Astrofisica dei corpi collassati

Stellar systems:

1. **White Dwarf** as main end–product of stellar evolution
   isolated
   in double (multiple) system
   magnetic WD
2. **Neutron Star** as end–product of stellar evolution / as accretion onto a WD
   pulsar
3. **Black Hole** as end–product of stellar evolution / as accretion onto a NS

Galactic systems:

4. **Super Massive Black Hole** at the centre of Galaxies
5. **GRB**

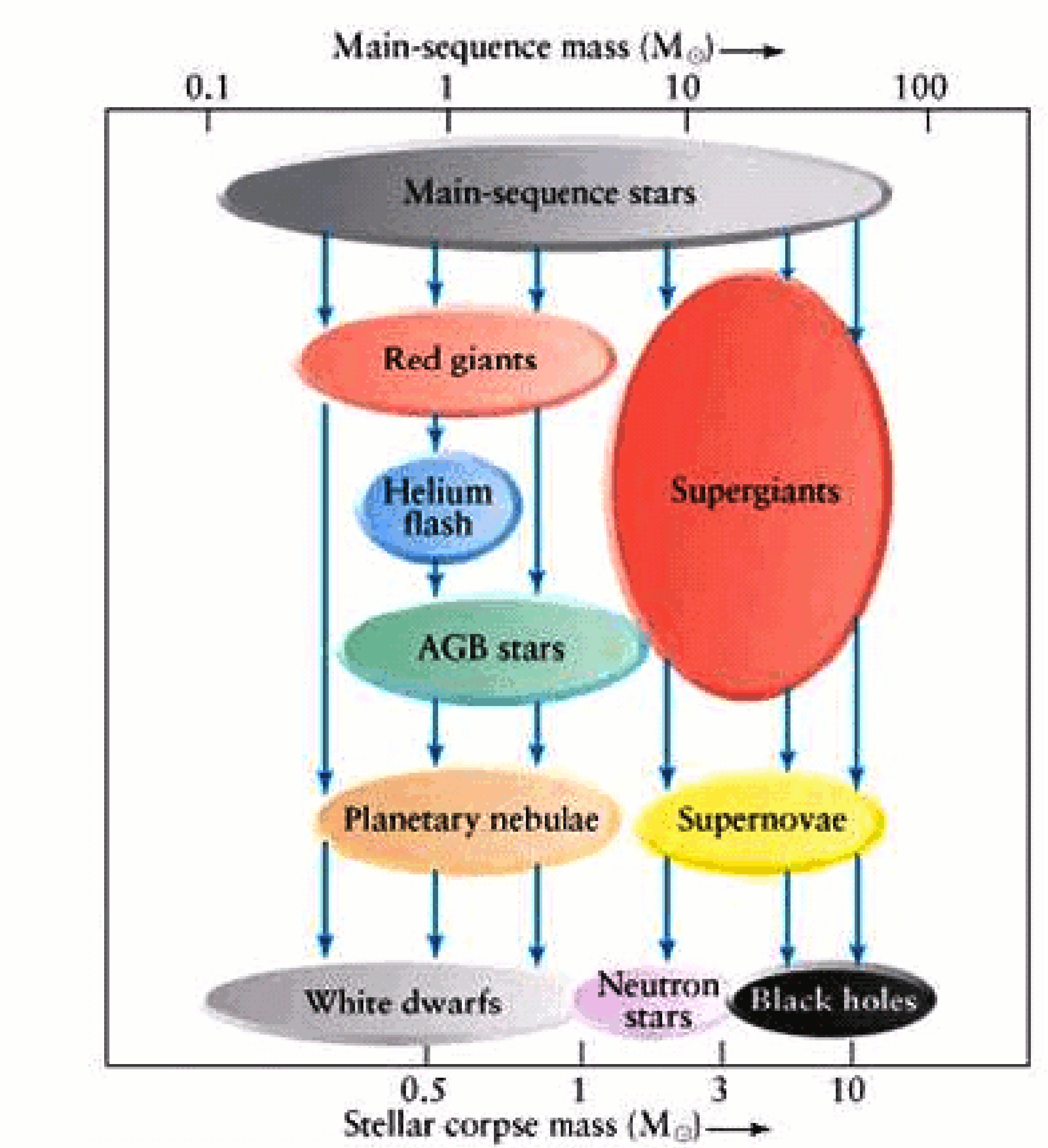
1. White Dwarf Stars

references/textbooks

M. Camenzind – ***Compact Objects in Astrophysics*** – *A&A Library, Springer (cap. 5)*

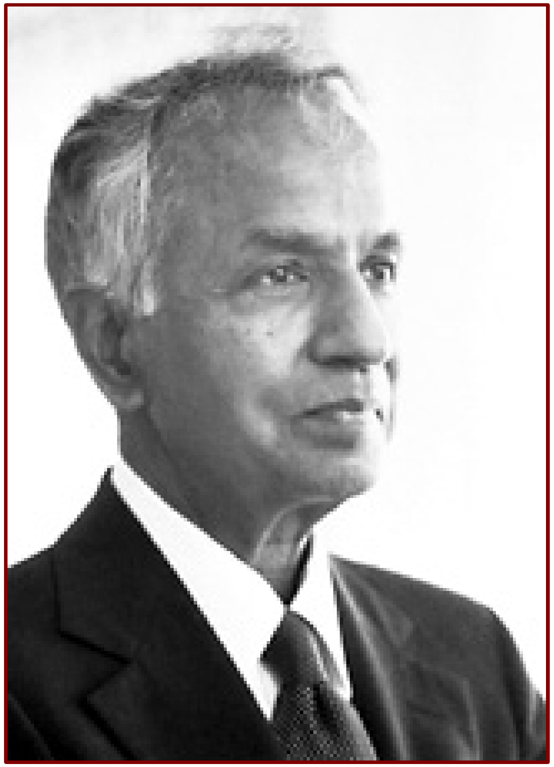
Liebert, 1980, ARA&A, vol. 18, p. 363-398
Weidemann V. 1990, ARA&A, vol. 28, p. 103-137
D'Antona F., Mazzitelli I., A 1990, ARA&A, vol. 28, p.139-181
Hansen B.M.S, Liebert J. 2003, ARA&A, vol. 41, p. 465-515

CO taxonomy



Main-sequence mass (M☉) →
0.1
1
10
100
Main-sequence stars
Red giants
Helium flash
AGB stars
Supergiants
Planetary nebulae
Supernovae
White dwarfs
Neutron stars
Black holes
0.5
1
3
10
Stellar corpse mass (M☉) →

Subrahmanyan Chandrasekhar *(1910-1995)*



1930 *student researcher at Cambridge (Fowler)*

1933 *1-yr at the Institut för Teoretisk Fysik in Copenhagen (@ P. A. M. Dirac – Nobel Prize right in 1933!)*

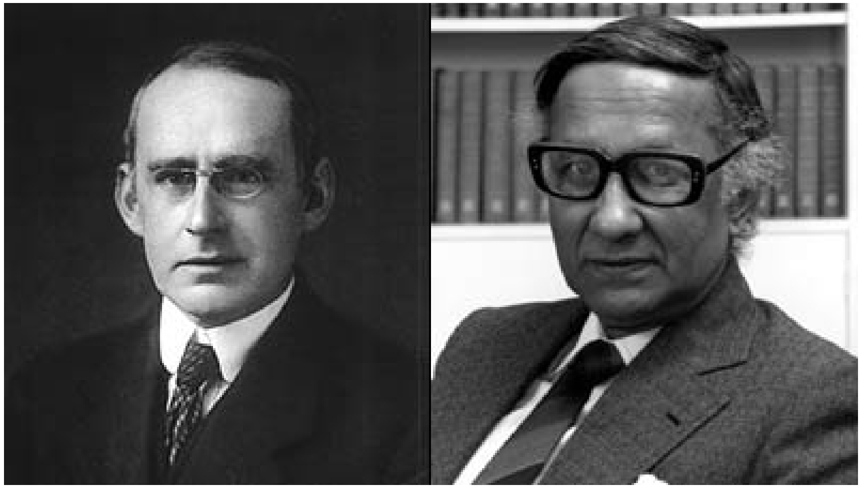
1933 *PhD degree at Cambridge*

1933-37 *Fellowship at Trinity College Cambdrige*

1937 *– University of Chicago*

1983 *Nobel Prize*

Collapsed stellar objects: electron degeneracy pressure balances gravitational forces



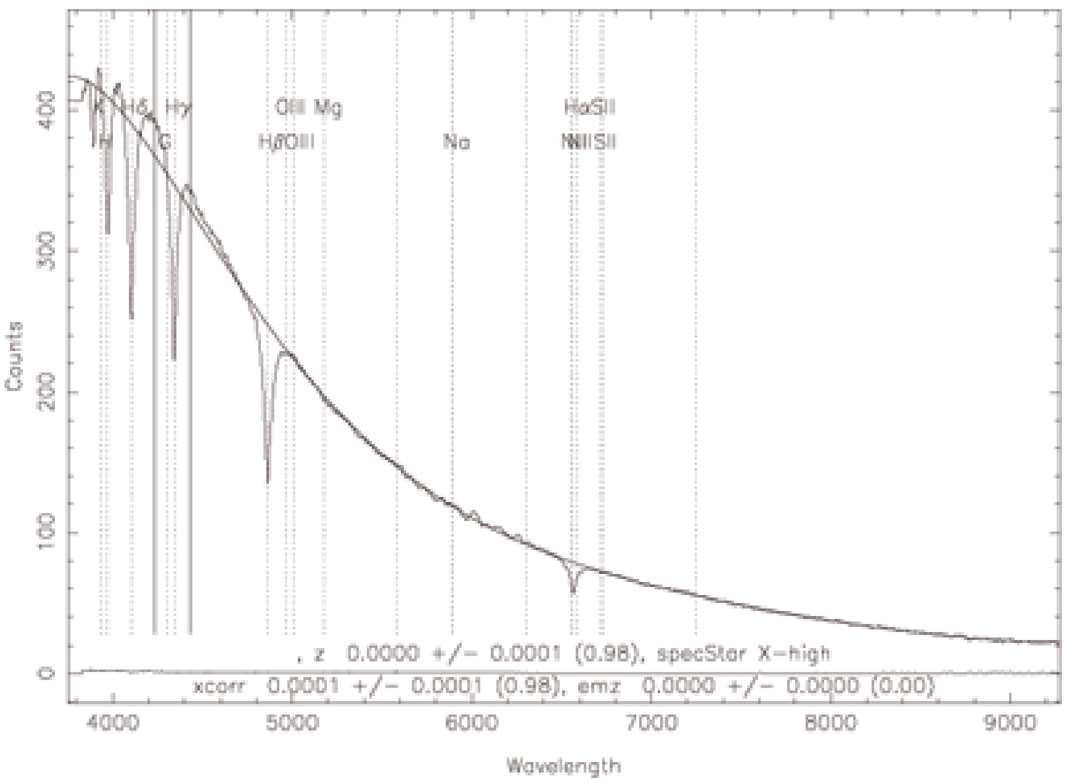
1935: Royal Astronomical Society Meeting

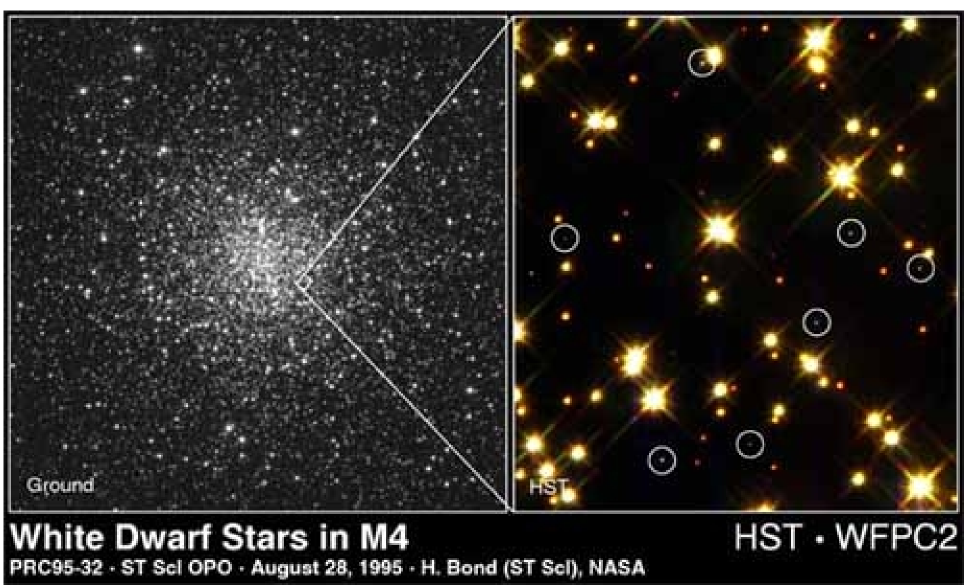
Arthur Eddington vs Subrahmanyan Chandrasekhar

Eddington said Chandrasekhar's ideas were "stellar buffoonery". Eddington thought stars ended their lives as lumps of metal called white dwarves.

The result of the dispute was that the science of astronomy was put on hold for thirty years. Chandrasekhar was hurt and left Cambridge University for the United States. He also changed his topic of research and it was three decades before his theory was proved right. Eddington died in 1944 and never retracted his attack on Chandrasekhar.

Taxonomy of Compact objects: *White Dwarfs*

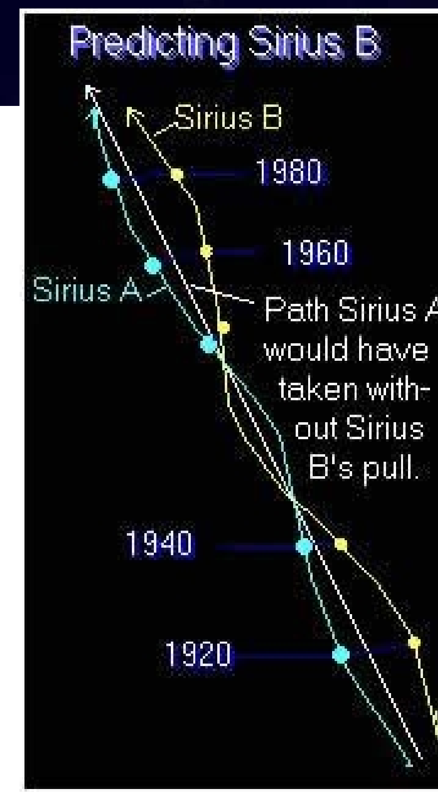




WDs

Sirius A (type A1)
H-fusing dwarf, Teff 9880 K, 26 times brighter than our Sun
Distance = 8.6 ly (2X Alpha Centauri) [5^{th} closest star system].
Metal rich (Fe ~ 3X our Sun) [elemental diffusion]
Radius = 1.75 R_sun   Mass = 2.12 Msun
equatorial rotation speed of 16 kilometers per second
Sirius rotates in under 5.5 of our days.

Sirius B = 8.44 mag, ~ 10,000 times fainter than Sirius A
T_eff 24,800 K. Mass = 1.03 Msun
Though typically separated from each other by a few seconds of arc,
Sirius B is terribly difficult to see inthe glare of Sirius A.
only 0.92 the size of Earth, the total luminosity (including its UV light)
just 2.4 percent that of our Sun.
The two orbit each other with a 50.1 year period at an
average distance of 19.8 Astronomical Units (~ Uranus's distance from our Sun)
large orbital eccentricity carrying them from 31.5 AU apart to 8.1 AU
They were closest in 1994 and will be again in 2044,
while they will be farthest apart in 2019.



WD form as end-product of stellar evolution

PNe are the signature of a borning WD

Progenitor mass: $M_{min} \leq M \leq M_{max}$

where M_{min} *is relatively well known* $= 0.8\text{-}1.0\ M_{sun}$

while M_{max} *is more uncertain (4-9* M_{sun}*) depending on the details of stellar evolution in the latest stages before (convective overshooting of C and O cores, H and He burning, semi-convection, ...) and during the PN phase (mass loss by wind and PN ejection)*

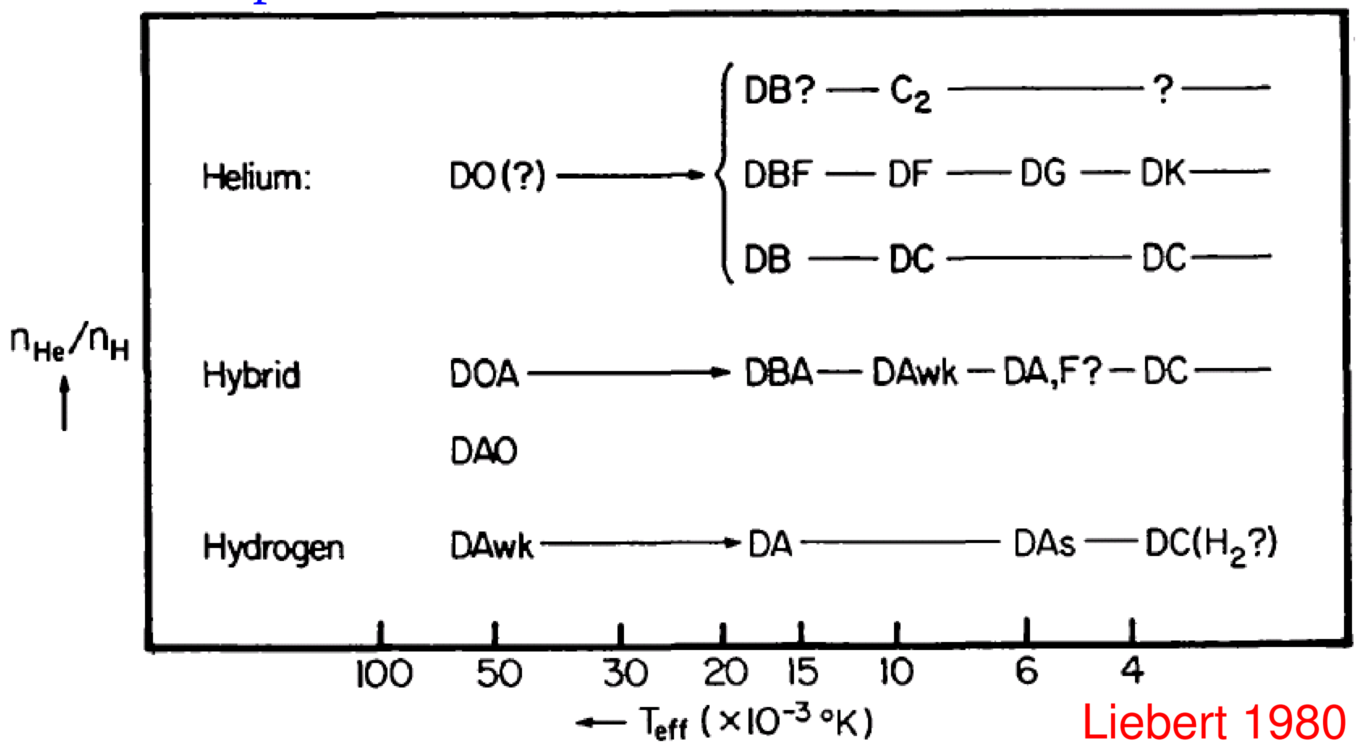
M_{WD} is also affected

Mostly dependent on the (lines in the) optical spectrum (~ mono-elemental)
i.e. the composition of the outer envelope

DA: strong H lines (80%)

DB: strong HeI lines

DO: strong HeII lines



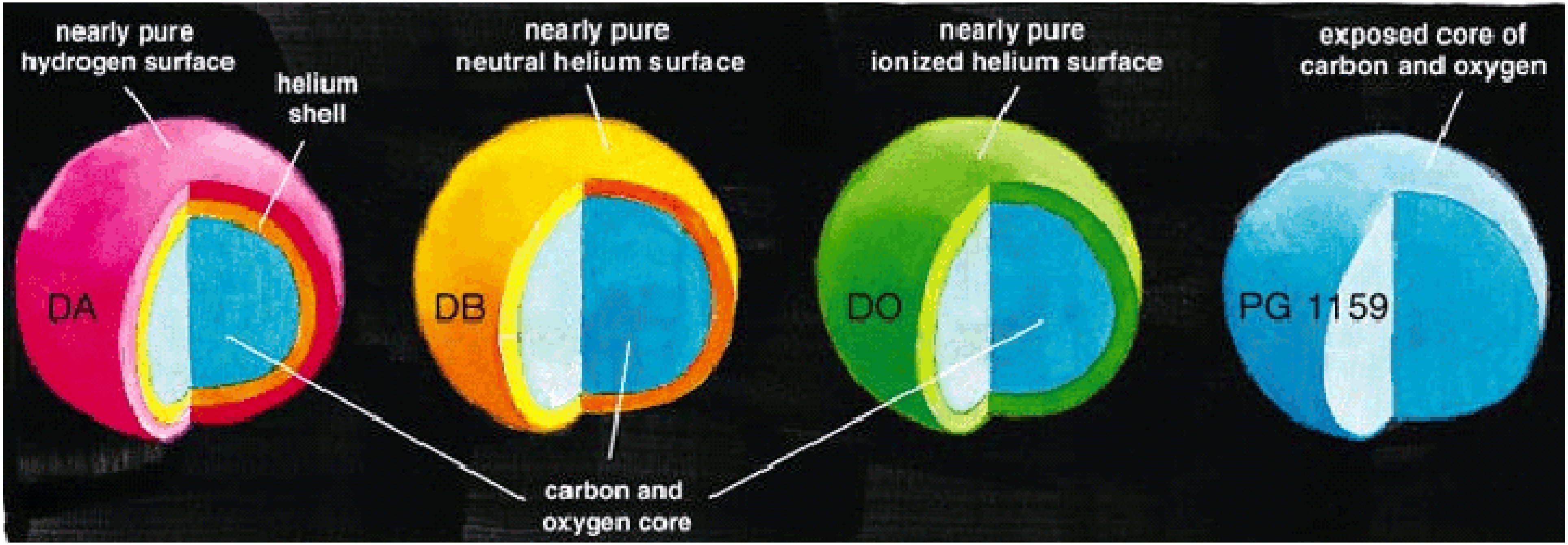
Liebert 1980

DC: continuous spectrum w/o significant lines

DZ: strong metal lines (no C)

DQ: strong C lines

Intermediate families: DAB, DQAB, ...



