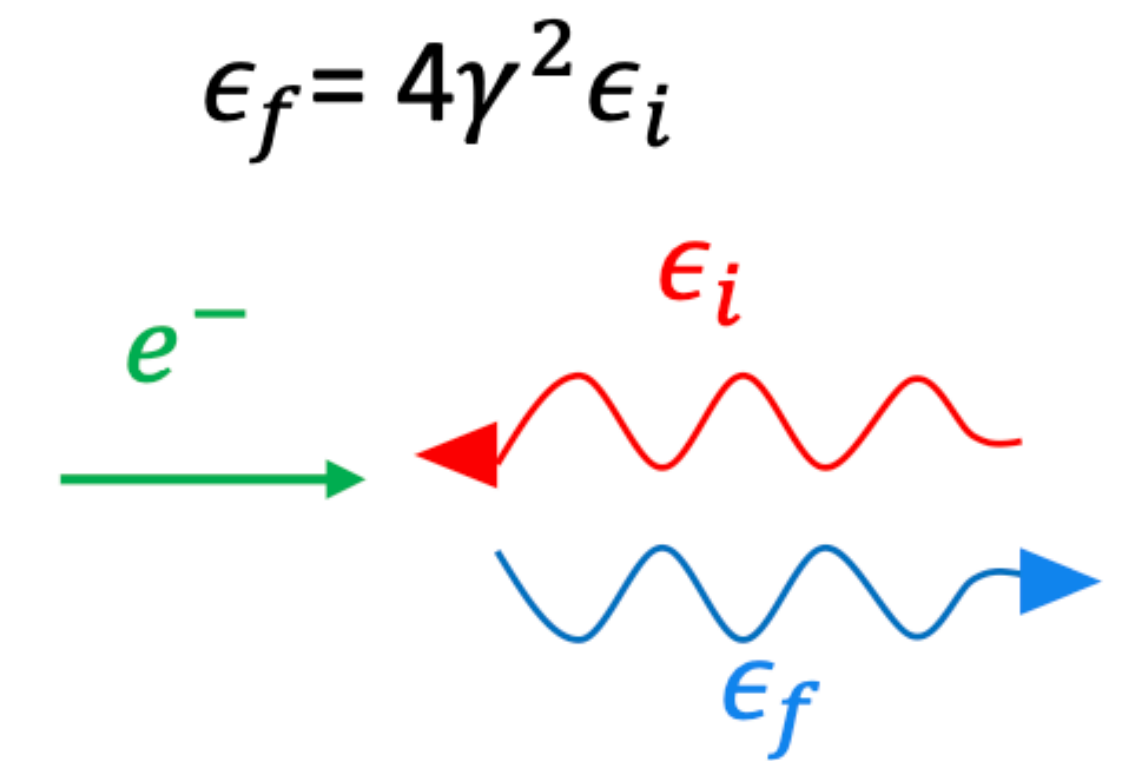
The probability of a “positive” scattering event is much larger and increases with γ

If we average over all scattering angles, the energy that can be gained is: $h\nu_f \approx \frac{4}{3} \gamma^2 h\nu_i$

implying a $\approx \gamma^2$ shift in frequency (see next slide)

The emitted energy is a Lorentz invariant, i.e. the same in both reference frames (laboratory and electron), and refers to what happens **after** the scattering (the first amplification is due to Doppler effect only). This represents the **energy lost by the electron**.

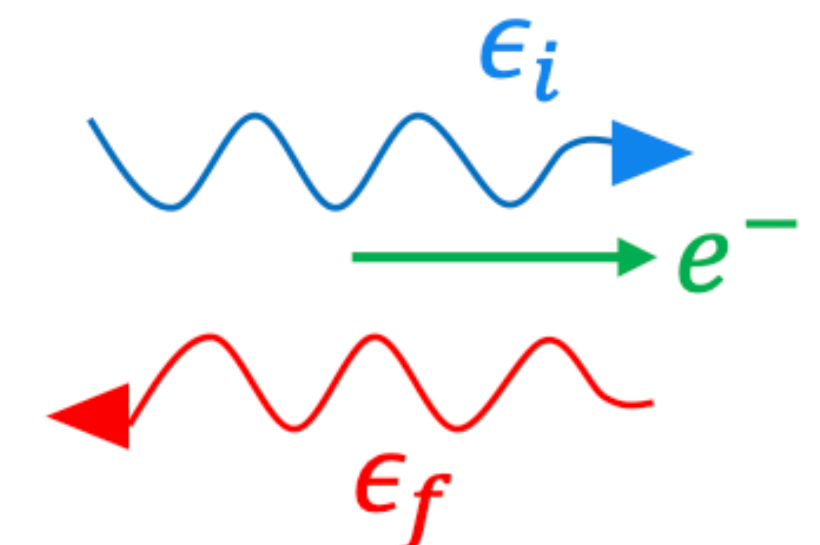
$$E' = h\nu' = h\nu \gamma (1 + \beta \cos \varphi) = E \gamma (1 + \beta \cos \varphi)$$



Head on, $\theta_i = \pi$

Photon maximum energy **gain** (top)
and maximum energy **loss** (bottom)

$$\epsilon_f = \epsilon_i / 4\gamma^2$$



Tail on, $\theta_i = 0$

Let's consider a region where there is a plasma of relativistic electrons and a radiation field:

in the electron framework, the photon energy is

$$\frac{d\epsilon}{dt} = \frac{d\epsilon'}{d\tau} = c\sigma_T \frac{\langle E_{\text{rad}}'^2 \rangle}{8\pi} = c\sigma_T \gamma^2 \langle (1 + \beta \cos \varphi)^2 \rangle \langle E_{\text{rad}}^2 \rangle$$

since the emitted energy is a Lorentz invariant. It is then possible to write

$$\langle E_{\text{rad}}^2 \rangle = U_{\text{ph}} \quad \text{and} \quad \langle (1 + \beta \cos \varphi)^2 \rangle = 1 + \frac{1}{3}\beta^2$$

Finally, the energy coming out the scattering region is

$$\frac{d\epsilon_{\text{out}}}{dt} = c\sigma_T \gamma^2 \langle (1 + \beta \cos \varphi)^2 \rangle \langle E_{\text{rad}}^2 \rangle = c\sigma_T U_{\text{ph}} \gamma^2 \left(1 + \frac{1}{3}\beta^2 \right)$$

To be compared with the energy (rate) of the seed photons. $\frac{d\epsilon_{\text{in}}}{dt} = c\sigma_T U_{\text{ph}}$

and the net effect of the Inverse Compton scattering is

$$\frac{d\epsilon_{\text{out}}}{dt} - \frac{d\epsilon_{\text{in}}}{dt} = \left(\frac{d\epsilon}{dt} \right)_{\text{ic}} = c\sigma_T U_{\text{ph}} \left[\gamma^2 \left(1 + \frac{1}{3}\beta^2 \right) - 1 \right]$$

$$\gamma^2 \left(1 + \frac{1}{3}\beta^2 \right) - 1 = \gamma^2 + \frac{1}{3}\gamma^2\beta^2 - 1$$

But

$$\begin{aligned} \gamma^2 - 1 &= \frac{1}{1 - \beta^2} - 1 = \frac{1 - 1 + \beta^2}{1 - \beta^2} = \\ &= \frac{\beta^2}{1 - \beta^2} = \beta^2 \gamma^2 \quad \text{then} \end{aligned}$$

$$\gamma^2\beta^2 + \frac{1}{3}\gamma^2\beta^2 = \frac{4}{3}\gamma^2\beta^2$$

Finally we can write the power provided (lost) by the relativistic electrons to energise the seed photons

$$\left(\frac{d\epsilon}{dt} \right)_{\text{ic}} = \frac{4}{3} c\sigma_T \gamma^2 \beta^2 U_{\text{ph}}$$

The IC losses $\left(\frac{d\epsilon}{dt}\right)_{ic} = \frac{4}{3}c\sigma_T\gamma^2\beta^2U_{ph}$ have to be added to the synchrotron

losses of the same relativistic electrons $\left(\frac{d\epsilon}{dt}\right)_{sync} = \frac{4}{3}c\sigma_T\gamma^2\beta^2U_B$

Both synchrotron and IC deplete the energy of the relativistic electrons at a rate given by

$$\left(\frac{d\epsilon}{dt}\right)_{sync+ic} = \frac{4}{3}c\sigma_T\gamma^2\beta^2[U_{ph} + U_B] = C_{sync}\epsilon^2[B^2 + 8\pi U_{rad}]$$