Layer thickness not to scale.

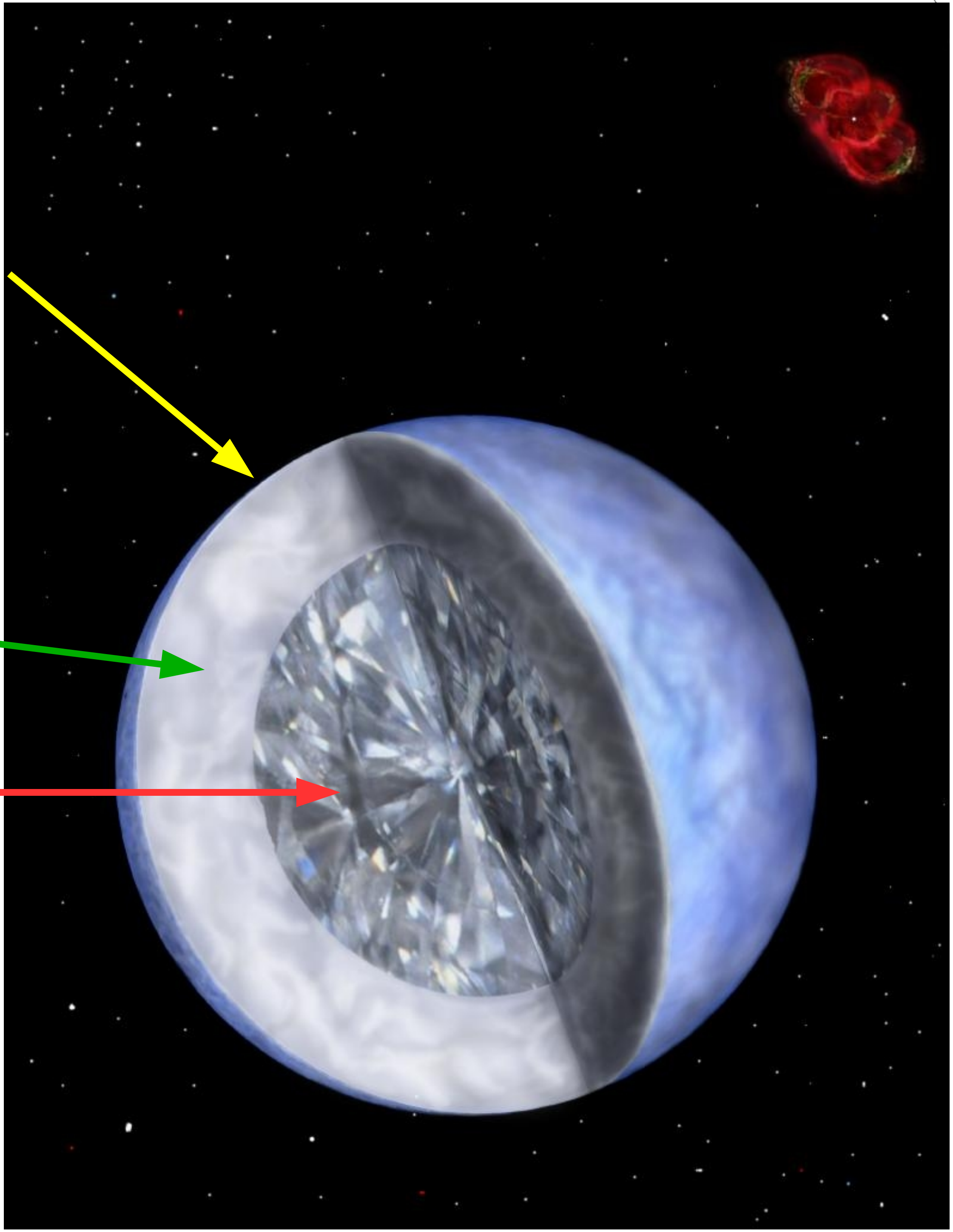
Youngest (i.e. hottest WDs) have a non degenerated envelope (providing EUV + X-ray emission):
Gravity makes heavier elements (nucleons) sink and leaves a very thin layer [... skyscraper size ...] of lighter elements (H+ and/or He++)
=> from spectroscopic observations

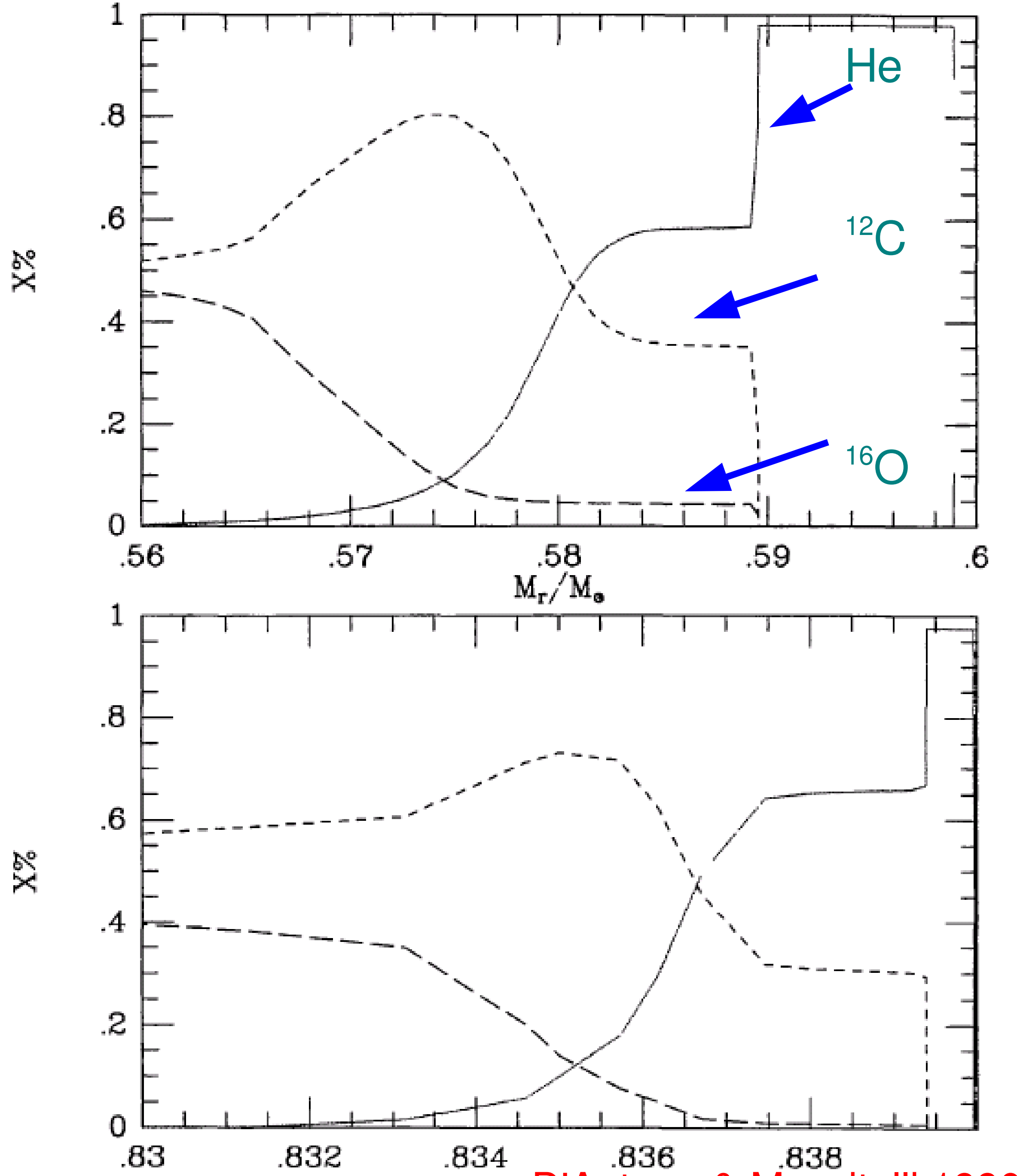
50 km layer (crust)

the remaining is a "crystalline lattice" of C and O atoms

The mass of the progenitor and that of the WD may alter this simple scheme

$^{12}C/^{16}O$ fraction depends on the late history of element burning within the progenitor





D'Antona & Mazzitelli 1990

*The **thickness of the H and He layers** determines the ultimate evolution (cooling) of a WD*

They are like insulating blankets on top of the isothermal core

The energy loss rate is a function of the opacity (and then strongly affected by chemical composition) within these layers.

If T_{eff} > 30000 K metals can reside within the atmosphere from radiative support.

At lower temperatures, gravitational settling makes such metal sink out of the atmosphere (decreasing the opacity)

Given the typical Luminosity of ~ 10^{-3} – 10^{-4} L_{sun}

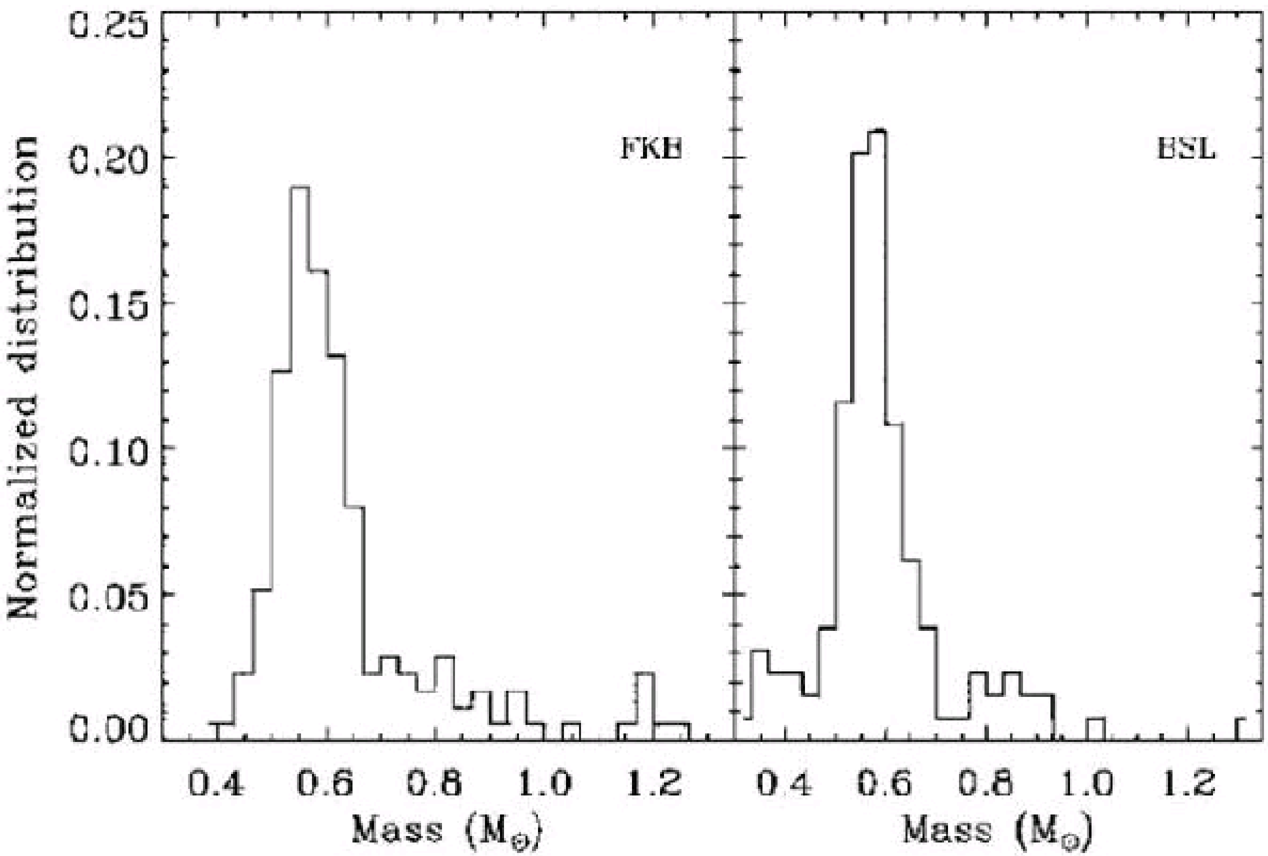
WDs are mainly observable in the solar vicinity

Old WD have ~ 10^{-5} L_{sun} (M_V ~ 17)

WDs are mainly observable in the very solar vicinity (hundreds pc)

hottest WI

Statistical *es, etc.*



Gravitational redshift/broadening:

$$\frac{\Delta\lambda}{\lambda} = \frac{a_g h}{c^2} = \frac{GM_{WD} h}{c^2 r^2}$$

ATLAS OF WHITE-DWARF OPTICAL SPECTRA 763

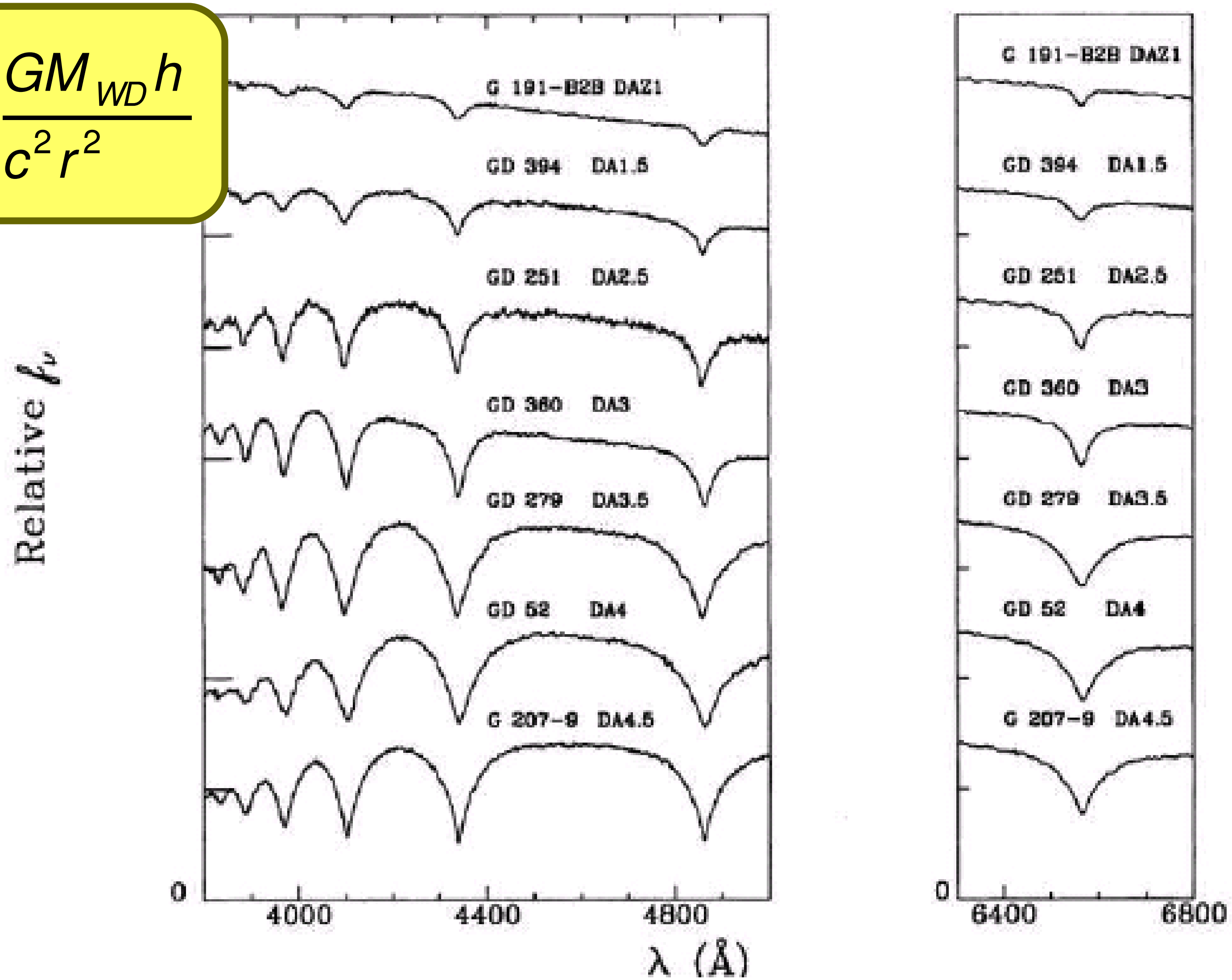


Figure 3.3: Spectra of DA white dwarfs look like A stars and are dominated by Balmer lines.