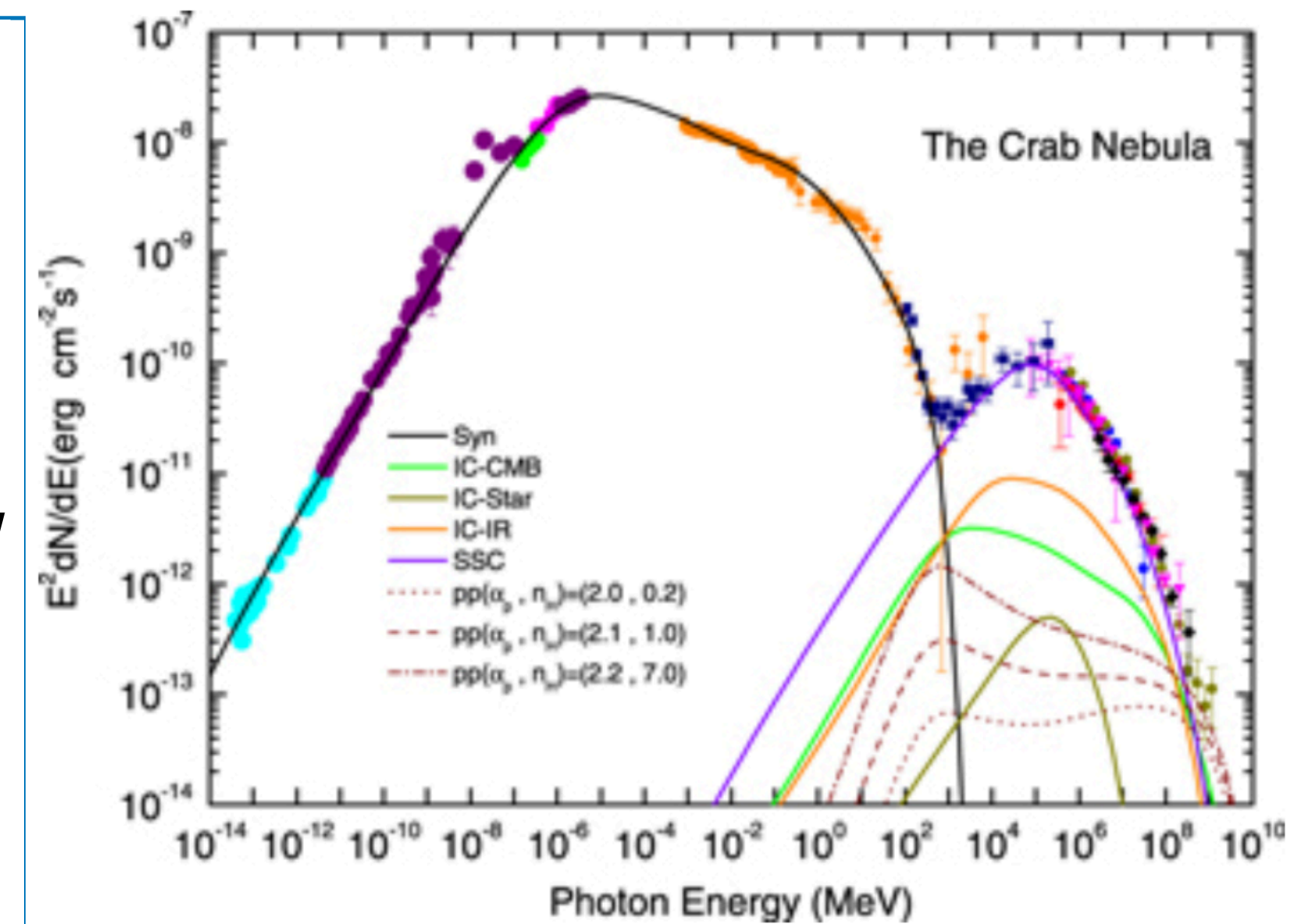
The two losses must be compared, to understand if there is a mechanism prevailing

over the other:

$$\frac{(d\epsilon/dt)_{ic}}{(d\epsilon/dt)_{sync}} = \frac{\frac{4}{3}c\sigma_T\gamma^2\beta^2U_{ph}}{\frac{4}{3}c\sigma_T\gamma^2\beta^2U_B} = \frac{U_{ph}}{U_B}$$

Consequently, the cooling time of a relativistic electron shortens significantly and becomes

$$t_{sync+ic}^* \approx \frac{1}{\epsilon^*} \frac{1}{B^2 + 8\pi U_{rad}} \approx \frac{3 \times 10^8}{(B^2/8\pi + U_{ph}) \text{ [eV cm}^{-3}\text{]}} \frac{1}{\epsilon^* \text{ [Gev]}} \text{ [yr]}$$



Energy spectrum of the Crab Nebula.
The first hump is from synchrotron,
the second comes from SSC

From Peng et al. 2022, ApJ, 926, 7

It is therefore straightforward that relativistic electron suffer at least two important radiative losses, that can be compared by means of the energy density of the magnetic field and that of the radiation field.

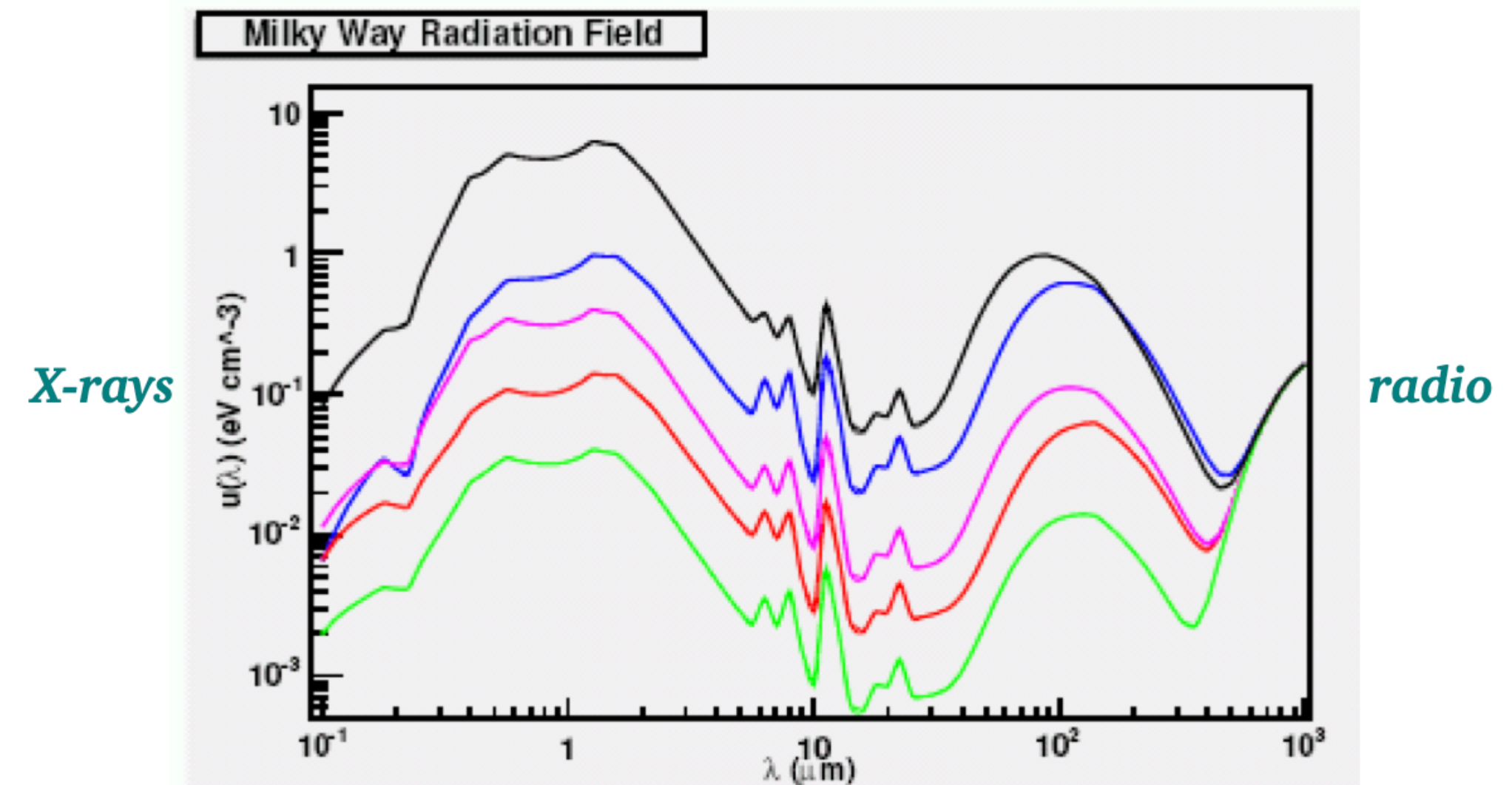
The magnetic field can be measured/estimated by various tools (turnover parameters in compact sources, "equipartition", etc.). What about U_{ph} ?