Total energy of the WD:

$$E = E_{nuc} + E_{g+th} + E_{\nu} + E_{cr}$$

It relies mostly on E_{th} *; depends on the initial central temperature* T_c *(~* 10^{48} *erg if* $T_c \sim 10^7$ *K)*

Gravitational energy is released at all stages
more relevant at the earliest stages then declines and may become relevant again in the final phases (up to 30% of the "final luminosity")

Crystallization releases of latent heat (kT each ion)

It becomes relevant when $E_{th} \sim E_{coulomb}$ *i.e.*

$$\frac{3}{2}kT = \frac{(Ze)^2}{r_i} \qquad r_i = \text{mean distance between ions}$$

Coulomb interactions are described by the Coulomb coupling constant

$$\Gamma = \frac{(Ze)^2}{a\,kT} \sim \frac{1}{\rho}$$

as time passes, ρ increases (i.e. r_i decreases) and T decreases

$\Gamma \ll 1$ *electrostatic interactions are ineffective in producing correlations between the ions ⇨ plasma behaves like a gas* ***(early phase)***

Γ increases *ions settle with short-range correlations without any significant structure (liquid)*

Γ gets large (>145-165) *thermal energy of ions gets negligible and they arrange in a body-centred-cubic periodic lattice structure which minimizes electrostatic interaction energy* ***(late phase)*** *and crystallization takes place*

Nuclear energy:
residual He burning in pre-WD (PPNe) stage (50% energy output),
some p-p burning coupled with some residual CNO nuclear burning
may reduce the M_H *from* 10^{-4} *to* 10^{-7} M_{SUN}
total energy output generally marginal

Neutrinos
Surface luminosity of the stars drops quickly, whereas the central temperature changes on a much longer time scale.
Neutrino rate and luminosity are linked to the central T (slow changes).
It is the driving mechanism for cooling the core
Neutrino luminosity steadily decreases until $L=10^{-1.5} L_{SUN}$ ($T_C = 3 \times 10^7$ K) *it fades away*

Interior:

Electron conduction:

High degree of degeneracy --> heat transfer by free electrons is effective
Large thermal conductivity (mainly electron-ion interactions, two phases: superfluid and crystalline lattice)
a proper treatment of the thermal conduction requires electron-electron, and ion-ion collisions, as well as crystallization

WD interior may be safely considered isothermal

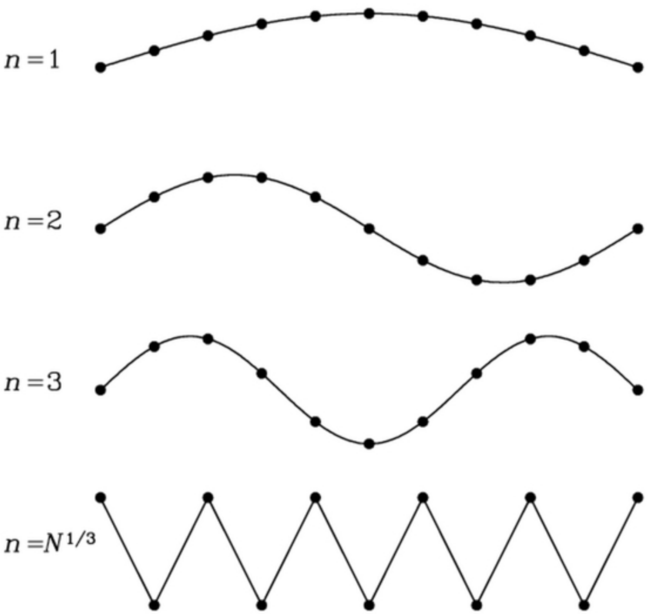
.... to the external envelope....
very complex problem:
consider a partially ionized, partially degenerated plasma subject to strong Coulomb and short-range interactions ---> very uncertain equation of state

possible presence of convection and mixing which both influence opacity which is proportional to the number of electrons in the non-degenerated atmosphere

energy loss rate is function of the opacity of the radiative parts of the envelope

Debye cooling (heat transfer)

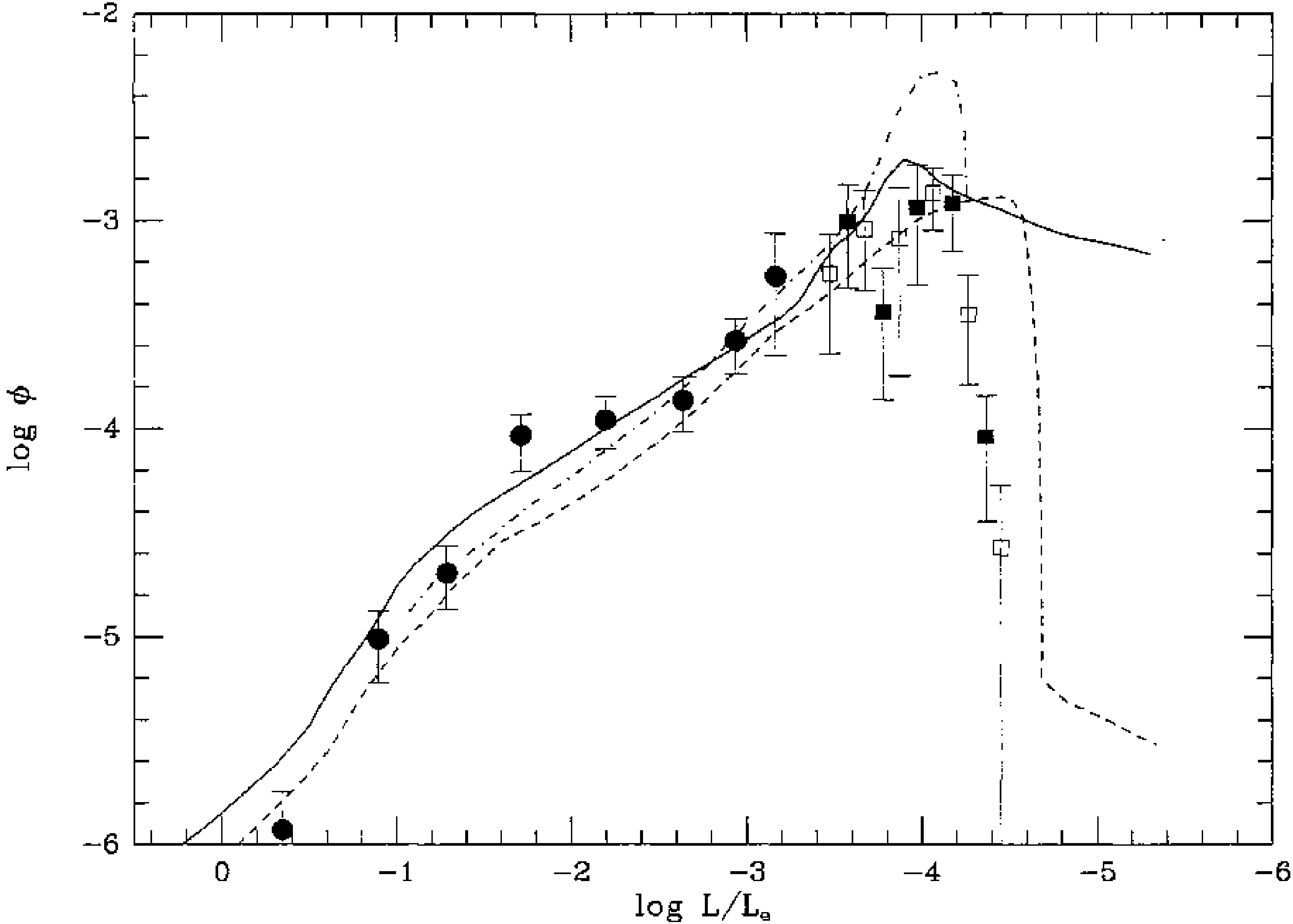
The Debye model is a solid-state equivalent of Planck's law of black body radiation, where one treats electromagnetic radiation as a gas of photons in a box. The Debye model treats atomic vibrations as phonons in a box (the box being the solid). Most of the calculation steps are identical.



Now, this is where Debye model and Planck's law of black body radiation differ. Unlike electromagnetic radiation in a box, there is a finite number of phonon energy states because a phonon cannot have infinite frequency. Its frequency is bound by the medium of its propagation -- the atomic lattice of the solid. Consider an illustration of a transverse phonon aside.

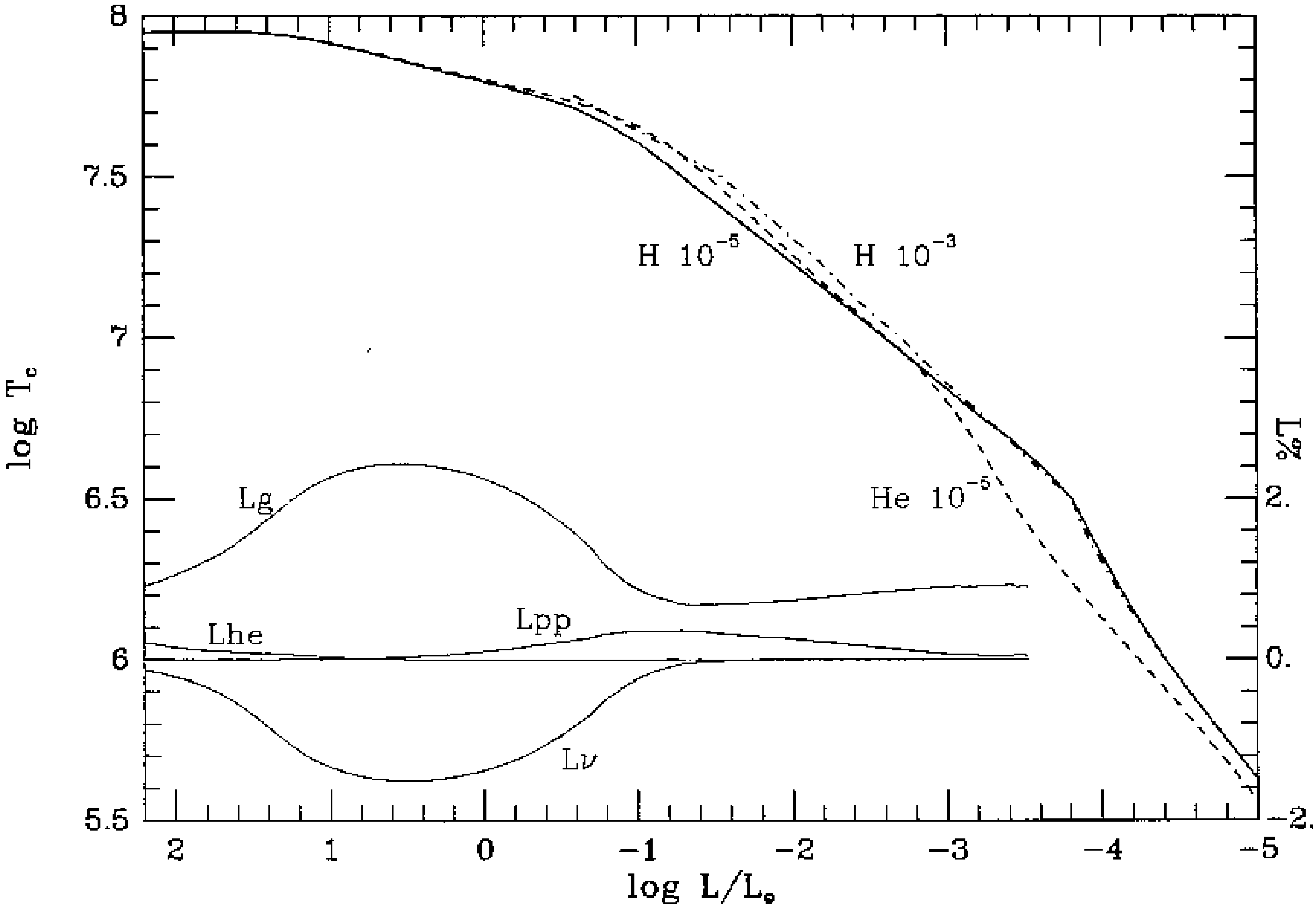
It is reasonable to assume that the minimum wavelength of a phonon is twice the atom separation, as shown in the figure. There are N atoms in a solid. Our solid is a cube, which means there are $\sqrt[3]{N}$ atoms per side. Atom separation is then given by $2L / \sqrt[3]{N}$, and the minimum wavelength is

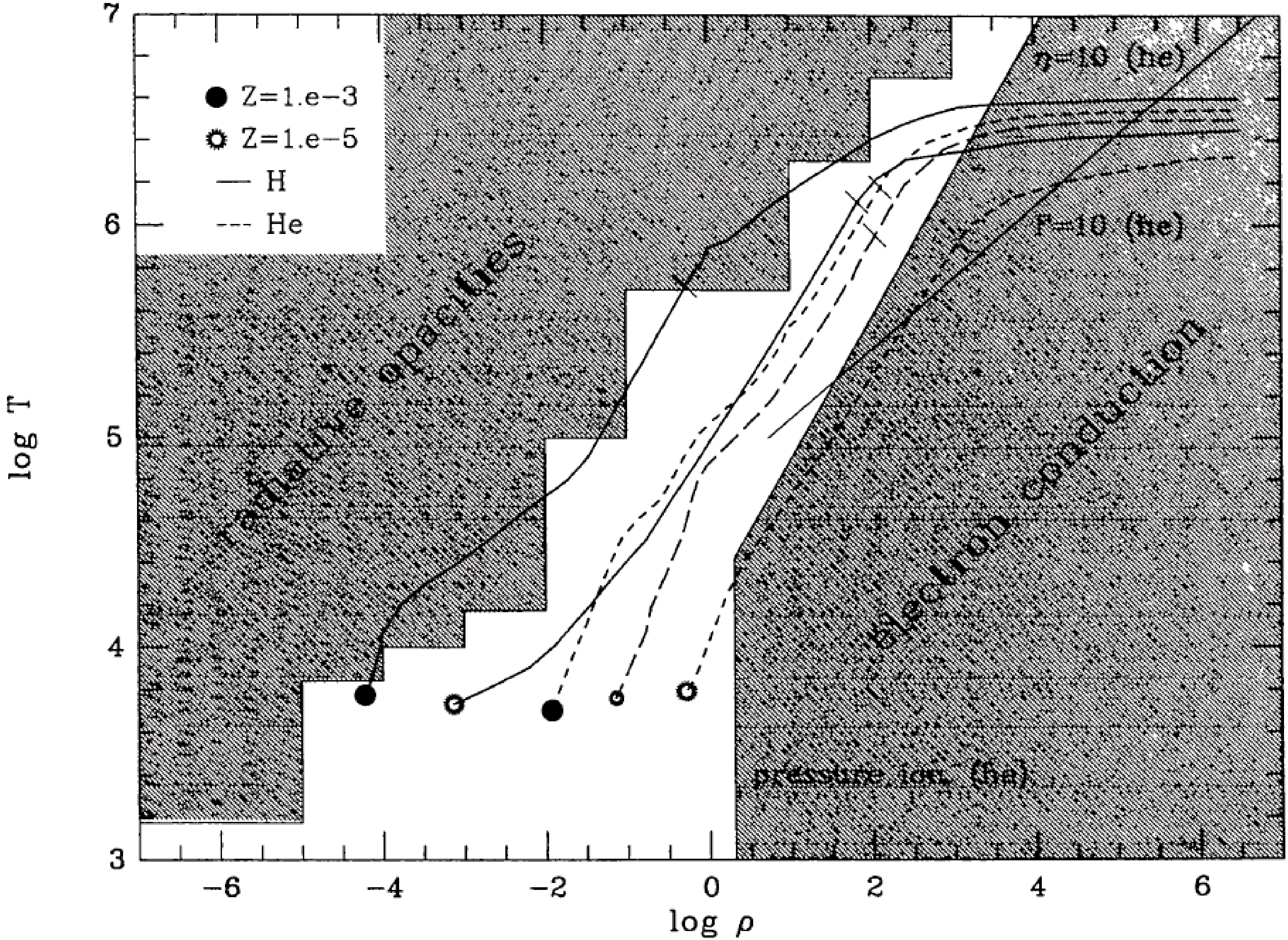
$$\lambda_{min} = \frac{2L}{\sqrt[3]{N}}$$



-2
-3
-4
-5
-6
log φ
0
-1
-2
-3
-4
-5
-6
log L/L☉

Cooling .vs. luminosity



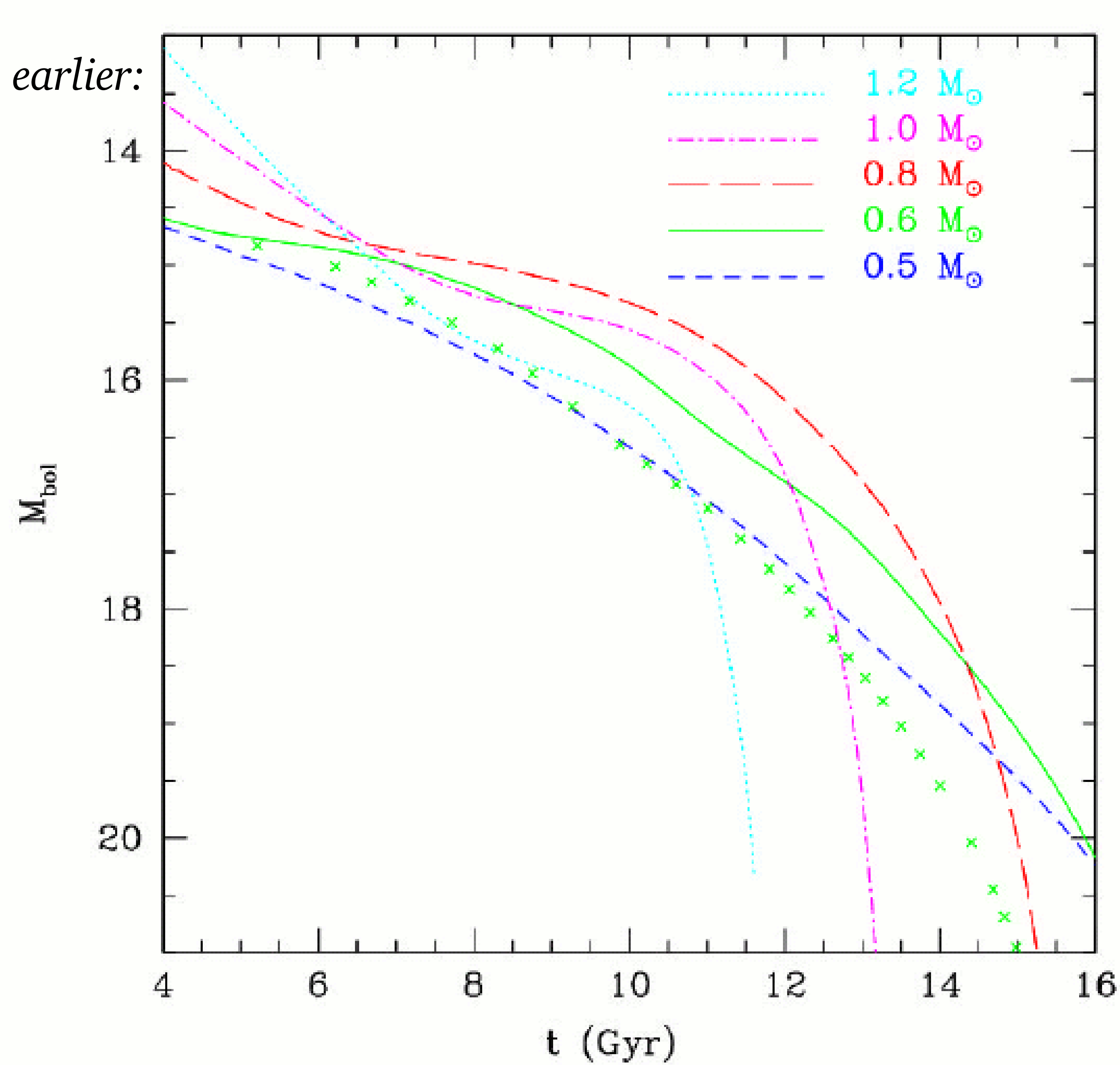

Z=1.e−3
Z=1.e−5
H
He
radiative opacities
electron conduction
η=10 (he)
Γ=10 (he)
pressure ion. (he)
log T
log ρ
7
6
5
4
3
−6
−4
−2
0
2
4
6

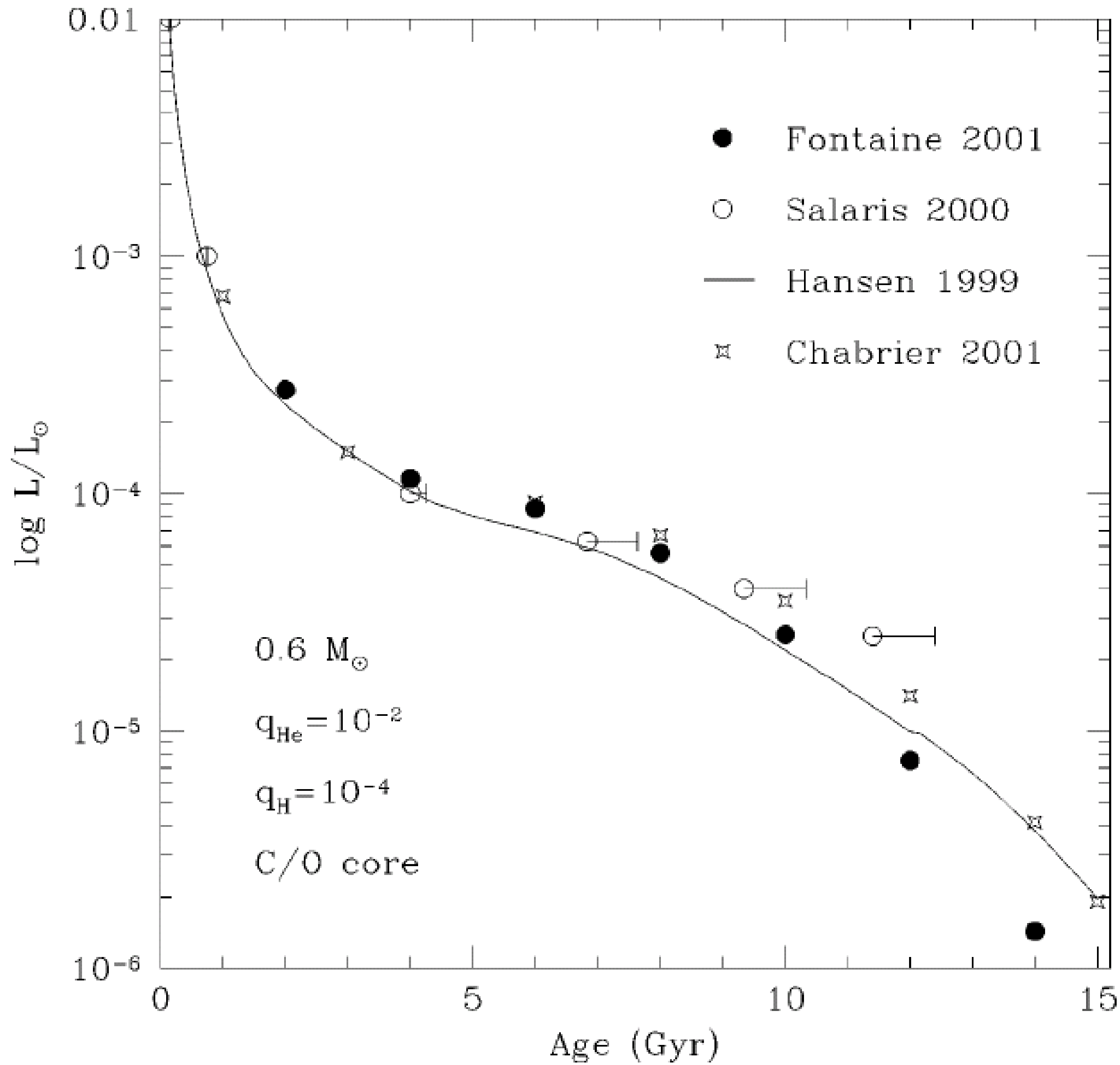
Debye cooling starts earlier in massive WD since occurs at higher T_{eff} and then at higher luminosities

heaviest WD stars fade earlier:

Debye cooling:

$$C_v \approx T_c^3$$




0.01
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
log L/L⊙
0
5
10
15
Age (Gyr)
Fontaine 2001
Salaris 2000
Hansen 1999
Chabrier 2001
0.6 M⊙
q_He=10⁻²
q_H=10⁻⁴
C/O core

WD in binary systems

Accreting objects with high luminosity:
Cataclysmic variables:
WD + main sequence (small mass) star donating H rich plasma through L1 organized in a disk

Gravitation at work in all cases

H field relevant in polars and

WD summary (1)

End-products of evolution of a 0.8 – 8 $M_\odot$ star:

After the PN phase the WD starts its long-term cooling progressively decreasing its L

Stratification very effective due to high gravity

Internal

Crystallization starts earlier in more massive WD due to the higher gravity (i.e. density)

The fate of matter leaving the binary system

Dwarf Novae:

Stellar wind with enhancements during/after outbursts

Classical Novae (& Recurrent Novae):

external shell (H-burning) ejected during/after outbursts
+ quiescent stellar wind

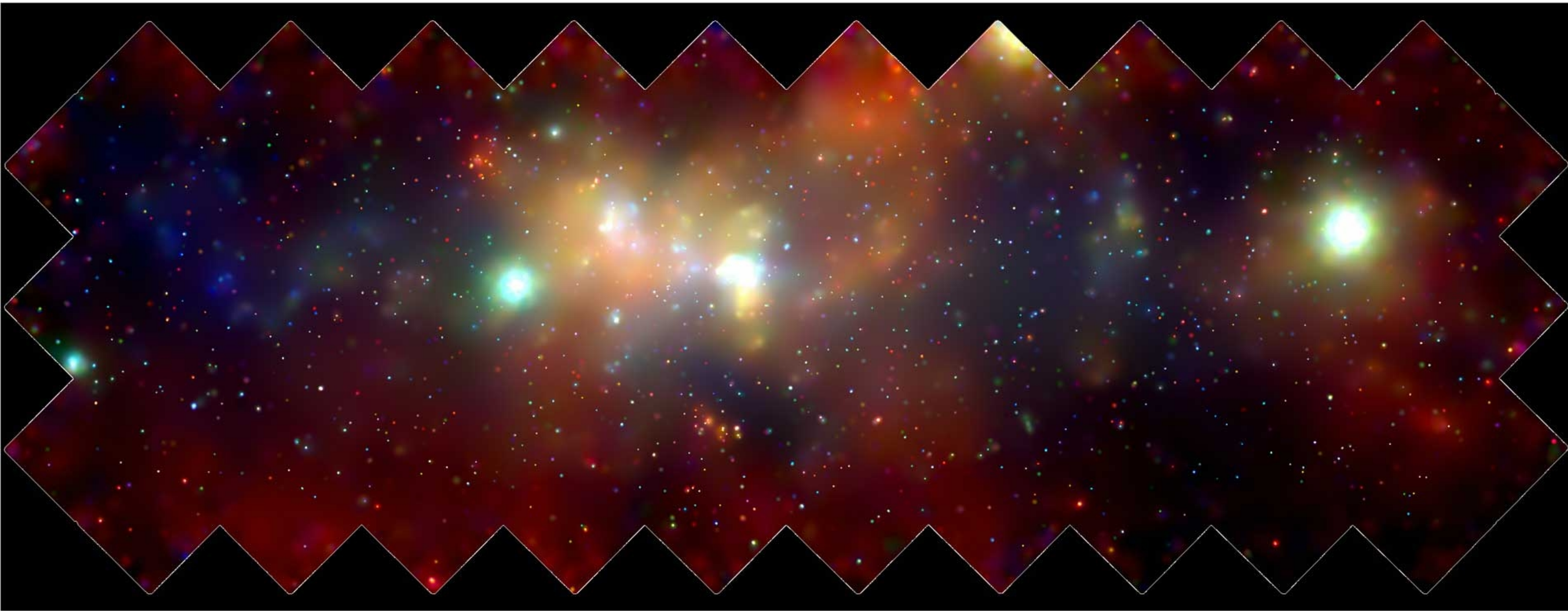
Polars:

Magnetically driven accretion, only very fast charged particles may leave the system
+ quiescent stellar wind

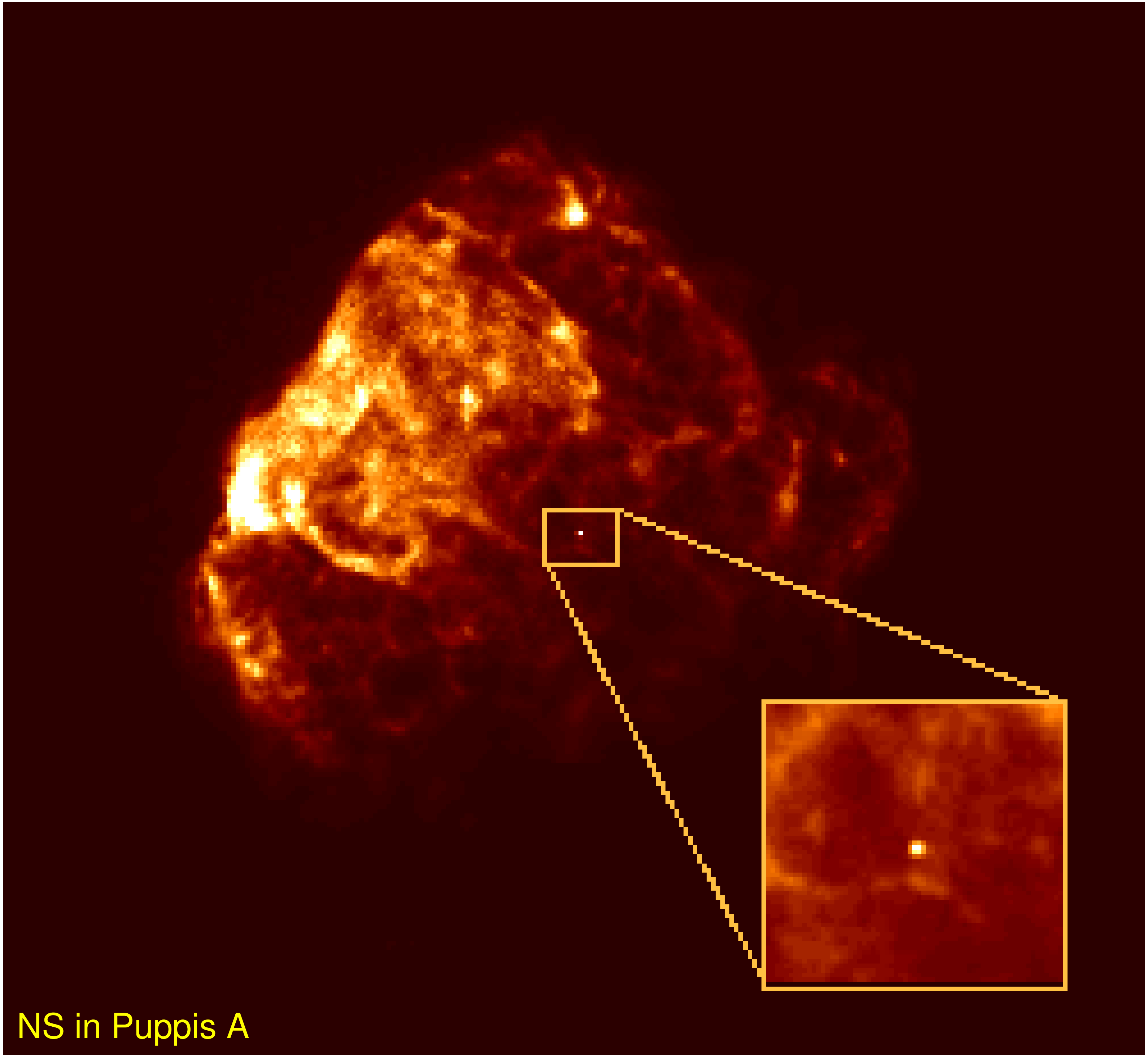
Point X-ray sources towards the Galactic centre

- Spotted diffuse emission is from SNR

- More diffuse bremsstrahlung from the Galaxy hot corona



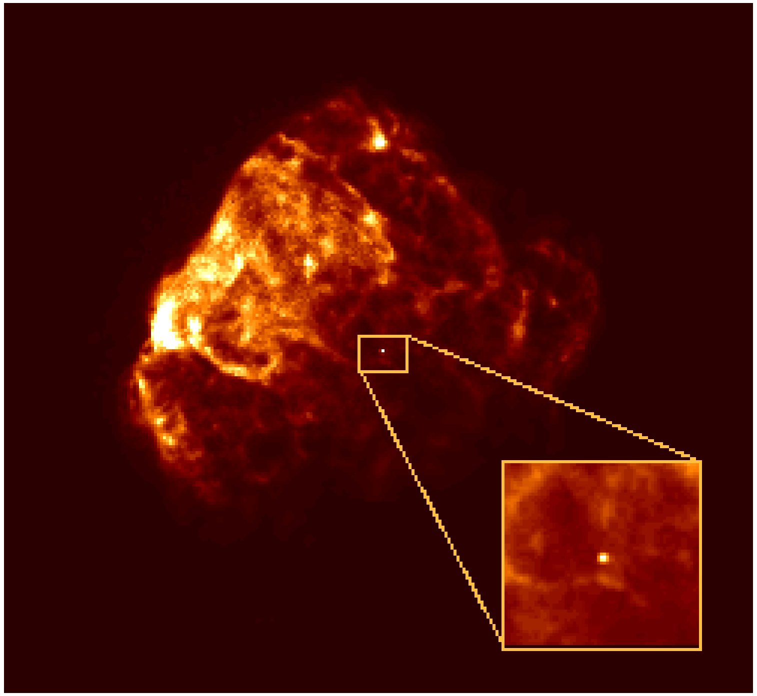
Neutron stars



Neutron stars

What happens in case the compact object is or gets more massive?

$$Mass \geq 1.4\, M_{sun}$$
$$Diameter \sim 20\, km$$



Neutron stars: compact objects created in the cores of massive stars during SN explosions.
The core collapses and crushes together every proton with a corresponding electron turning each electron-proton pair into a neutron.
The neutrons can often stop the collapse and remain as a neutron star.

Neutron stars can be observed occasionally:
Puppis A – above – an extremely small and hot star within a SNR.
They are more likely seen when they are a pulsar or part of an X-ray binary.

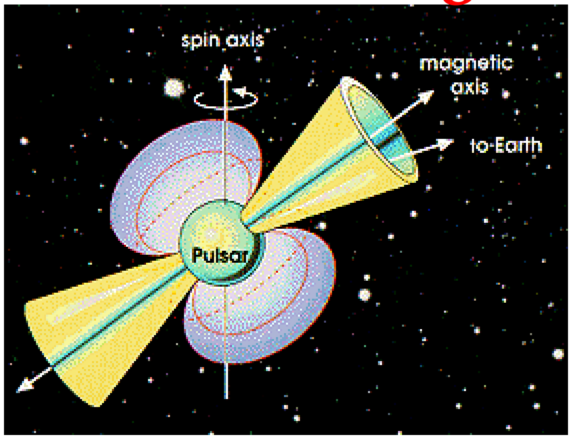
Chandrasekhar (1931) *WDs collapse at > 1.44 Msun*

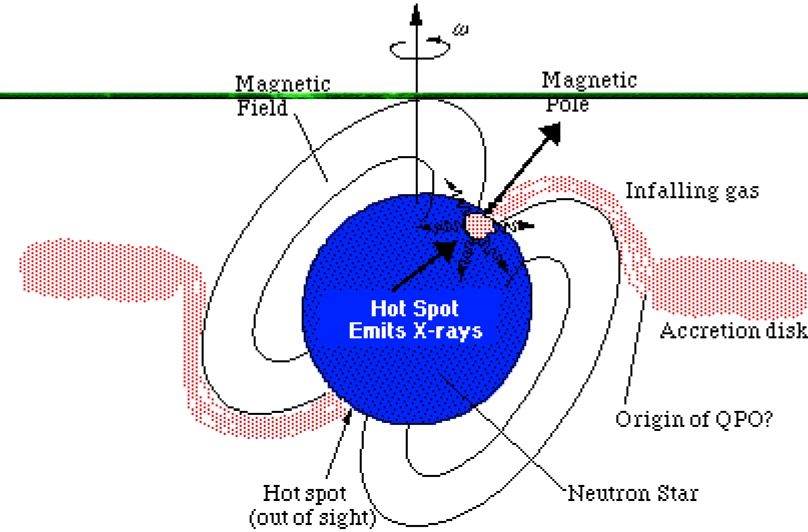
Baade & Zwicky (1934) *proposed: existence of neutron stars*
their formation in supernovae
radius of approximately 10 km

Oppenheimer & Volkoff (1939)
Evaluated first equation of state
Mass > 1.4 Msun
Radius ~ 10 km
Density > 10^{14} *g cm* $^{-3}$

Pacini (1967) *proposed:*
electromagnetic waves from rotating neutron stars
such a star may power the Crab nebula
i.e. ***PULSARS***

PULSARS: a digression





Neutron stars are about 10 km in diameter and have the mass of about 1.4 times that of our Sun. This means that a neutron star is so dense that on Earth, one teaspoonful would weigh a billion tons! Because of its small size and high density,

a neutron star possesses a surface gravitational field about 300,000 times that of Earth.

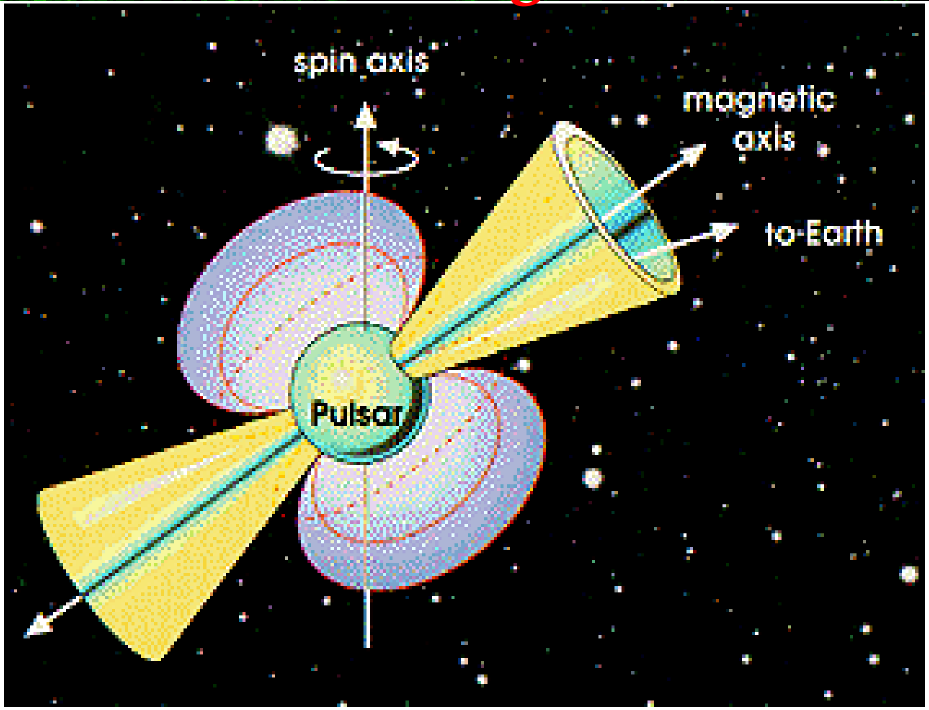
Neutron stars are one of the possible ends for a star. *They result from massive stars which have mass greater than 4 to 8 times that of our sun*. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion. This explosion blows off the outer layers of a star into a beautiful supernova remnant. The central region of the star collapses under gravity. It collapses so much that protons and electrons combine to form neutrons. Hence the name "neutron star".

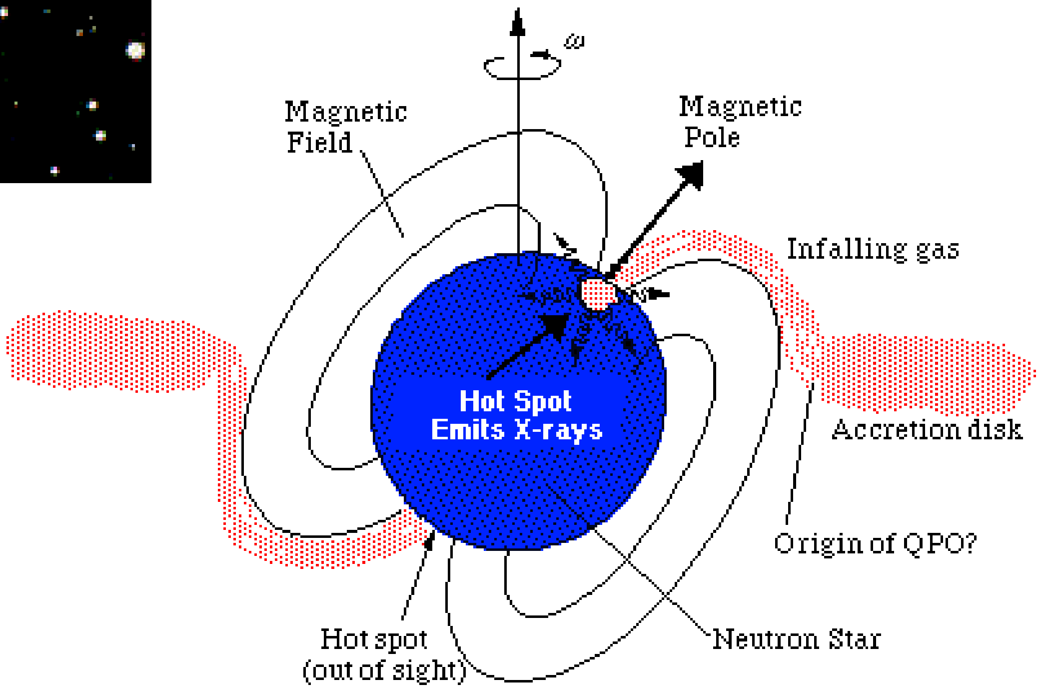
Neutron stars may appear in supernova remnants or in x-ray binaries with a normal star.