From
$$\frac{(d\epsilon/dt)_{ic}}{(d\epsilon/dt)_{sync}} = \frac{U_{ph}}{U_B}$$

assuming a power-law distribution of relativistic electrons, $(d\epsilon/dt)_{ic}$ provides the X-ray luminosity (that can be measured!), while $(d\epsilon/dt)_{sync}$ determines the radio luminosity. If we know U_{ph} , then we can determine U_B .

If U_{ph} is from the CMB, we know exactly its value $[0.26 \text{ eV cm}^{-3}]$

Evaluation of U_{rad} : as taken in a number of selected locations in/around the Milky Way.



Spatial variation of the total radiation field throughout the Galaxy.

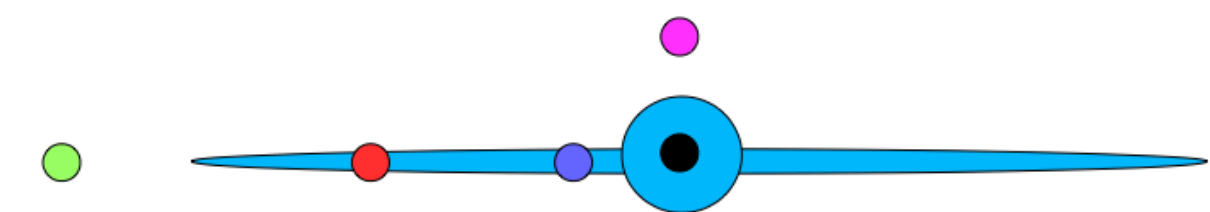
Black line : total radiation field for GC.

Magenta line : total radiation field for $R=0$, $z=5$ kpc,

Blue line : total radiation field for $R=4$, $z=0$ kpc,

Red line : total radiation field for $R=12$, $z=0$ kpc,

Green line : total radiation field for $R=20$, $z=0$ kpc.



This diagram will be discussed a bit further when considering the ISM

- Below 1 mm, all the profiles converge to the CMB.

Note that being the energy density, the number of seed photons is obtained dividing the values by the energy of the photons. There are large quantities of low energy photons, and a small number of high energy ones. At large distances from galaxies (e.g. 100 kpc) the seed photons are this of the CMB radiation.

Worked example:

In the radio galaxy Her A, the radio lobes are far away from the host galaxy. Therefore the radiation field for the IC is provided by the CMB radiation, and the IC losses can be predicted, and compared to those from synchrotron.

Often the IC losses are compared to those from synchrotron, using an "equivalent magnetic field" defined as

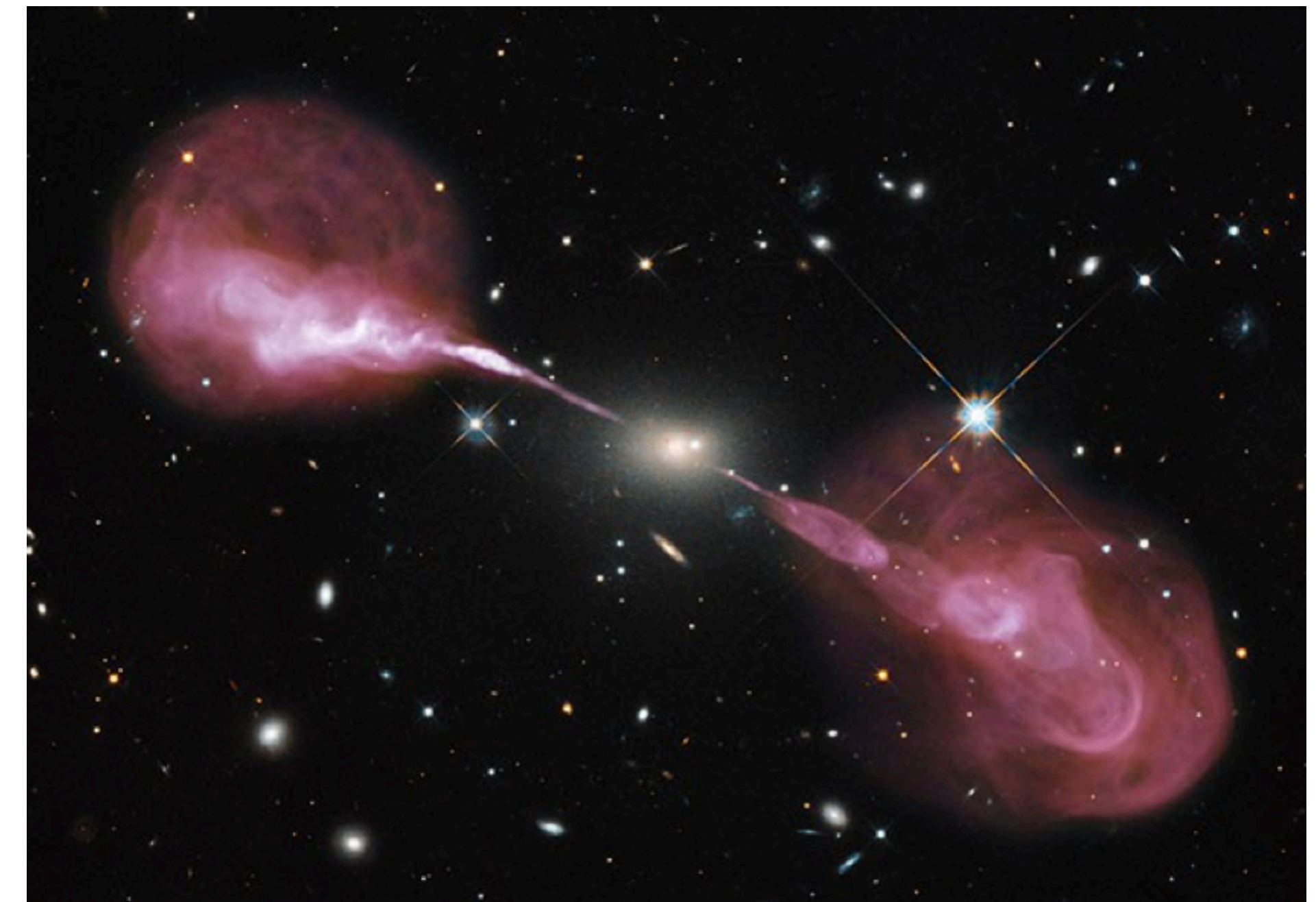
$$H_{\text{CMB}} = 3.25 (1+z)^2 \mu\text{G}$$

It is such that $H_{\text{CMB}}^2 / 8 \pi = 0.26 \text{ eV cm}^{-3}$

Note that it is redshift dependent! At $z=1$, H_{CMB} is 4 times stronger

For Her A, we have $H_{\text{CMB}} = 3.25 (1+0.155)^2 \mu\text{G} = 4.34 \mu\text{G}$

If the local magnetic field is larger than $4.34 \mu\text{G}$, synchrotron losses prevail, the contrary happens if the field is weaker. Remember that what we observe is the current status, and does not take into consideration the past history of the field and of the radio source.



Hercules A (3C348) @ $z=0.155$ – JVLA radio image at 1.4 GHz

Note that the brightness is not constant, but regions where more radio photons are produced it is likely that the field is higher than in regions with lower surface brightness.

The study of the magnetic field in various regions helps in the interpretation of the formation and evolution of the radio source.

In compact radio sources U_{ph} should take into consideration also the contribution of the synchrotron photons

Let's consider a spherical bubble uniformly filled by magnetised relativistic plasma isotropically radiating, with radius R , at a distance D . The integrated radio luminosity is

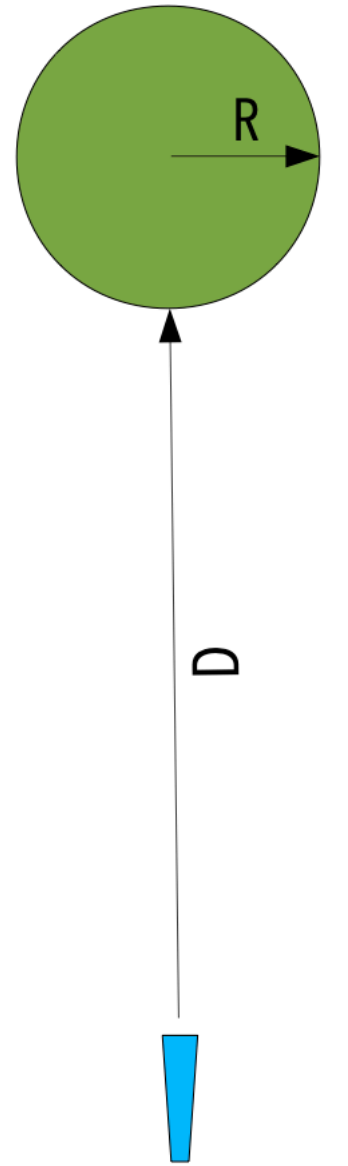
$$L_{sync} = 4\pi D^2 \int S(\nu) d\nu \approx 4\pi D^2 S_{max} \nu_c f(\alpha)$$

Where we have assumed a power-law radio spectrum with a characteristic flux density S_{max} up to a frequency ν_c , and $f(\alpha)$ is a factor ≈ 1 taking into account the spectral shape/slope.