When a neutron star is in an x-ray binary, astronomers are able to measure its mass.

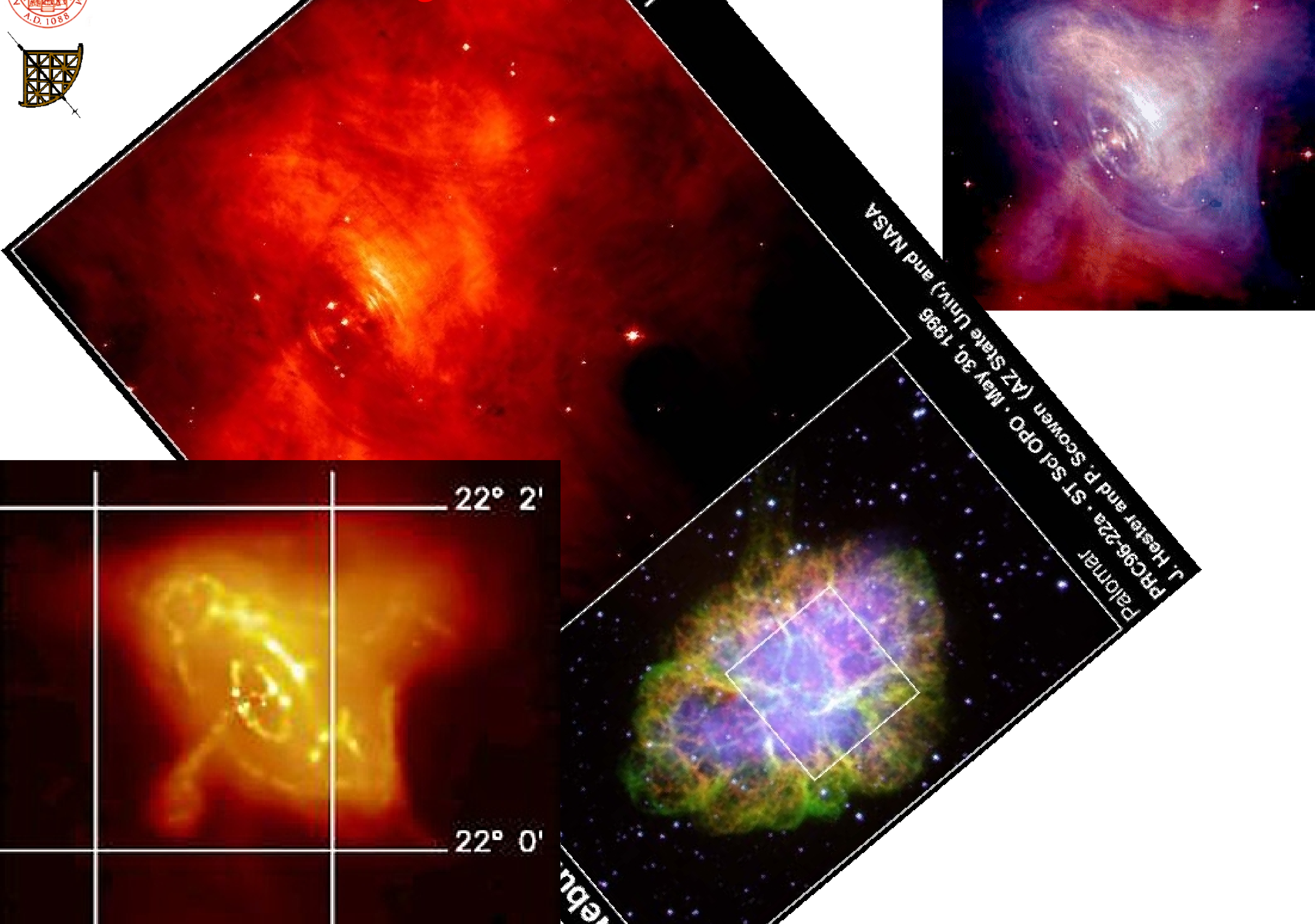
From a number of such x-ray binaries, neutron stars have been found to have masses of about 1.4 times the mass of the sun. Astronomers can often use this fact to determine whether an unknown object in an x-ray binary is a neutron star or a black hole, since black holes are more massive than neutron stars.

PULSARS: a digression

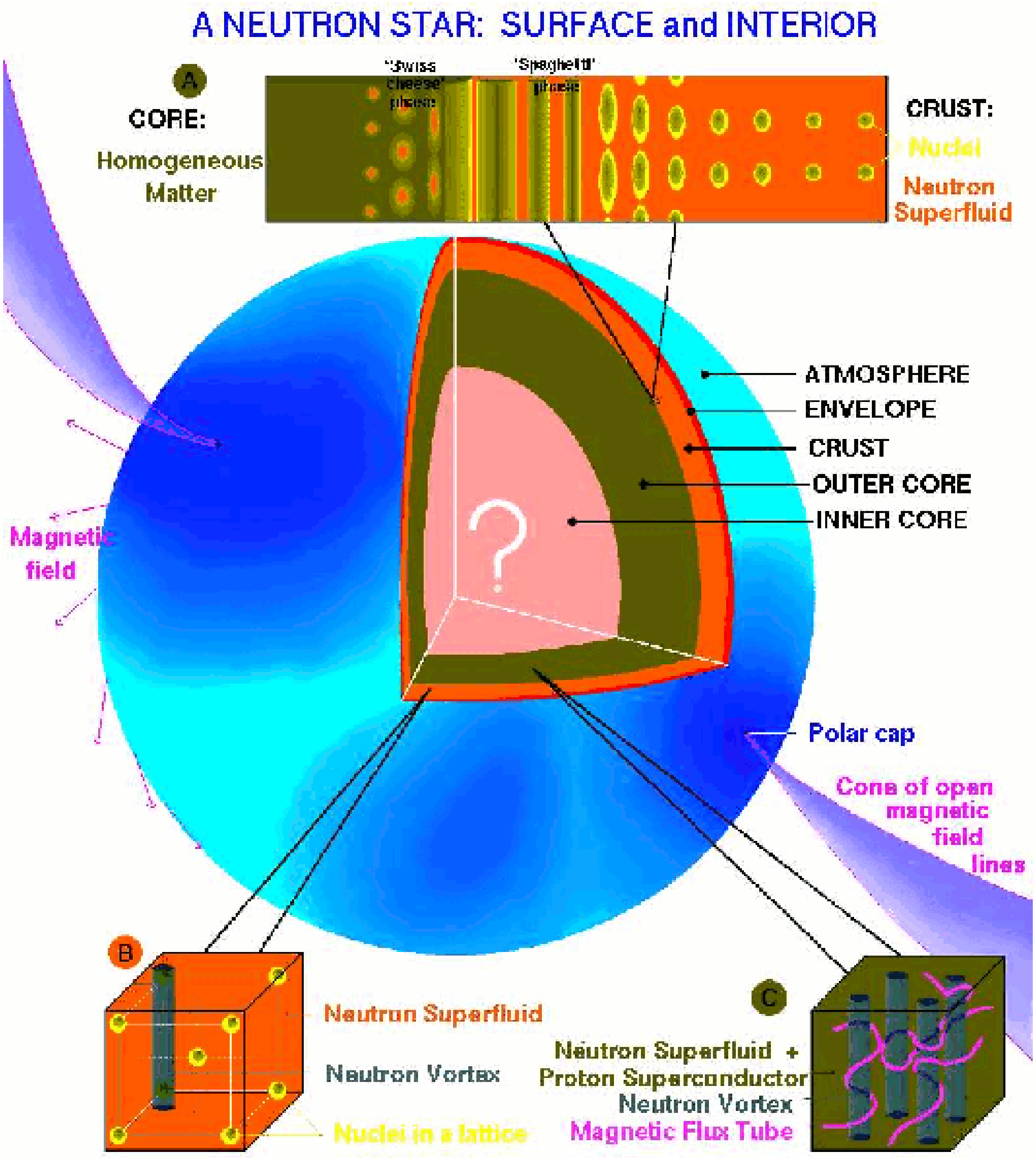




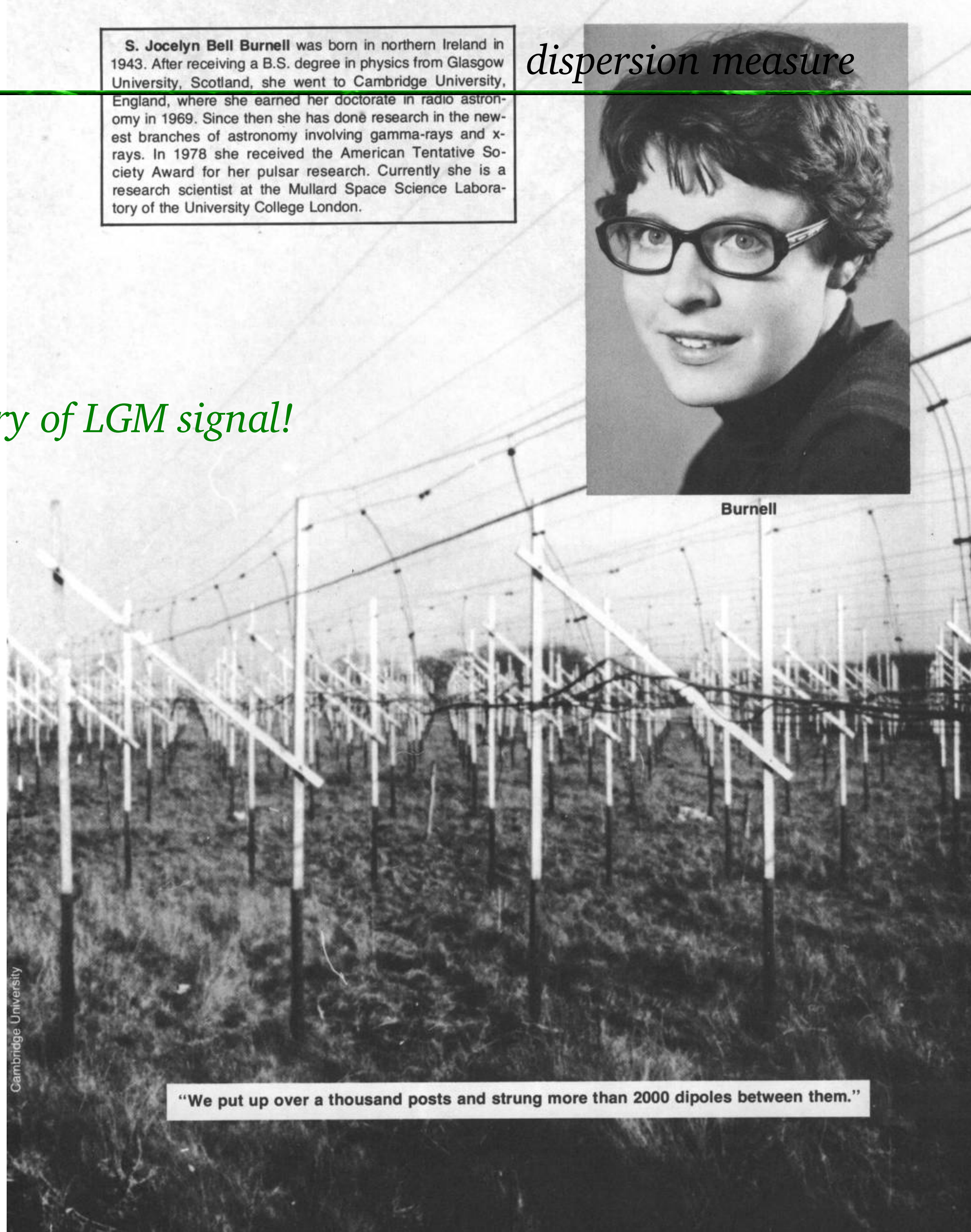
PULSARS: a digression



Historical notes:



1967: discovery of LGM signal!



S. Jocelyn Bell Burnell was born in northern Ireland in 1943. After receiving a B.S. degree in physics from Glasgow University, Scotland, she went to Cambridge University, England, where she earned her doctorate in radio astronomy in 1969. Since then she has done research in the newest branches of astronomy involving gamma-rays and x-rays. In 1978 she received the American Tentative Society Award for her pulsar research. Currently she is a research scientist at the Mullard Space Science Laboratory of the University College London.

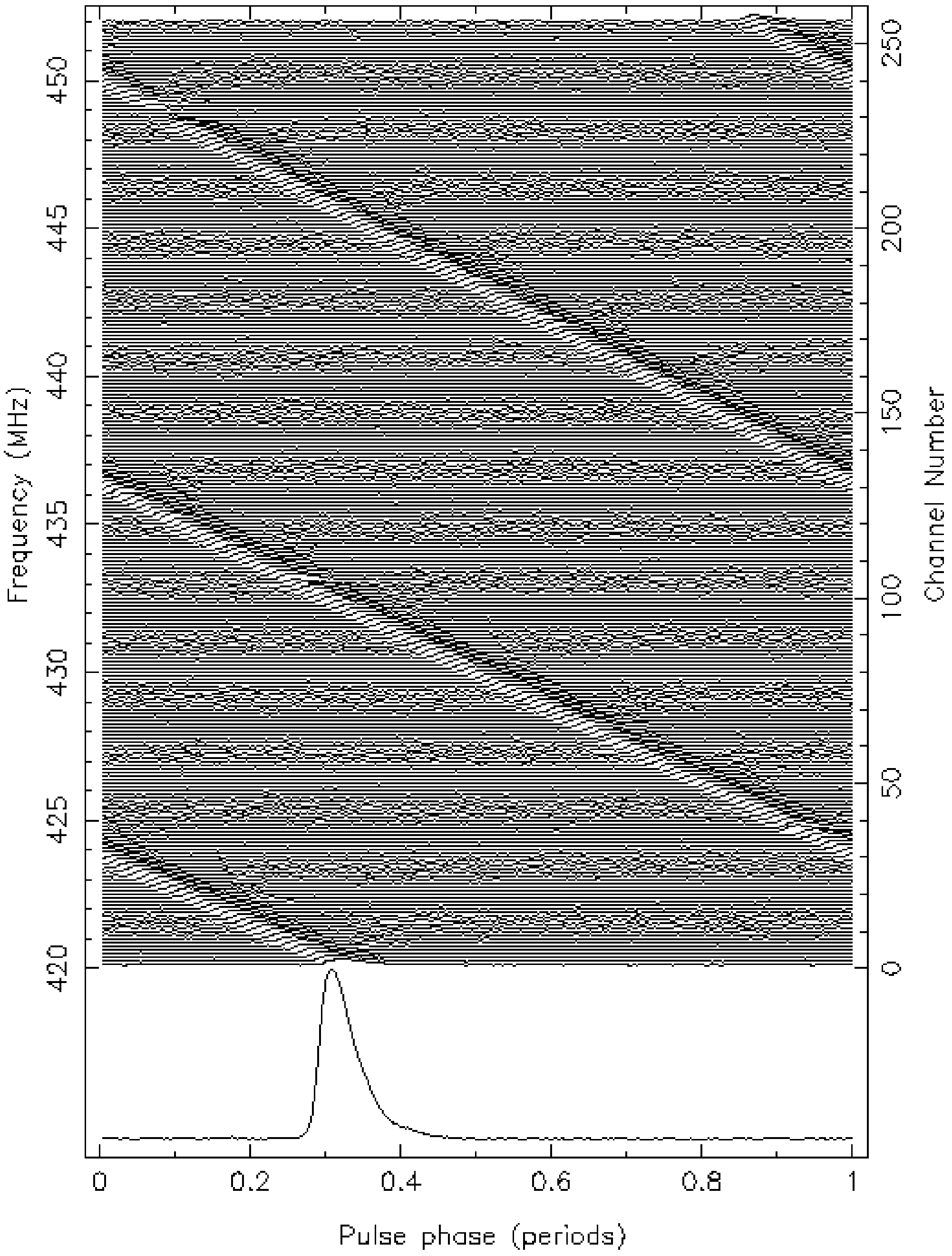
Burnell

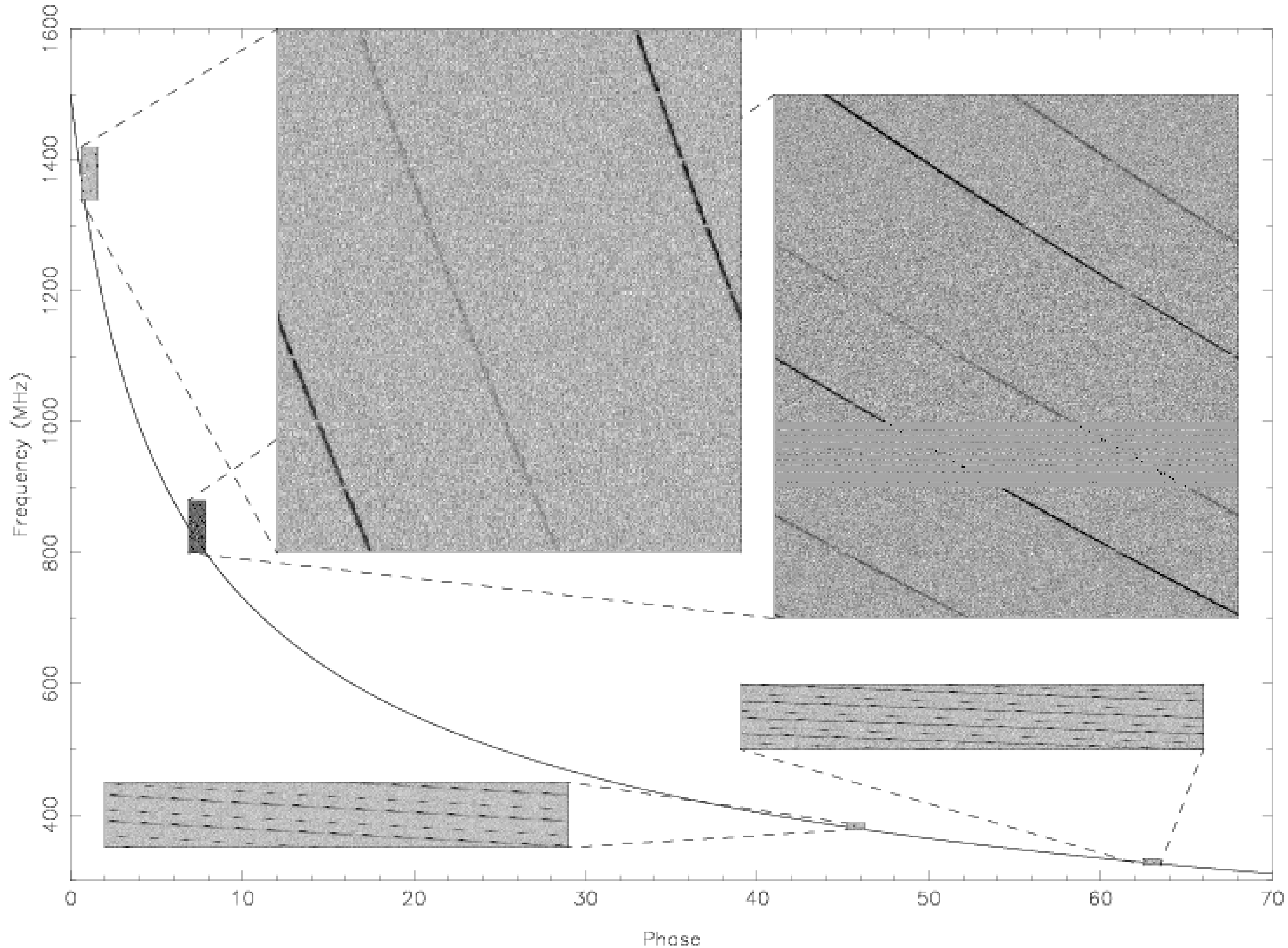
"We put up over a thousand posts and strung more than 2000 dipoles between them."