Assuming that the photons diffuse isotropically out of the surface of the bubble at a speed $c/\sqrt{3}$, then the radiation field is $L_{sync} = U_{ph} \cdot 4\pi R^2 \frac{c}{\sqrt{3}}$

Then $U_{ph} = \frac{\sqrt{3} \cdot L_{sync}}{4\pi c R^2}$ is the radiation field for SSC.

$$\frac{L_{IC}}{L_{sync}} = \frac{U_{ph}}{B^2/8\pi} = \frac{2\sqrt{3} \cdot L_{sync}}{cB^2 R^2} \approx \frac{2\sqrt{3} \cdot 4\pi D^2 S_{max} \nu_c f(\alpha)}{cB^2 R^2} \approx \frac{S_{max} \nu_c f(\alpha)}{cB^2 \theta^2}$$



Note that the brightness is not constant, but regions where more radio photons are produced it is likely that the field is higher than in regions with lower surface brightness.

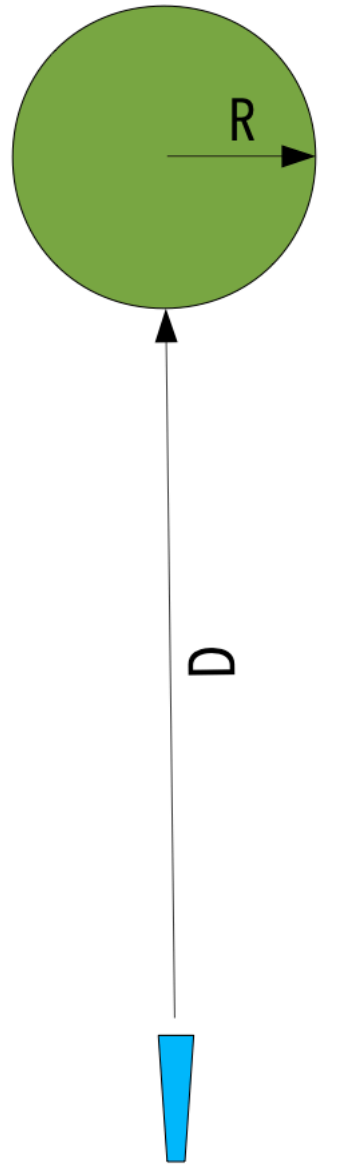
The study of the magnetic field in various

$$\approx \frac{S_{\max} \nu_c f(\alpha)}{c B^2 \theta^2} = \frac{S_{\max}}{\theta^2 \nu_{\max}^2} \frac{\nu_{\max}^2}{B^2} \frac{\nu_c}{c^2} f(\alpha) = \frac{S_{\max}}{\theta^2 \nu_{\max}} \left(\frac{S_{\max}}{\theta^2 \nu_{\max}} \right)^4 \frac{\nu_c}{c} f(\alpha)$$

That can be expressed in terms of brightness temperature in the R-J regime for a BB emitter:

$$\frac{L_{\text{IC}}}{L_{\text{sync}}} = \left(\frac{T_{\text{Bmax}}}{10^{12} \text{ K}} \right)^5 \frac{\nu_c}{\text{GHz}} f(\alpha)$$

At GHz frequencies, for $T_{\text{Bmax}} > 10^{12} \text{ K}$, the IC (i.e. X-ray Luminosity) would dominate over the radio emission. For some radio sources, T_{Bmax} in some compact regions exceeds the aforementioned limit, without reaching the predicted X-ray luminosity. This is interpreted as due to an incorrect measure of the intrinsic source radio luminosity due to relativistic beaming associated to bulk motion of relativistic plasma bubbles moving at very high velocities along the LoS. Such sources belong to the class of blazars.



VLBI Image of 3c279

Compton & Inverse Compton are the two limiting cases where there is energy transfer "photon $\Leftrightarrow$ electron" and there are astrophysical bodies in which the two processes are mixed together

We want to evaluate if there is a substantial change in the spectrum of the seed photons in case of multiple scattering with hot (but non-relativistic) electrons in thermal equilibrium.

- Let's consider a large volume (uniformly) full of thermal plasma along with a sea of seed photons

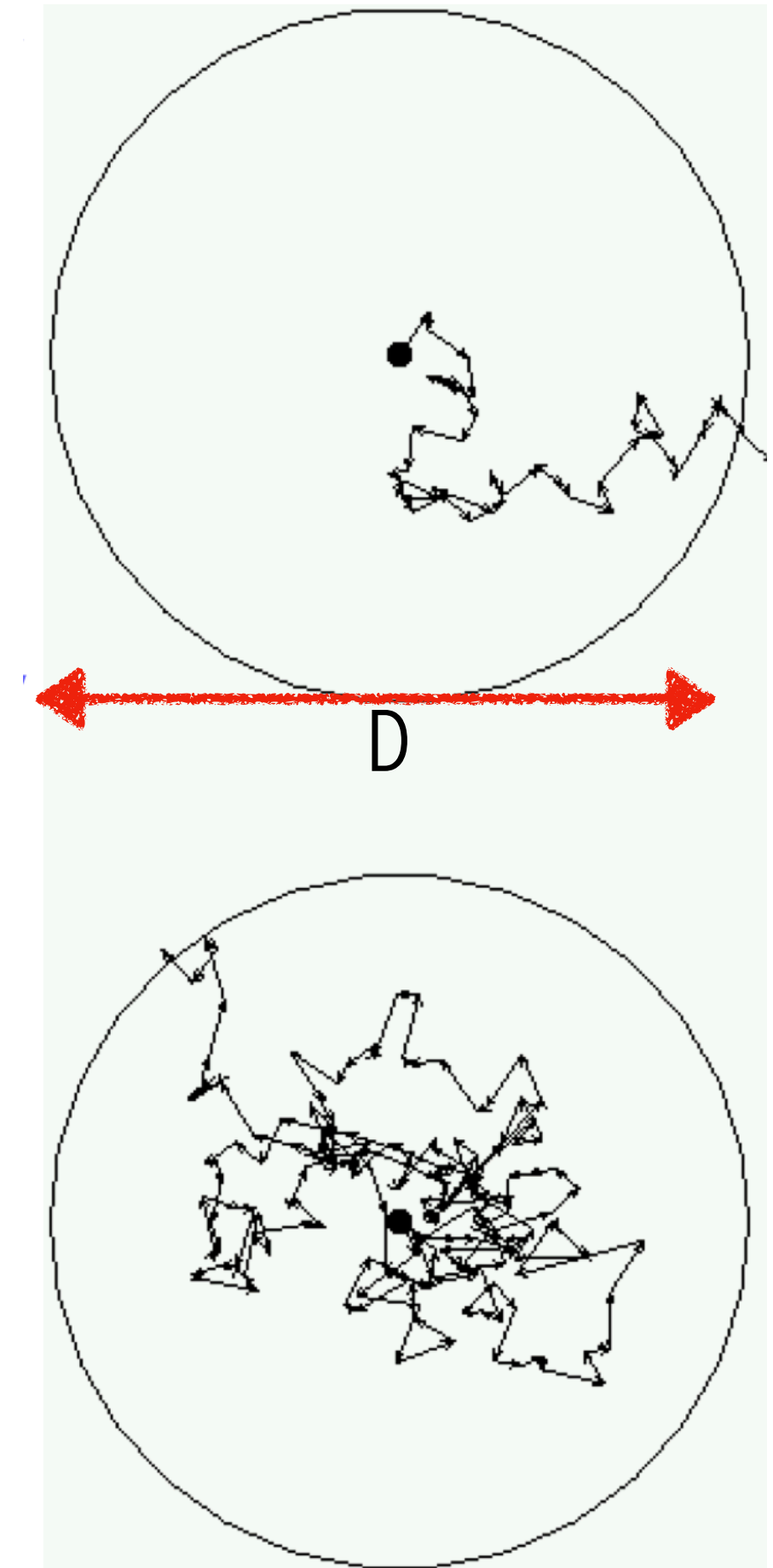
$\Rightarrow$ and let's consider the process from the point of view of the photons

** There are interactions where energy is transferred from photons to electrons (**C scattering**)

$$\Delta h\nu = h(\nu_i - \nu_f) = h\left(\frac{c}{\lambda_i} - \frac{c}{\lambda_f}\right) = hc\left(\frac{\lambda_i - \lambda_f}{\lambda_i\lambda_f}\right) = \frac{hc}{\lambda_i\lambda_f} \frac{h}{m_e c} (1 - \cos \theta) = \frac{h^2 \nu_i \nu_f}{m_e c^2} (1 - \cos \theta)$$

Then the average energy loss for photons via Compton $\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{ph} \approx - \frac{h\nu}{m_e c^2}$

it has been taken negative to show the energy balance from the photon point of view



(Non astrophysical) Example of a multiple scattering within a spherical volume (D is the diameter). Changes in the photon trajectory are signatures of scattering processes.

**** IC interactions** transfer energy from the electrons to the photons (positive energy balance).

Let's consider a thermal distribution $\frac{1}{2}m_e\langle v^2 \rangle = \frac{3}{2}kT \rightarrow v^2 \approx \frac{3kT}{m_e}$,

then since the IC energy transfer is $\left(\frac{d\epsilon}{dt}\right)_{ic} \sim \frac{4}{3}\gamma^2\beta^2$

$$\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{el} \approx \frac{4}{3} \left(\frac{v}{c} \right)^2 = \frac{4}{3c^2} \frac{3kT}{m_e} = \frac{4kT}{m_e c^2}$$

Therefore the net energy transfer (as seen by the photons) is $\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle = \frac{(4kT - h\nu)}{m_e c^2}$

- If $4kT \ll h\nu$ photons loose energy and electrons (i.e. the plasma) heats up
- If $4kT \gg h\nu$ photons gain energy and electrons (i.e. the plasma) cools down

There is no net energy transfer when $4kT = h\nu$.

The the first two cases, the process with end when electrons and photons reach equilibrium (saturation) and the photon distribution does not change anymore.

Let's consider a region where $4kT \gg h\nu$

Electrons loose energy at a rate $\left\langle \frac{\Delta \varepsilon}{\varepsilon} \right\rangle \simeq \frac{4kT}{m_e c^2}$

The total number of scattering defines the **opacity** of the medium: $\tau_e = n_e \sigma_T D = \frac{D}{l_{\text{mfp}}}$

where there is the definition of the **mean free path**. $l_{\text{mfp}} = \frac{1}{n_e \sigma_T}$.

The **total number of collisions** in the system is $N = \left(\frac{D}{l_{\text{mfp}}} \right)^2 = \tau^2$

Then, the total energy gain $\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{\text{tot}} \simeq \frac{4kT}{m_e c^2} \cdot N = \frac{4kT}{m_e c^2} \cdot N dt$

Integrating in $d\nu$ and dt we get $\ln h\nu(t) - \ln h\nu_0 = \frac{4kT}{m_e c^2} \cdot Nt$

Where N =number of collision in unit time, then

$$\rightarrow h\nu(t) = h\nu_0 e^{(4kT/m_e c^2) \cdot Nt} = h\nu_0 e^{(4kT/m_e c^2) \cdot N} = h\nu_0 e^{4y}$$

In the former equation, we have introduced the "Compton y-parameter"

$$y = \frac{kT}{m_e c^2} \cdot N = \frac{kT n_e^2 \sigma_T^2 D^2}{m_e c^2}$$

whose value determines the level of "Comptonisation".

The spectrum of the seed photons is substantially modified when $\nu_f = \nu(t) > \nu_0$, namely

$$y \geq \frac{1}{4} \rightarrow \frac{kT}{m_e c^2} \cdot N = \frac{kT n_e^2 \sigma_T^2 D^2}{m_e c^2} \geq 1$$

When comptonisation is strong, the energy of the scattered photons saturate (when equilibrium is achieved) at

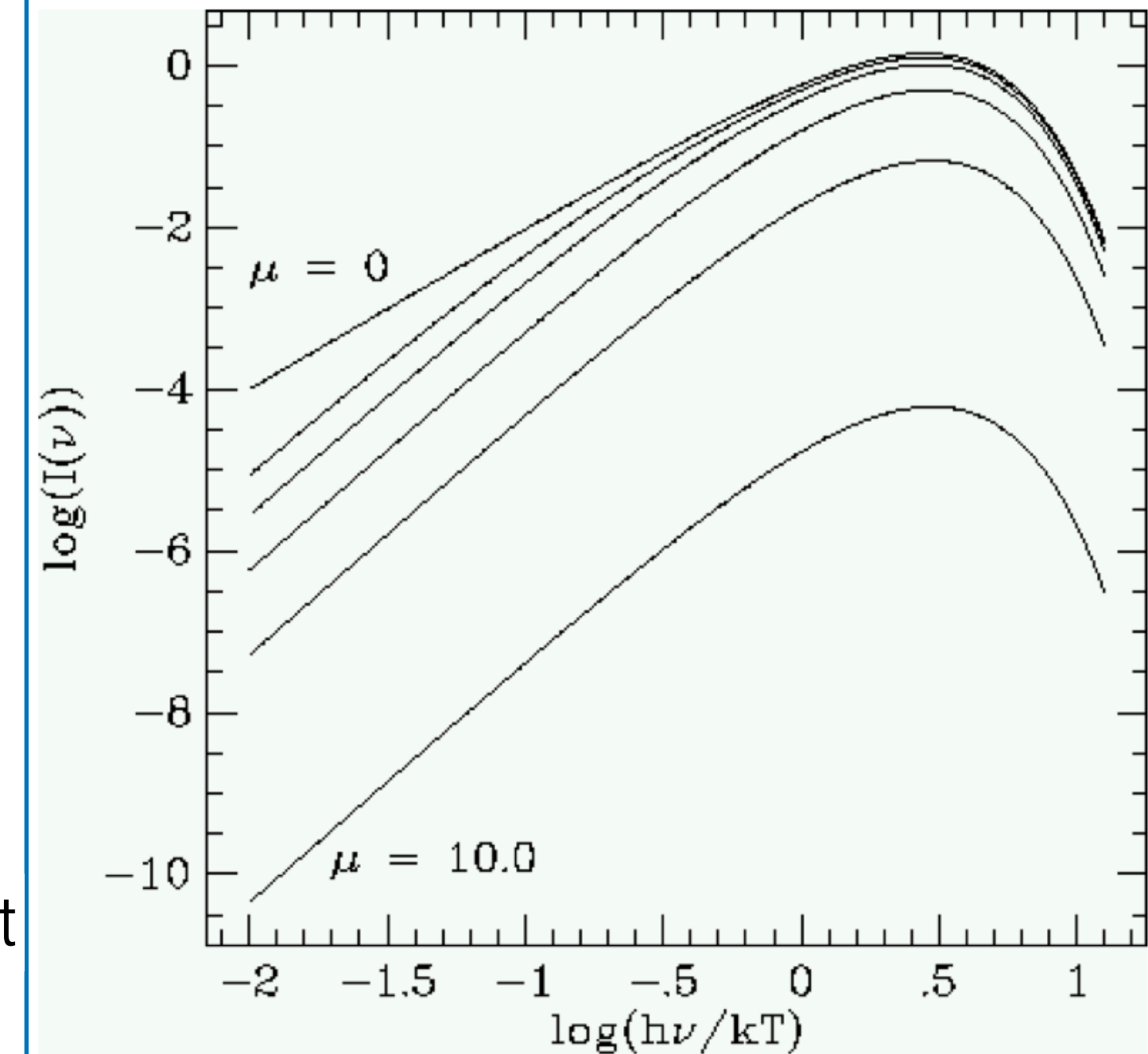
$$h\nu_f = 4kT \quad \text{i.e.} \quad y = \frac{1}{4} \ln\left(\frac{4kT}{h\nu_0}\right)$$

which, in turn, sets a condition on the opacity $\tau_e = \left[\ln\left(\frac{4kT}{h\nu_0}\right) \frac{m_e c^2}{4kT} \right]$

when the modified photon spectrum follows the Bose-Einstein distribution

$$u(\nu)d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{(h\nu/kT)+\mu} - 1}$$

In case $\frac{h\nu}{kT} + \mu \gg 1 \rightarrow u(\nu)d\nu = \frac{8\pi h \nu^3}{c^3} e^{-(h\nu/kT)} e^{-\mu}$ which is the Wien Law modified by $e^{-\mu}$



1. μ is termed "chemical potential"
2. The probability that an atom will emit a photon into a particular final state is increased by a factor of $(n+1)$ if there are already n photons in that state. Photons are spin -1 bosons (Feynman+1965)

The exact description of the photon scattering (i.e. find the observed spectrum originated by various values of the y -parameter) is better obtained in the phase-space through the Kompaneets' equation:

If we define $n = \frac{u(\nu)c^3}{8\pi h\nu^3}$ and $x = \frac{h\nu}{kT}$ $\frac{\partial n}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(n + n^2 + \frac{\partial n}{\partial x} \right) \right]$

In particular, n stands for the recoil effect, n^2 is for induced/stimulated emission and $\frac{\partial n}{\partial x}$ represents the outcome of the Doppler effect.