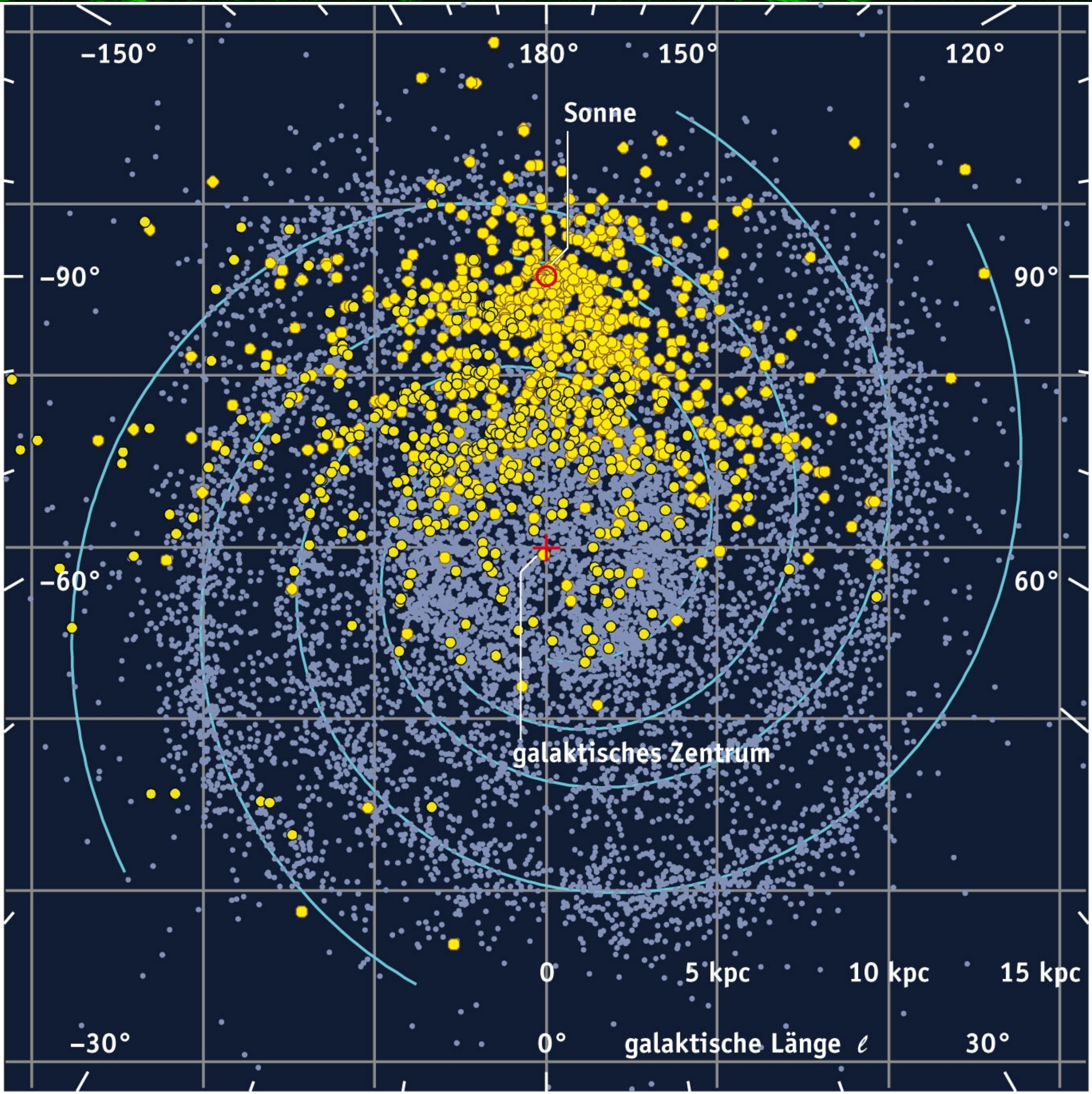
Cambridge University

$$n_r(\nu) \simeq \sqrt{1 - \frac{4\pi e^2}{m_e}\frac{n_e}{(2\pi\nu)^2}} =$$

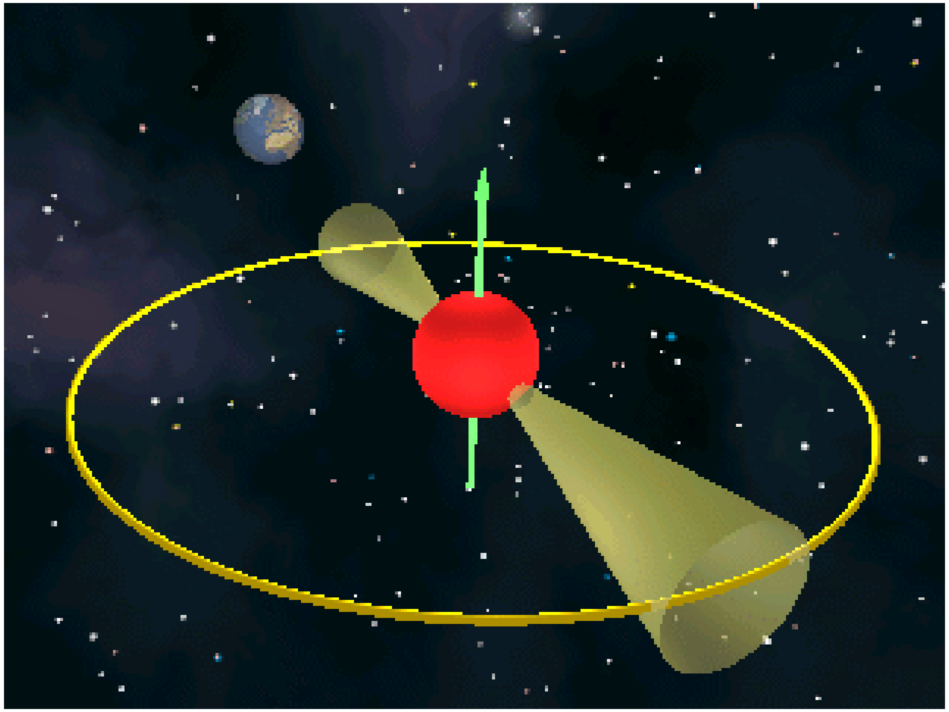
$$= \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2}$$




Frequency (MHz)
1600
1400
1200
1000
800
600
400
0
10
20
30
40
50
60
70
Phase


-150°
180°
150°
120°
Sonne
-90°
90°
-60°
60°
galaktisches Zentrum
0
5 kpc
10 kpc
15 kpc
-30°
0°
galaktische Länge ℓ
30°

Name	P (sec)	(# turns in 1 sec)
PSR B0329+54	P=0.714519	1.4
PSR B0833-45=The Vela Pulsar	P=0.089	11
PSR B0531+21=The Crab Pulsar	P=0.033	30
PSR J0437-4715	P=0.0057	174
PSR B1937+21	P=0.00155780644887275	642
PSR J1748-2446	P=0.0013966	716



estimated from actual observation of P its derivative

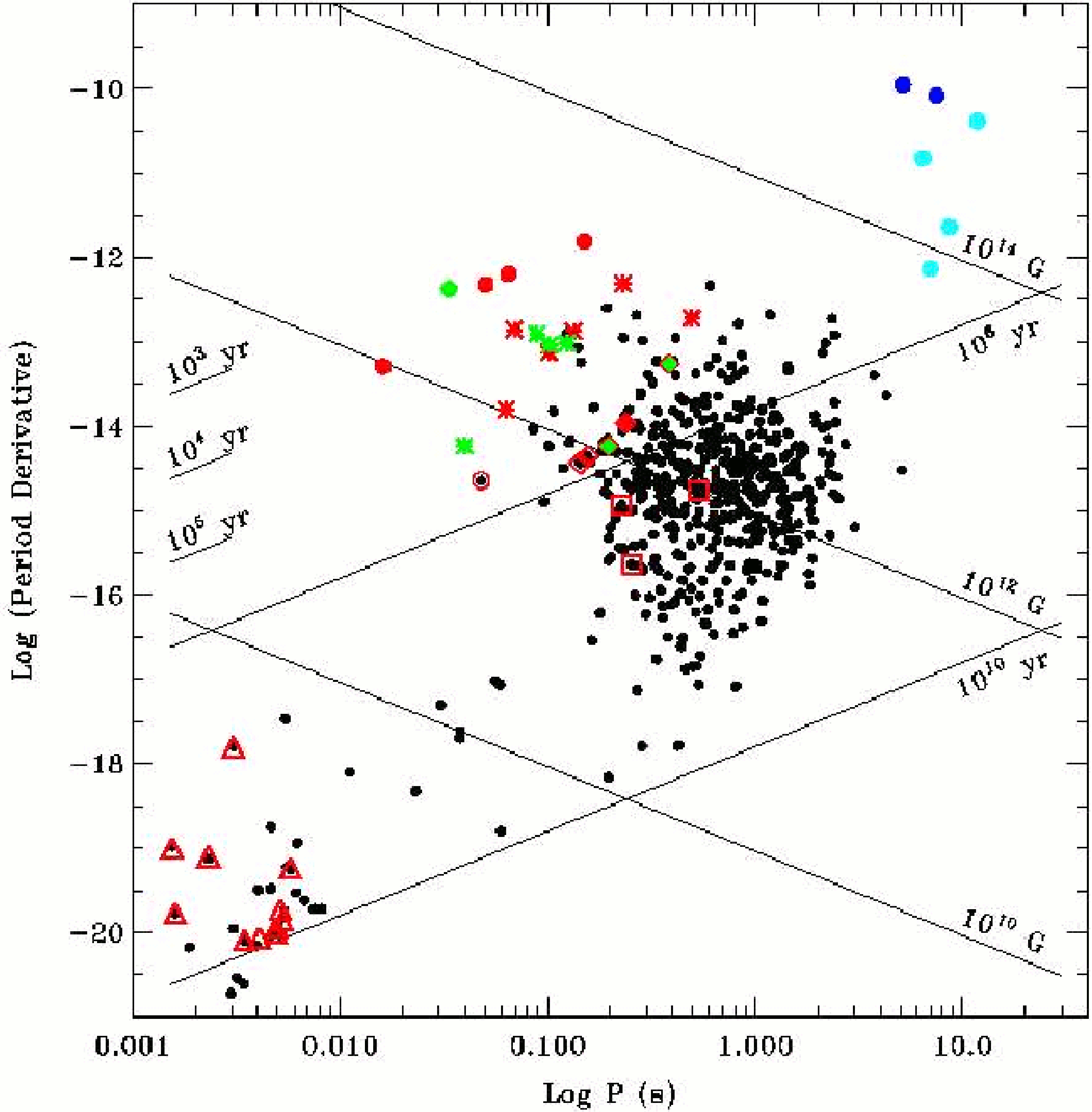
$$\dot{P} = \frac{dP}{dt} = a P^{-1}$$

$$\int_{P_o}^{P} P \, dP = a \int_{o}^{\tau} dt$$

$$\frac{P^2}{2} - \frac{P_o^2}{2} = a \tau$$

$$\tau \simeq \frac{P^2}{2a} \simeq \frac{P}{2 \dot{P}}$$

rifare per bene dopo il rotatore obliquo



−10
−12
−14
−16
−18
−20
Log (Period Derivative)
0.001
0.010
0.100
1.000
10.0
Log P (s)
10³ yr
10⁴ yr
10⁵ yr
10⁶ yr
10¹⁰ yr
10¹⁴ G
10¹² G
10¹⁰ G

about 2000.

If we consider the spatial distribution (distance) of the known pulsars we get an estimate of the total number $\sim$ 150 thousands in our galaxy

if we attribute an average age of 5 million years we infer the formation of a pulsar every 30-120 yr

consistent with the formation rate of SNR

However, we must correct the number for:
– more distant and weak sources may be under-represented
– pulsars are revealed if the beam is at a small angle to the LOS
– the magnetic field decays with age, then old pulsars are underepresented

$$-\frac{dK_{NS}}{dt} = \frac{d\varepsilon}{dt} = \frac{2}{3}\frac{1}{c^3}\left(\frac{d^2\vec{m}}{dt^2}\right)^2 = \frac{2}{3}\frac{1}{c^3}\omega_{NS}^4(\vec{m}_o \sin\alpha)^2$$

$$K_{NS} = \frac{1}{2} I_{NS}\omega_{NS}^2 \qquad I_{NS} = \frac{2}{5} M_{NS} R_{NS}^2$$

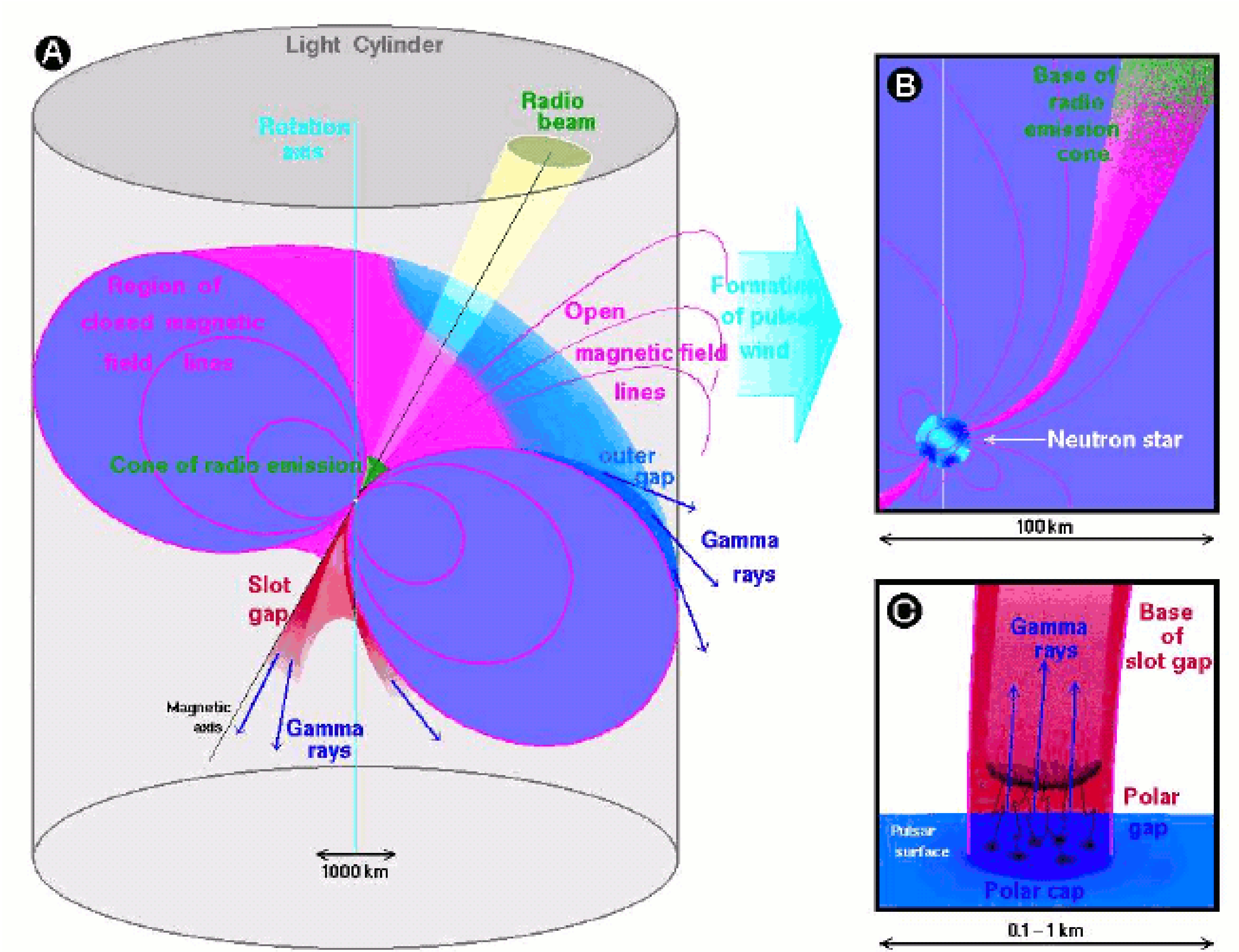
$$\frac{2}{3}\frac{1}{c^3}\omega_{NS}^4(\vec{m}_o^2 \sin\alpha)^2 = -I_{NS}\omega_{NS}\dot{\omega}_{NS}$$

$$\dot{\omega}_{NS} = -\frac{2}{3}\frac{1}{c^3}\omega_{NS}^3\frac{(\vec{m}_o \sin\alpha)^2}{I_{NS}}$$

$$|\vec{m}_o| \sim H_{NS} R_{NS}^3$$

$$H_{NS} \leftarrow\rightarrow P\,\dot{P}$$

$$H_{NS} = \sqrt{\frac{3c^3}{8\pi^2}\frac{I_{NS}}{R_{NS}^6} P\,\dot{P}}$$

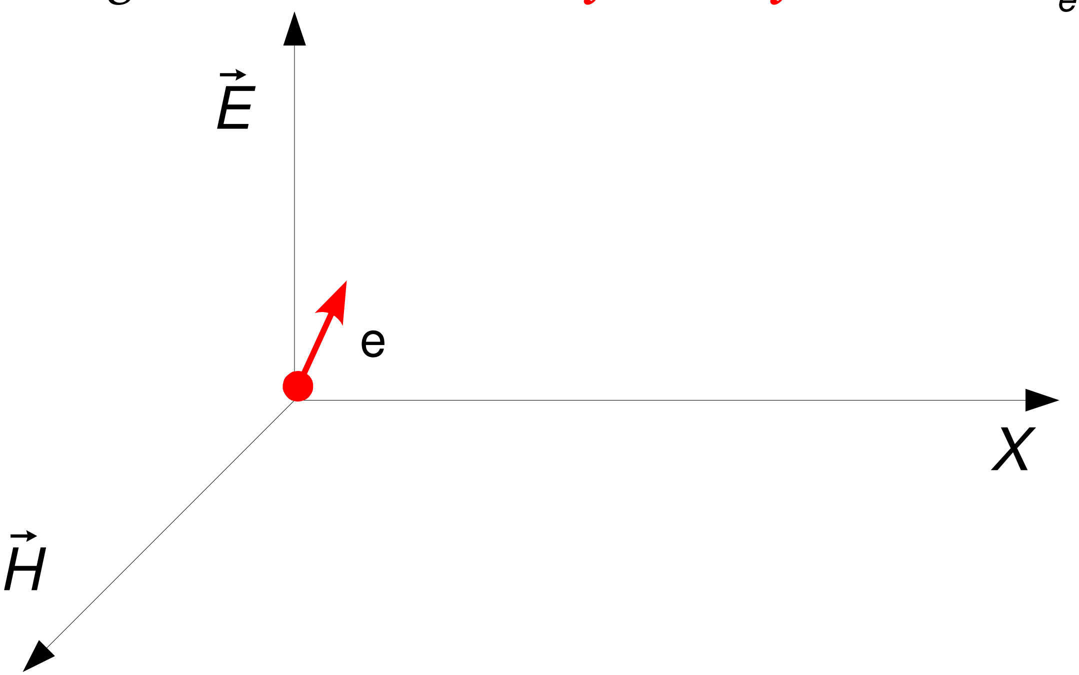

A
Light Cylinder
Radio beam
Rotation axis
Region of closed magnetic field lines
Open magnetic field lines
outer gap
Cone of radio emission
Gamma rays
Slot gap
Magnetic axis
Gamma rays
1000 km
B
Base of radio emission cone
Neutron star
100 km
C
Gamma rays
Base of slot gap
Polar gap
Pulsar surface
Polar cap
0.1 – 1 km