The emission spectrum comes out as a power-law with slope $3+m$, where $m = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{1}{y}}$

Comptonisation: occurs in various [small] celestial bodies/regions like accretions disks (i.e. relatively high free electron densities) in

- X-ray binaries (XRBs)
- micro quasars
- Active Galactic Nuclei (AGNs), etc.

It is the basis of the Sunayev – Zeldovich (S-Z) effect in clusters of galaxies, where the volume involved is very large and the electron densities are quite low.



Alexander Kompaneets

Pills of (thermal) SZ effect.

Galaxy clusters have hot thermal electrons (10^8 °K) and the high energy tail of the distribution can make IC interactions with the CMB photons.

The net energy transfer is modest, given that we are in a non relativistic regime.

In a typical galaxy cluster $n_e \sim 10^{-3} \text{ cm}^{-3}$, the diameter can be chosen $D \sim 5 \text{ Mpc}$

The opacity to the SZ effect is

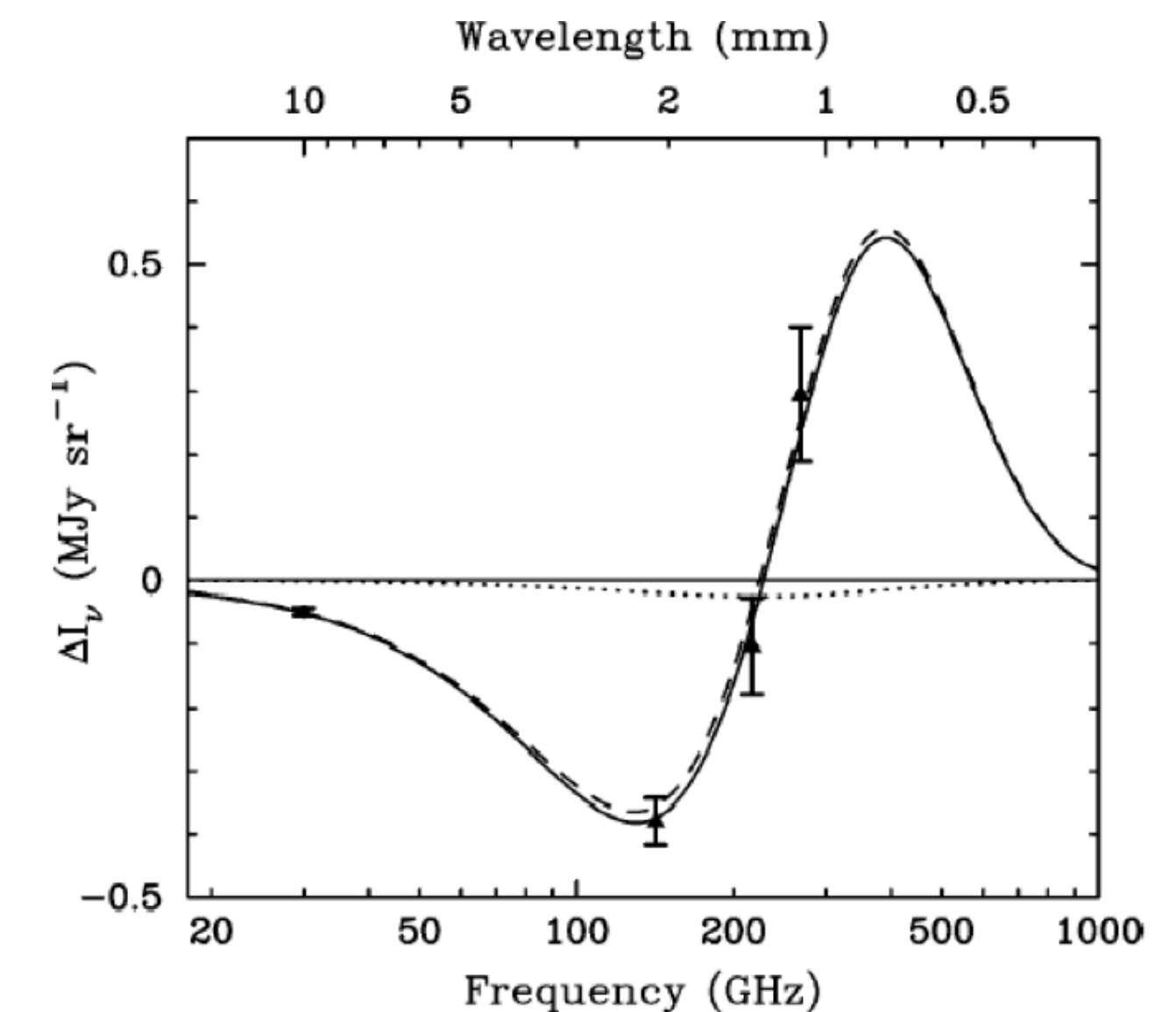
$$\tau_{\text{SZ}} = n_e \sigma_T D = 10^{-3} \text{ cm}^{-3} \cdot 6.65 \times 10^{-25} \text{ cm}^2 \cdot 5 \times 10^6 \times 3.086 \times 10^{18} \text{ cm} = 10^{-2}$$

Therefore a CMB photon has a probability of $\sim 1\%$ of interacting with a high energy electron of the Intra Cluster Medium (ICM).

Given the large number of electrons and photons, the number of processes is large, despite the energy gain is rather modest. The diagram on the right shows the amplitude of the SZE.

Consequently the BB spectrum of the CMB is modified, with a slight shift to higher energies

A modern review on this process can be found in Carlstrom + 2002 ARAA, vol 40, p.643-680

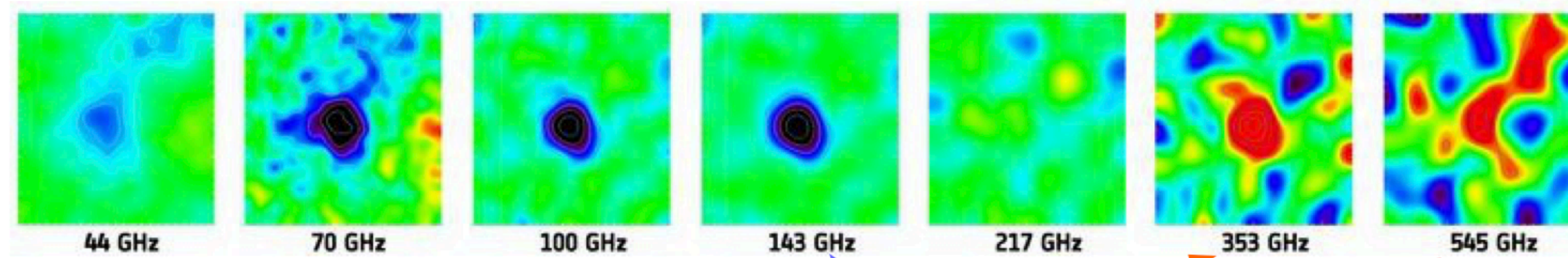
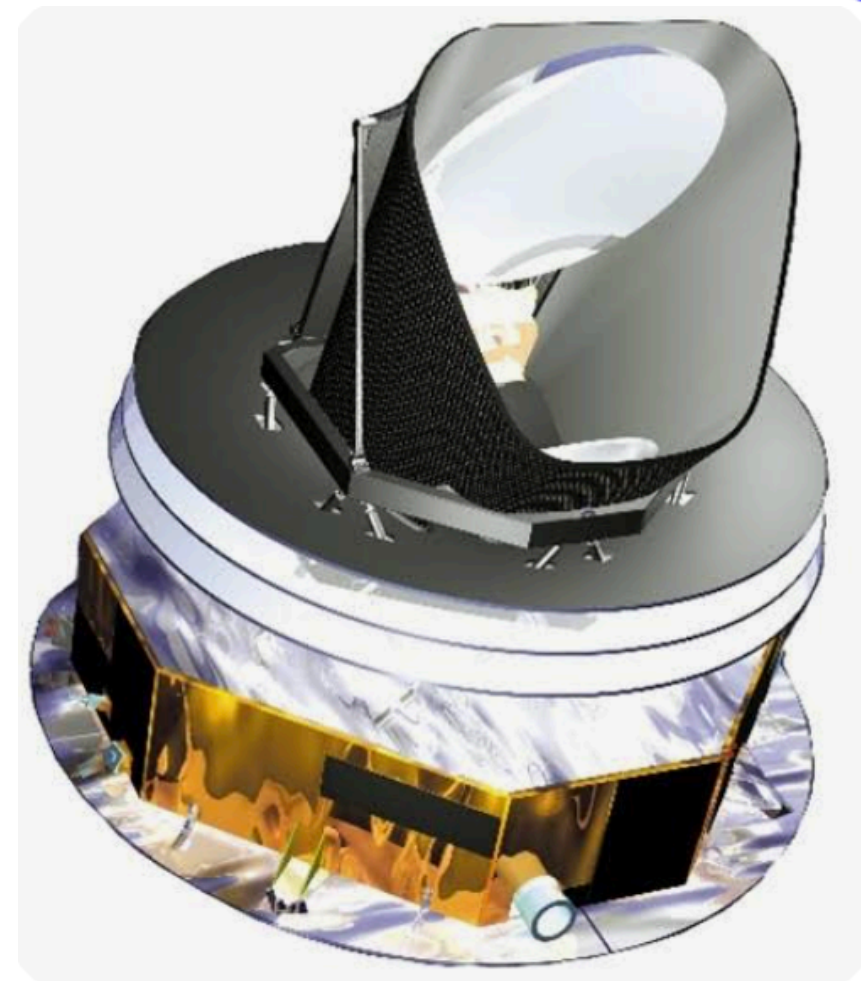


Beyond the thermal SZ, there is a smaller contribution due to the relative (peculiar) motion of the cluster in the Hubble flow. This is known as **kinetic** SZ, whose amplitude is much smaller than the thermal one.

Example: How the thermal SZ signal is measured in a cluster of galaxies.

Abell 2319 ($z=0.055$, distance ~ 250 Mpc) as seen by the Planck Satellite in the microwaves, over a range of frequencies.

↓ Planck measures the sky temperature of the CMB radiation. Green represents regions where is BB curve is the expected one. Blue show lower temperatures, red higher temperatures. The cluster region appears cooler below the cross-over frequency and hotter above it.



The Planck satellite

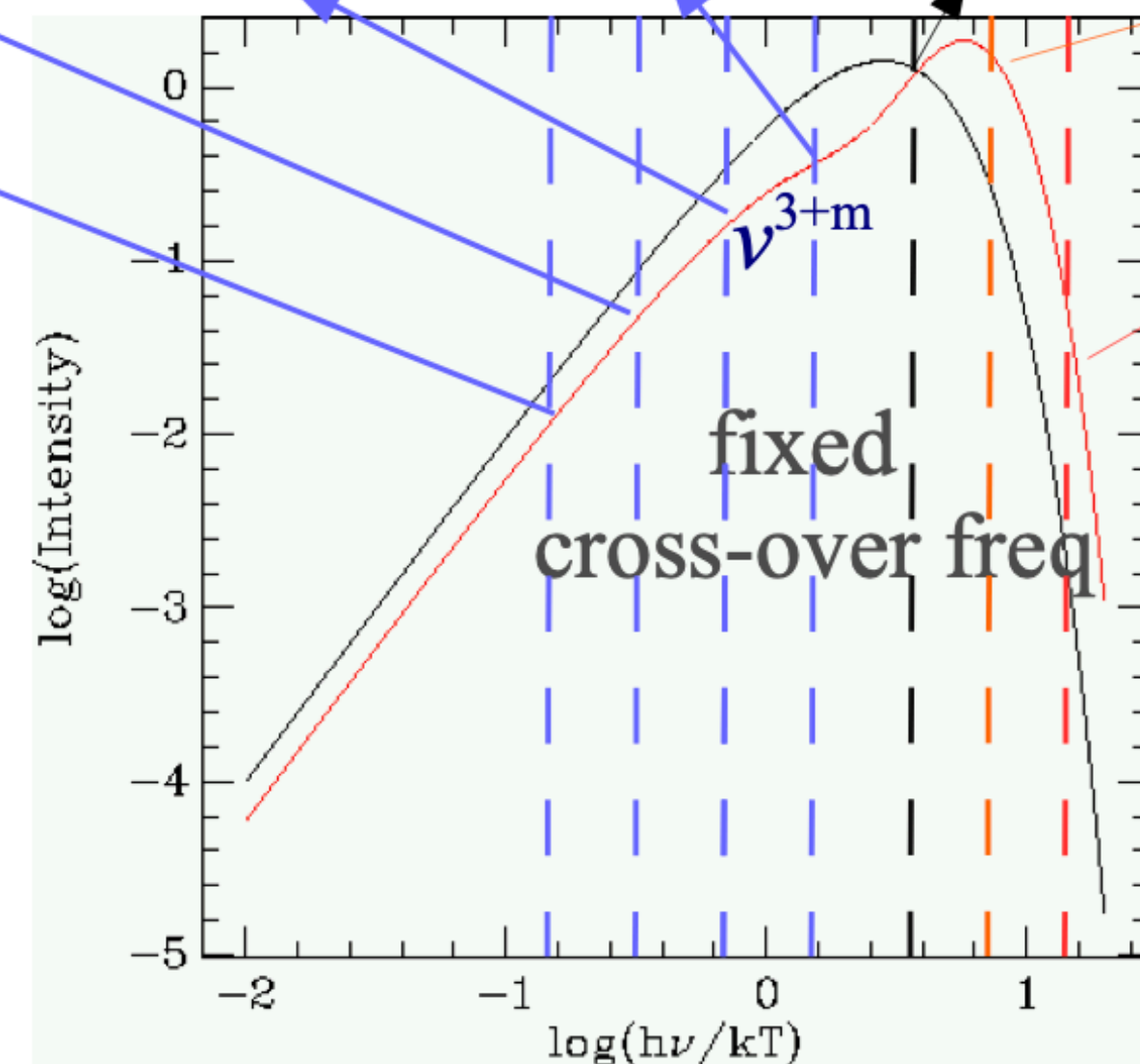
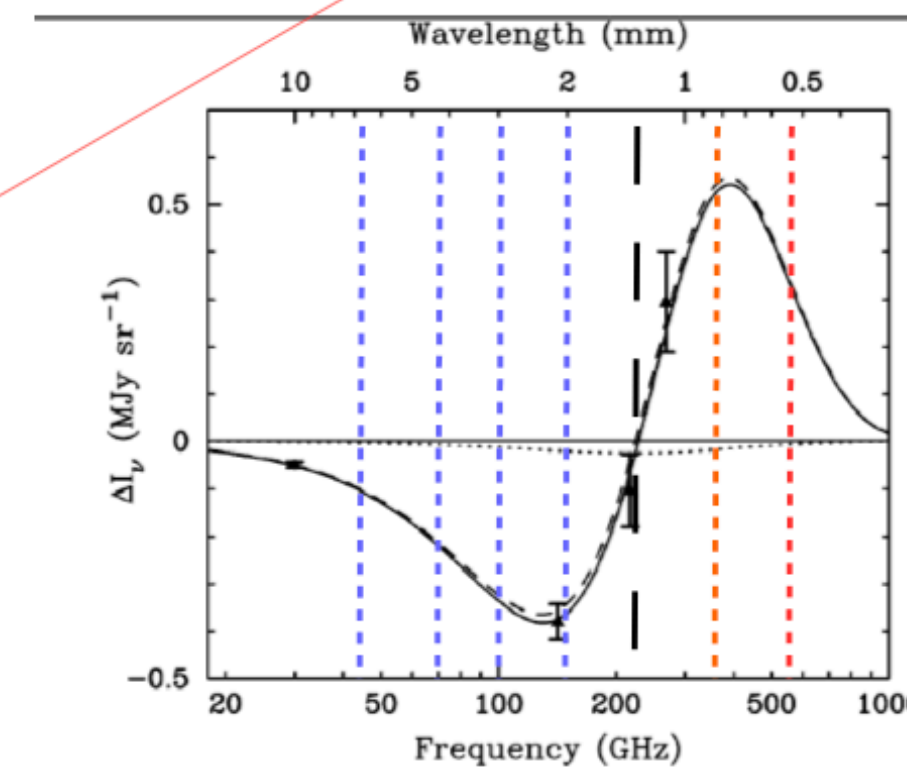
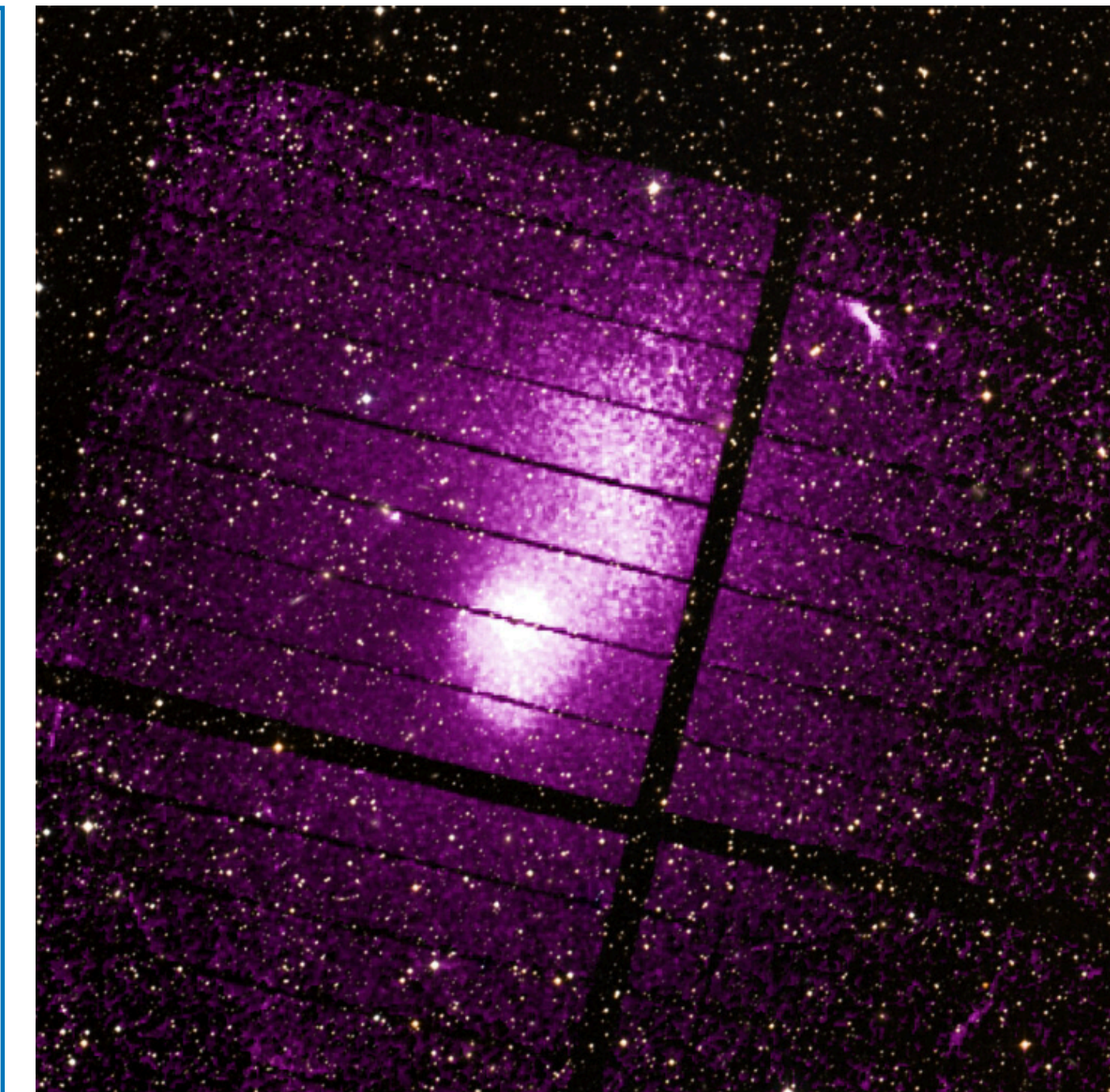


Figure 4 The measured SZE spectrum of Abell 2163. The data point at 30 GHz is from the Berkeley-Illinois-Maryland-Association (BIMA) array (LaRoque et al. 2002), at 140 GHz it is the weighted average of Diabolo and SuZIE measurements (Desert et al. 1998, Holzapfel et al. 1997) (filled squares), and at 218 GHz and 270 GHz from SuZIE (Holzapfel et al. 1997) (filled triangles). The best fit thermal and kinetic SZE spectra are shown by the dashed and dotted lines, respectively, with the spectra of the combined effect shown by the solid line. The limits on the Compton y -parameter and the peculiar velocity are $y_0 = 3.56^{+0.41+0.27}_{-0.59-0.41-0.19} \times 10^{-4}$ and $v_p = 410^{+1038+460}_{-559-140}$ km s $^{-1}$, respectively, with statistical followed by systematic uncertainties at 68% confidence (Holzapfel et al. 1997, LaRoque et al. 2002).



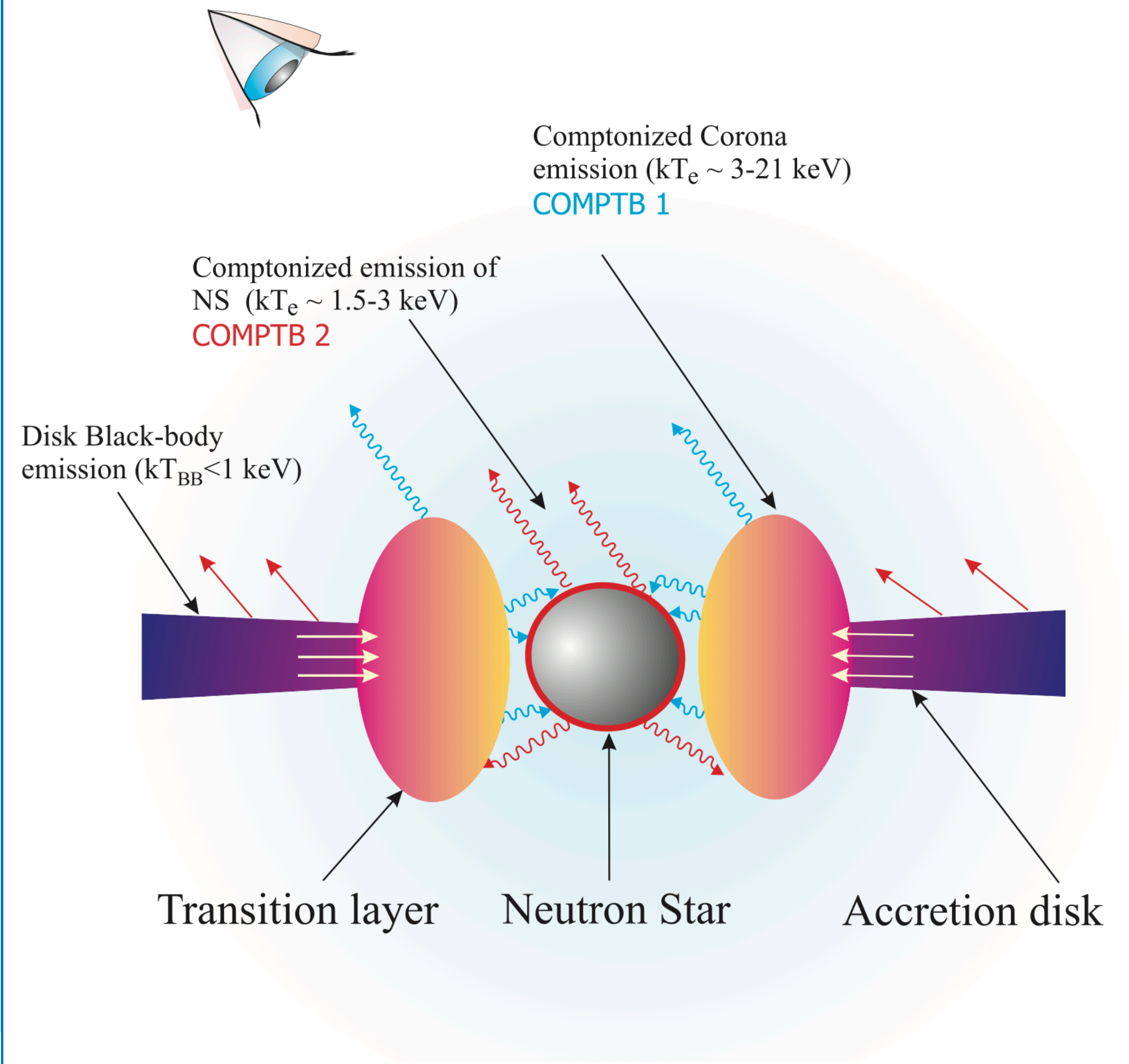
Abell 2319 as seen by the satellite XRISM overlaid to an optical image (galaxies in the centre are not visible due to the high X-ray brightness). The field is a few Mpc across (at the redshift of the cluster)

→ Remind that the amplitude of the SZ effect is independent of redshift. In fact, intensities scale as 1 over distance square, i.e. z^2 , but the energy density of the CMB increases at the same rate, leaving the intensity of the SZ signal unaltered.

Another **example** of Comptonisation in an accreting collapsed body (in this case a neutron star – NS –, similar ingredients in Active Galactic Nuclei)

A suggested geometry of GX 340+0. Disk and NS soft photons are upscattered off the hotter plasma of the Transition Layer (TL) located between the accretion disk and NS surface. Some fraction of these photons is seen directly by the Earth observer. Red and blue photon trajectories correspond to soft and hard (upscattered) photons, respectively. In our model, two Comptonisation components are considered: the first one (**COMPTB 1**), with "seed" (disk) photon temperature $T_{s1} \leq 1$ keV and the CC electron temperature kT_e varies from 3 keV to 21 keV. The second Comptonised component (**COMPTB 2** with $T_{s2} \sim 1.5$ keV) and kT_e are related to the NS surface and its inner part of the TL (boundary) layer, respectively.

On the right, Fig 1 in Seifina, Titarchuk & Filippo Frontera, 2013



Here the scale is very small, consider that the NS is ~ 10 km in radius