Pulsar with $P = 1$ *s rotates at* $\nu = 1$ *Hz and emits radiation with* $\lambda = c/\nu \sim 3\ 10^{10}$ *cm*

At a given distance (e.g. λ *) the energy flux of the emitted e-m wave is = (rotational) kinetic energy loss, and we can infer the value of the electric field*

$$\frac{|\vec{E}^2|}{8\pi} \times c = -\frac{1}{4\pi\lambda^2}\frac{dT_{NS}}{dt}$$

a charge in λ*,* ***in its own reference frame****:* $m_e \dot{\vec{v}} = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}\right)$



Let's write

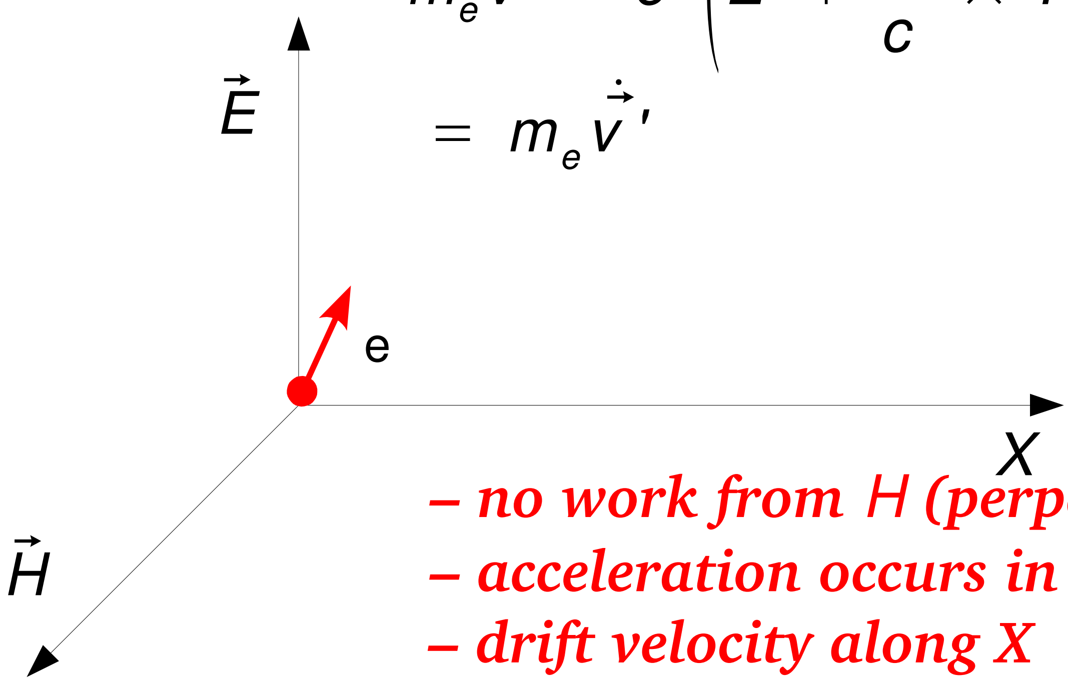
$$\vec{v}_X = \frac{c}{H^2}\left(\vec{E} \times \vec{H}\right) \qquad (constant\,, E\; and\; H\; stationary)$$

$$\vec{v}\,' = \vec{v} - \vec{v}_X \qquad i.e. \qquad \vec{v} = \vec{v}_X + \vec{v}\,' = \frac{c}{H^2}\left(\vec{E} \times \vec{H}\right) + \vec{v}\,'$$

$$given \;\; that \qquad \frac{\vec{v}_X}{c} \times \vec{H} = \frac{c}{H^2}\left(\vec{E} \times \vec{H}\right) \times \vec{H} = -\vec{E}\,\frac{H^2}{H^2} = -\vec{E}$$

$$m_e \dot{\vec{v}} = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}\right) = e\left(\cancel{\vec{E}} - \cancel{\vec{E}} + \frac{\vec{v}\,'}{c} \times \vec{H}\right) =$$

$$= m_e \dot{\vec{v}}\,'$$



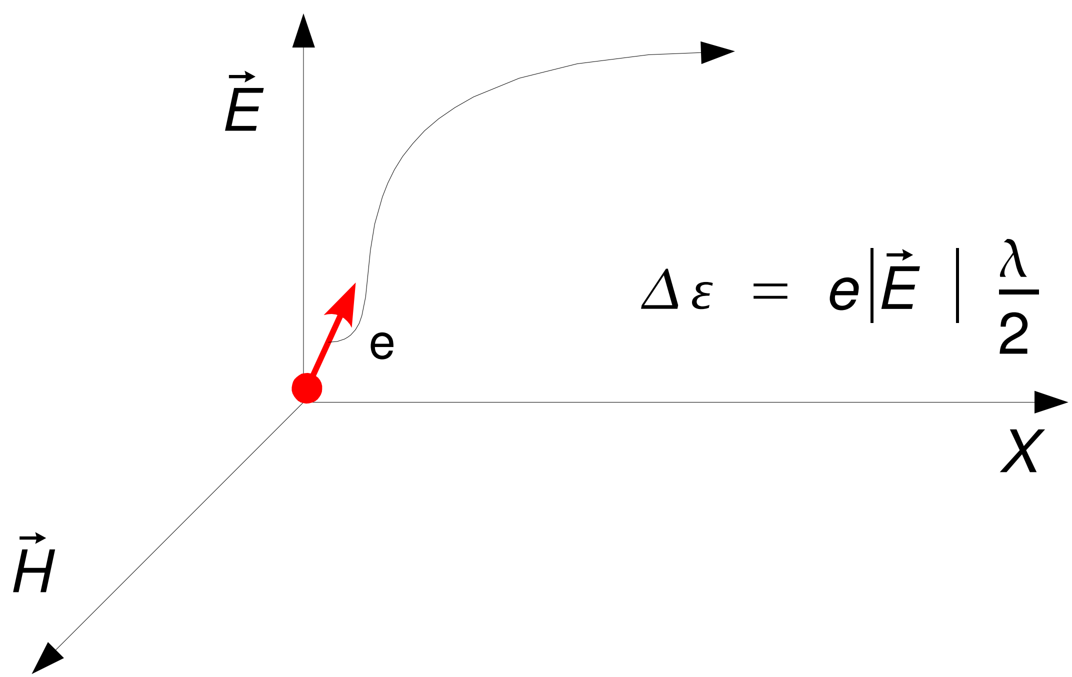
– ***no work from*** H ***(perpendicular to*** v***) (***ΔK ***from*** E ***filed)***
– ***acceleration occurs in the E,X plane***
– ***drift velocity along X***

Pulsar with $P = 1$ s rotates at $\nu = 1$ Hz and emits radiation with $\lambda = c/\nu \sim 3\ 10^{10}$ cm

Let' determine $\vec{v}_X = \frac{c}{H^2}\left(\vec{E} \times \vec{H}\right)$ then $\left|\vec{v}_X\right| = c\frac{\left|\vec{E}\right|}{H} \approx c$

for a plane wave (fields have the same intensity)
Such a speed is reached in a very short time and the acceleration comes from the electric field

$$\tau_X \approx \frac{v_X}{a} = v_X \frac{m_e}{eE} \approx \frac{cm_e}{eH} = \tau_L \ll \frac{2\pi}{\omega_{NS}}$$



$$\Delta\varepsilon = e\left|\vec{E}\right| \frac{\lambda}{2} \sim 10^{14} eV \qquad \left[\left|\vec{E}\right| = 10^6\ Volt\ sec^{-1}\right]$$

n_e < 10^-4 -> rotatore obliquo (1 Hz > v_plasma)

$$\vec{H} = H_{pole} R_{NS}^3 \left(\frac{\cos\theta\,\vec{r}}{R^3} + \frac{\sin\theta\,\vec{\theta}}{2R^3} \right) 1$$

HNS || omegaNS

NS is rotating then in its interior strong E fields are generated as effect of the Lorentz force inducing a charge separation

$$\vec{E} + \frac{1}{c} (\vec{\omega_{NS}} \times \vec{R}) \times \vec{H} = 0$$

$$\rho_e = \frac{1}{4\pi} \nabla \cdot \vec{E} = -\frac{1}{2\pi c} \vec{\omega_{NS}} \cdot \vec{H}$$

– charges at poles and + at equator if >0

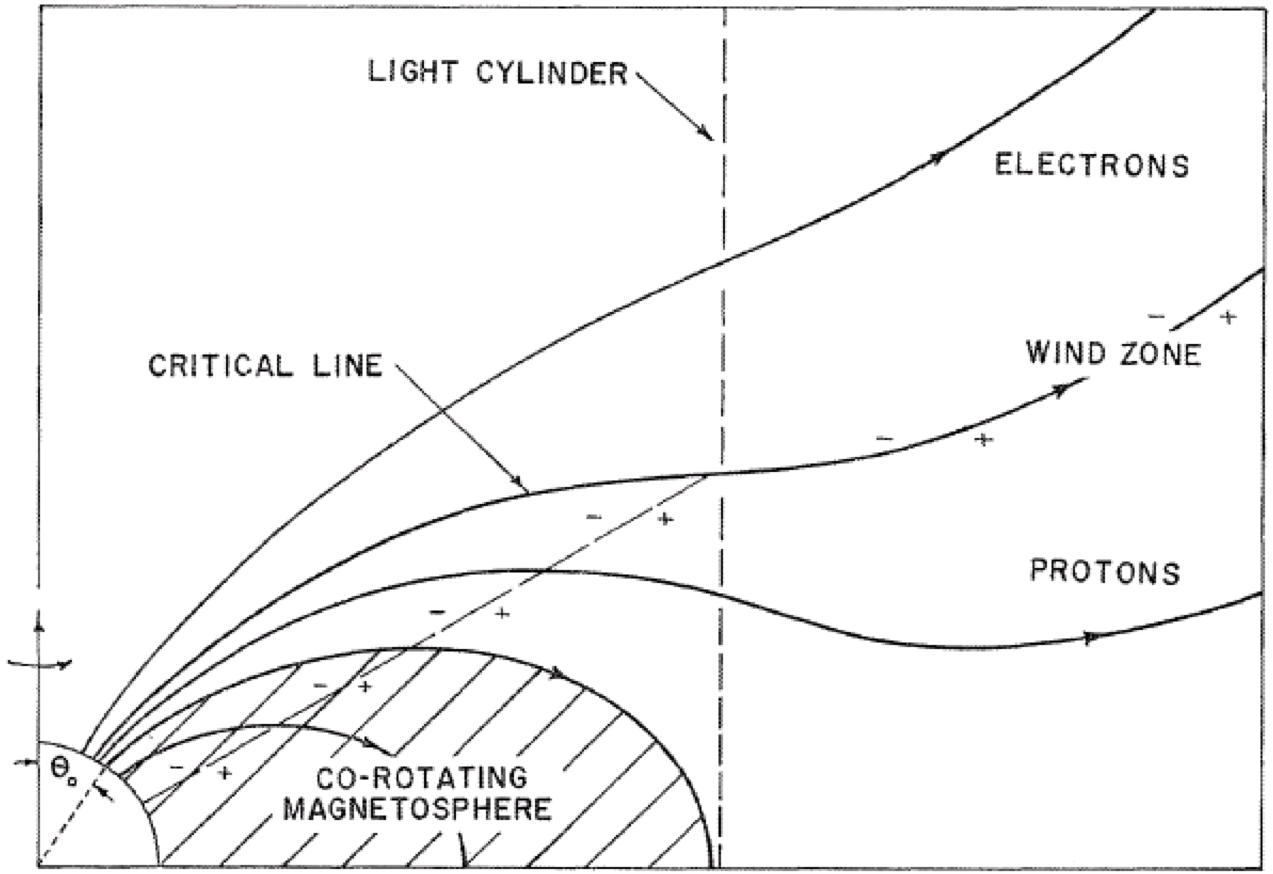
implies that E and H are perpendicular

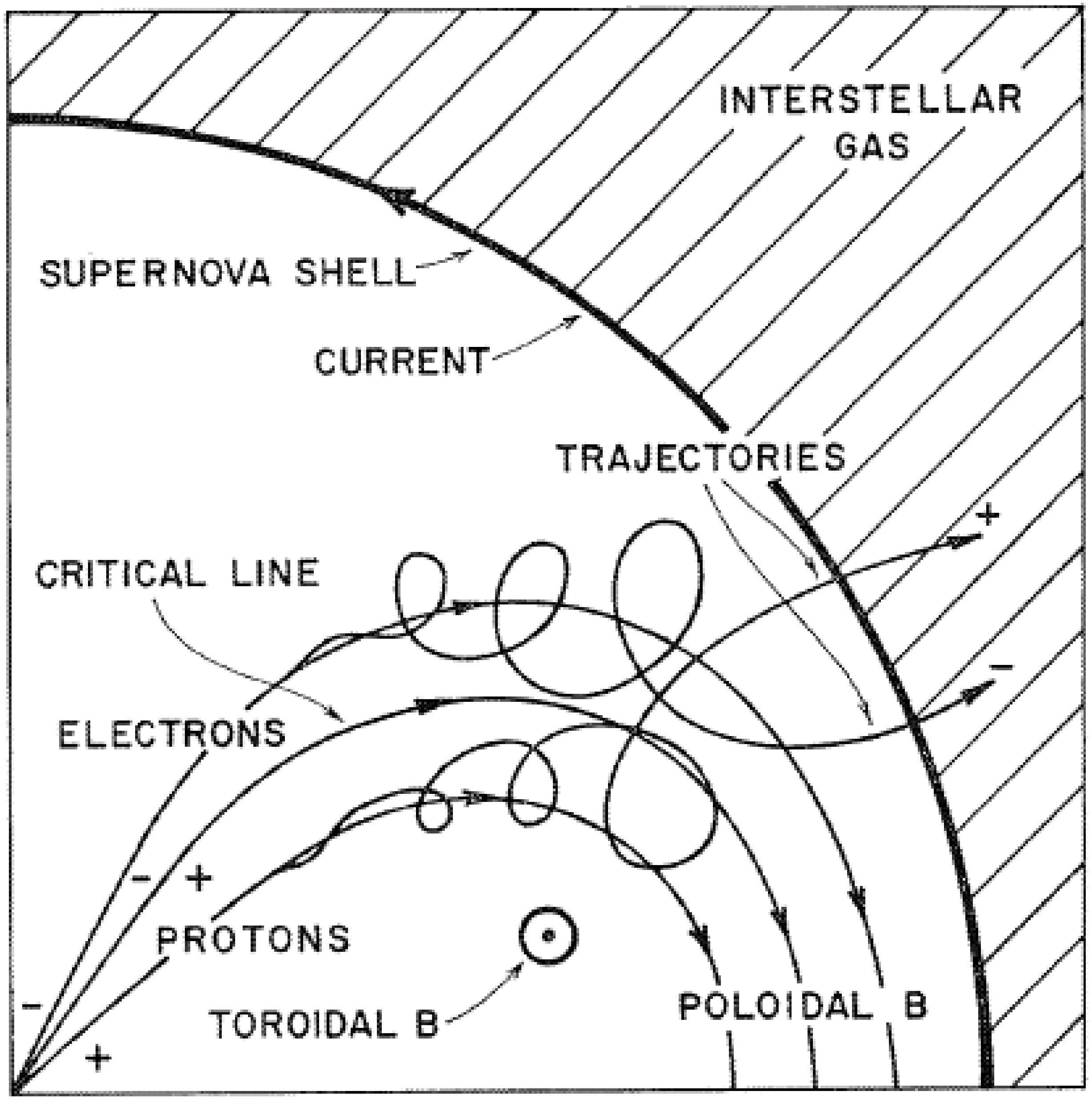
hollow cone

n_e < 10^-4 -> rotatore obliquo (1 Hz > v_plasma)

HNS || omegaNS

$$\vec{H} = H_{pole} R_{NS}^3 \left(\frac{\cos\theta \, \vec{r}}{R^3} + \frac{\sin\theta \, \vec{\theta}}{2R^3} \right) 1$$




INTERSTELLAR GAS
SUPERNOVA SHELL
CURRENT
TRAJECTORIES
CRITICAL LINE
ELECTRONS
PROTONS
TOROIDAL B
POLOIDAL B
+
−

$\vec{\omega}_{NS}$ *antiparallel* $\vec{H}$

+ at the poles and – at the equator (bound!)

A pulsar to start up needs:

a high energy photon (γ-rays!) > 1 MeV interact with another soft photon within the H field and originates leptonic pairs: e^+/e^-

these pairs are then accelerated by the E field in the polar cap and produce high energy curvature radiation capable in turn to originate new pairs, and so on

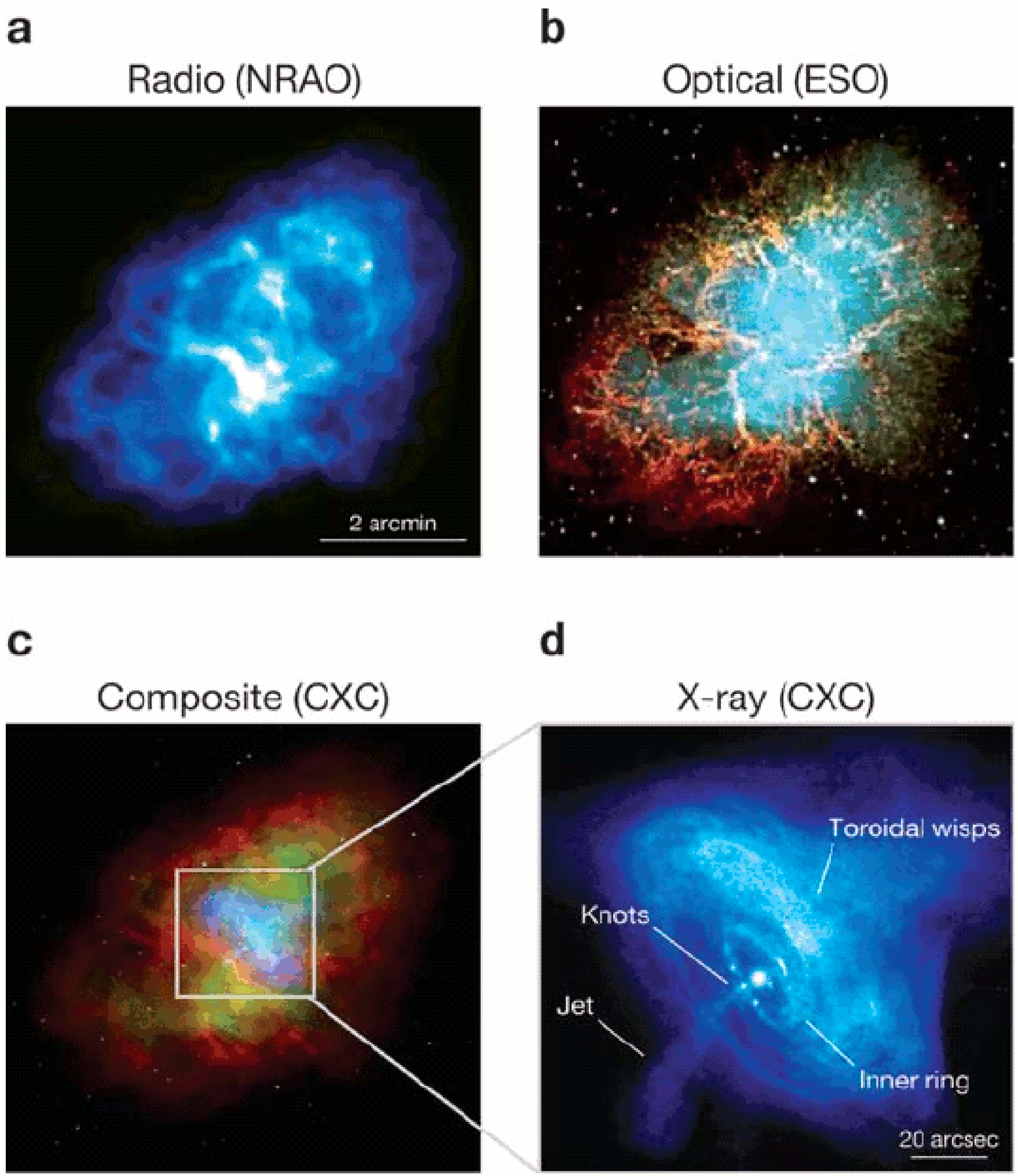
needs that:

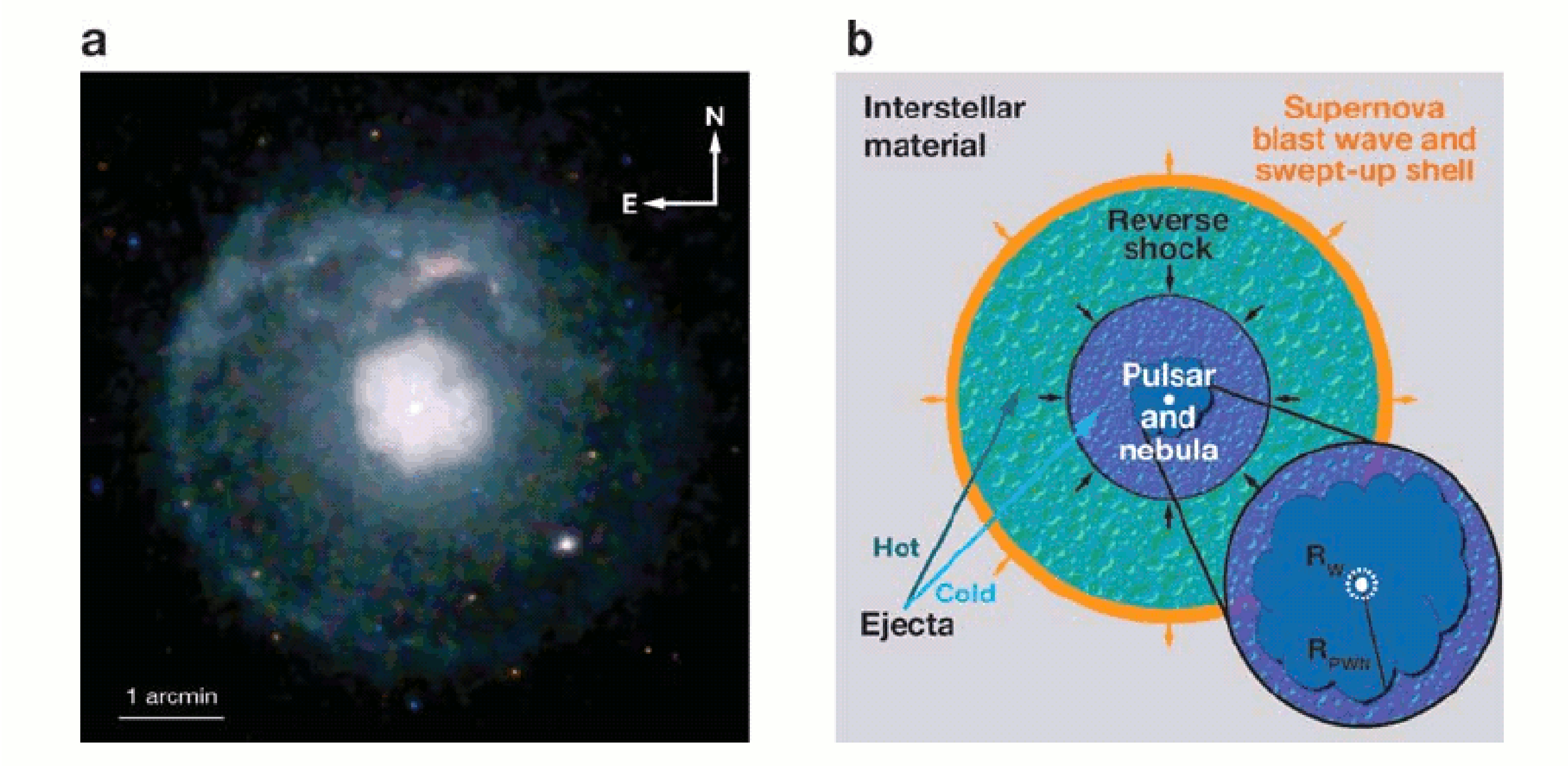
the mean free path of γ-rays is shorter than the H scale height

e^+/e^- pairs can be accelerated enough to produce γ-ray radiation themselves

from these requirements it comes

$$\left(\frac{H}{10^{12}\,G}\right)\left(\frac{P^{-2}}{sec}\right) \geq 0.2$$


a
Radio (NRAO)
2 arcmin
b
Optical (ESO)
c
Composite (CXC)
d
X-ray (CXC)
Toroidal wisps
Knots
Jet
Inner ring
20 arcsec


a
N
E
1 arcmin
b
Interstellar material
Supernova blast wave and swept-up shell
Reverse shock
Pulsar and nebula
Hot
Cold
Ejecta
R_w
R_{PWN}

Brightness temperature in the radio band as high as 10^{23} *K*

particles are grouped in small "packets" with size l, containing N particles each.

as long as $l \ll \lambda$ *that package acts like a single particle with mass*

$m' = N l^3 m_e$

and charge

$q' \sim N l^3 m$

$$\frac{m' c^2 \gamma}{k} = \frac{N l^3 (m_e c^2 \gamma)}{k} > T_B > 10^{22}$$

Periastron precession

$$\dot{\omega} = 3 \left(\frac{P_b}{2\pi}\right)^{-5/3} (T_\odot M)^{2/3} (1 - e^2)^{-1}$$

Gravitational redshift

$$\gamma = e \left(\frac{P_b}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3} m_2 (m_1 + 2m_2)$$

Orbital decay

$$\dot{P}_b = -\frac{192\pi}{5} \left(\frac{P_b}{2\pi}\right)^{-5/3} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) (1 - e^2)^{-7/2} T_\odot^{5/3} m_1 m_2 M^{-1/3}$$

range of Shapiro Delay

$$r = T_\odot m_2$$

shape of Shapiro Delay

$$s = x \left(\frac{P_b}{2\pi}\right)^{-2/3} T_\odot^{-1/3} M^{2/3} m_2^{-1}.$$

$$T_O = \frac{GM_O}{c^3} = 4.925490947\,\mu s \qquad s = \sin i$$

$$M = m_1 + m_2 \quad (m_1 = m_{NS};\ m_2 = m_{comp}) \quad in\ M_O$$

Post Keplerian (PK) parameters

$$\dot{\omega} = 3 \left(\frac{P_b}{2\pi}\right)^{-5/3} (T_\odot M)^{2/3} (1-e^2)^{-1}$$

$$\gamma = e \left(\frac{P_b}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3} m_2 (m_1 + 2m_2)$$

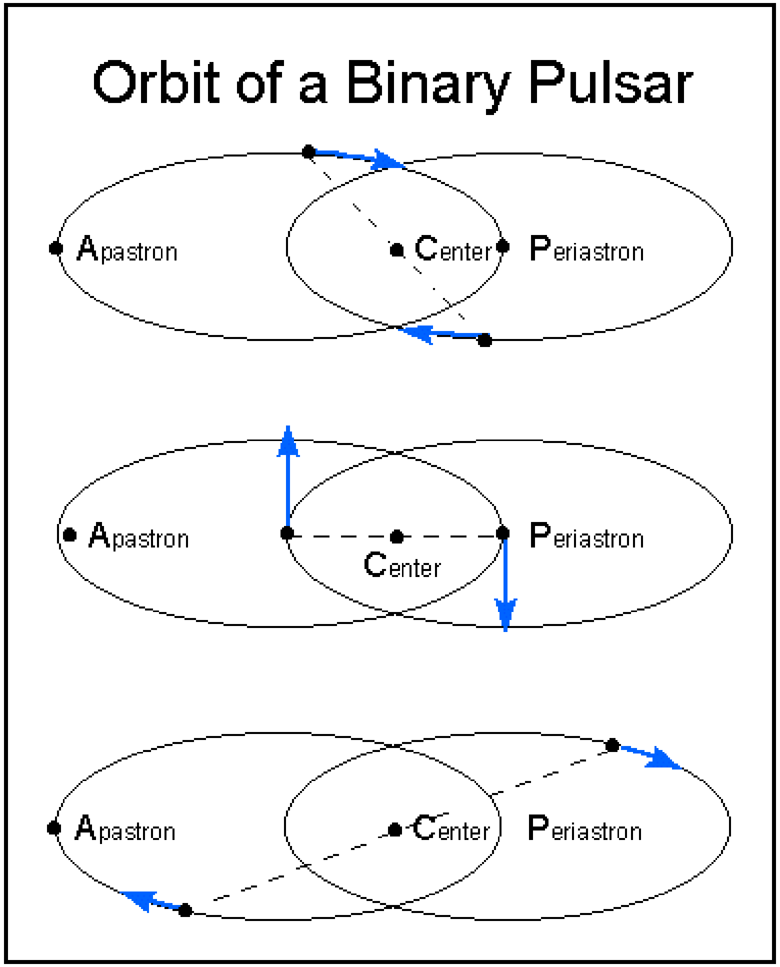
$$\dot{P}_b = -\frac{192\pi}{5} \left(\frac{P_b}{2\pi}\right)^{-5/3} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) (1-e^2)^{-7/2} T_\odot^{5/3} m_1 m_2 M^{-1/3}$$

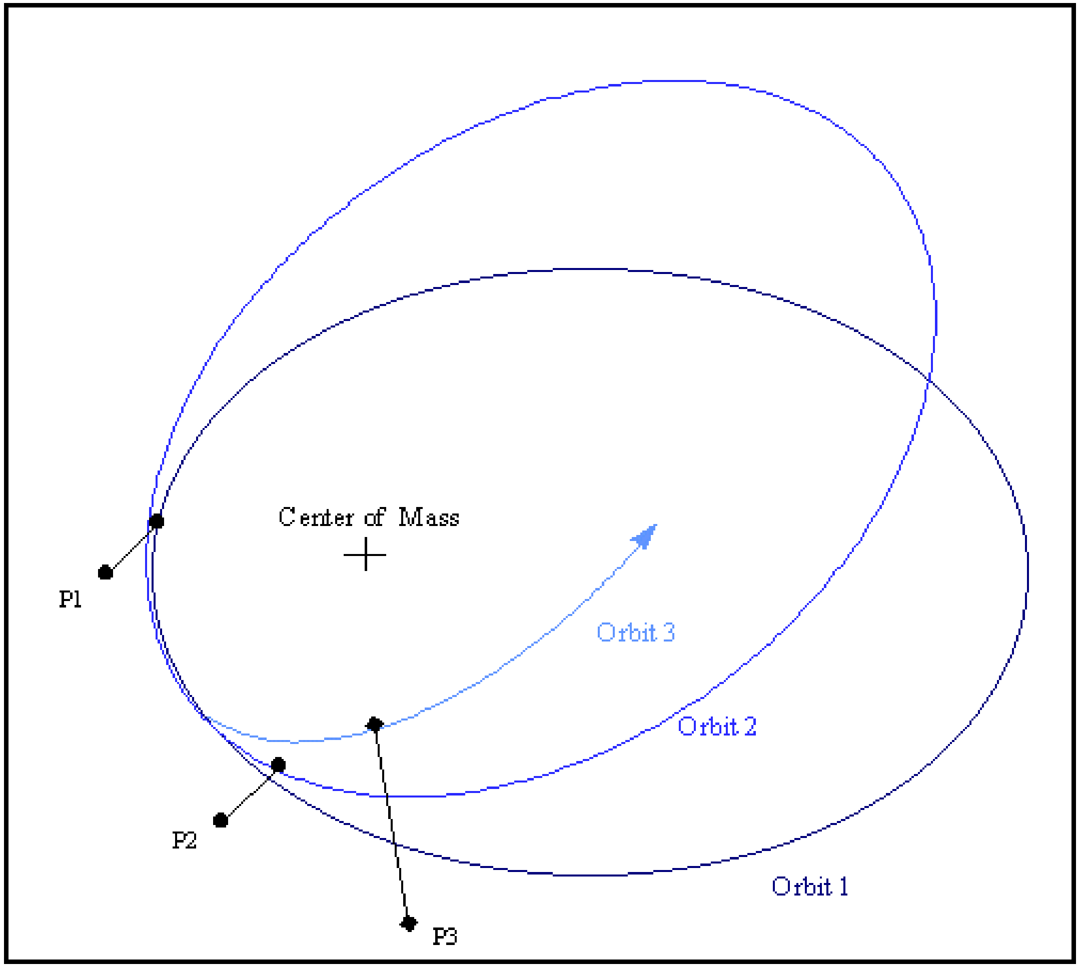
$$r = T_\odot m_2$$

$$s = x \left(\frac{P_b}{2\pi}\right)^{-2/3} T_\odot^{-1/3} M^{2/3} m_2^{-1}.$$

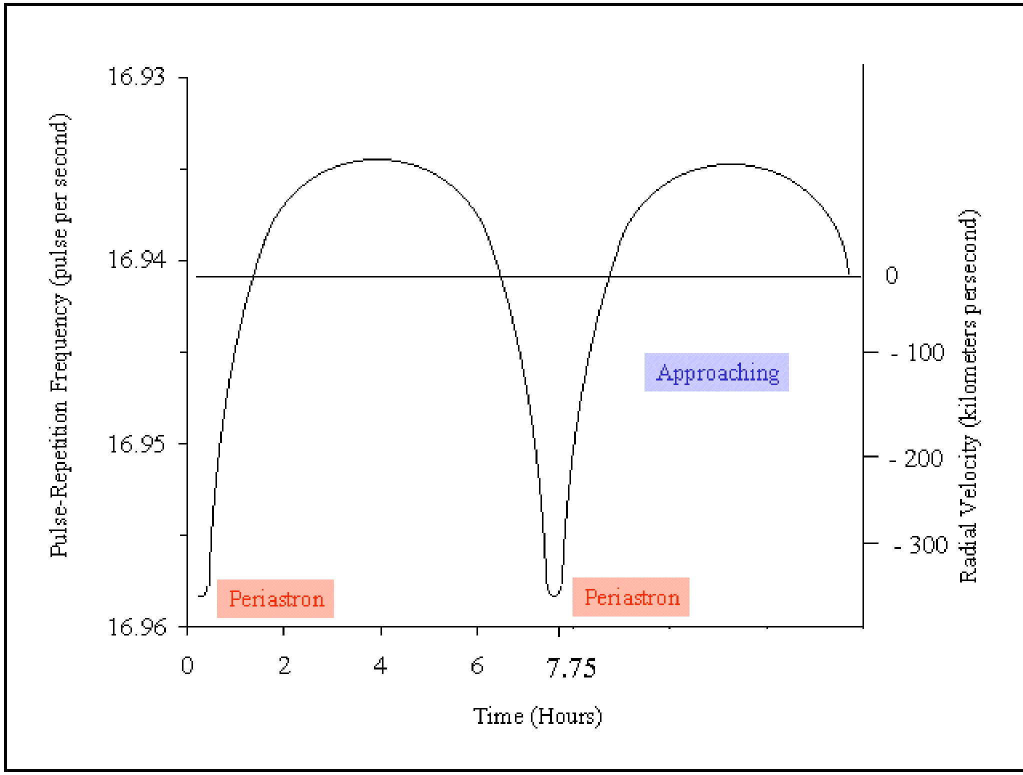
5 post-Keplerian (PK) parameters: the rate of periastron advance 'ω, the so-called relativistic γ (i.e. the Einstein term corresponding to time dilation and gravitational redshift), the orbital period decay Pb, and the Shapiro delay terms r (range) and s (shape).

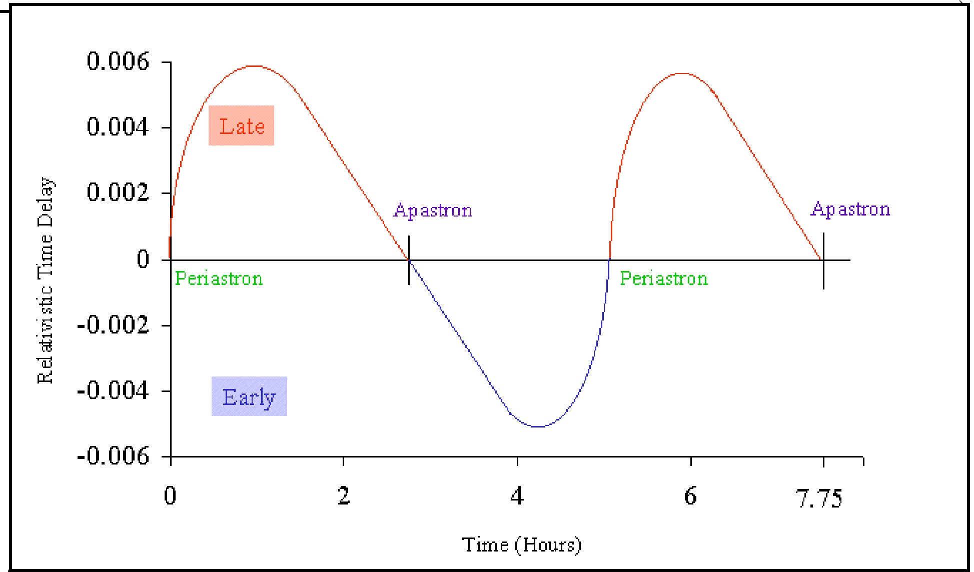
If any two of these PK parameters are measured, the masses of the pulsar and its companion can be determined. If more than two are measured, each additional PK parameter yields a different test of a gravitational theory.

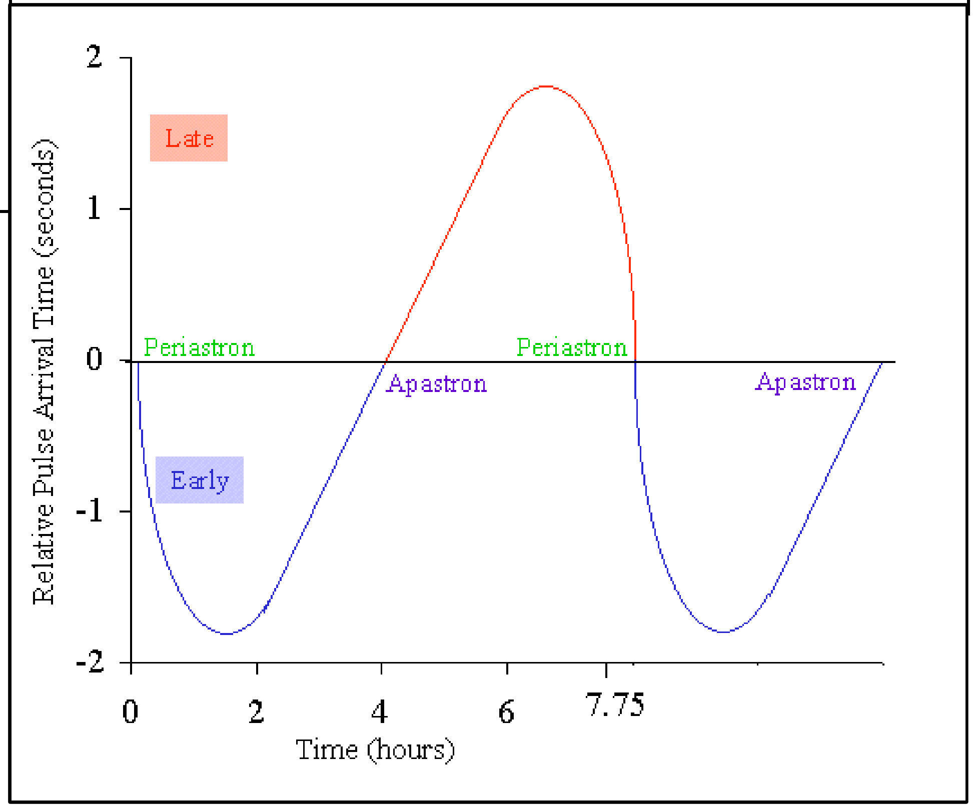

Orbit of a Binary Pulsar
Apastron
Center
Periastron
Apastron
Center
Periastron
Apastron
Center
Periastron


Center of Mass
P1
Orbit 3
Orbit 2
P2
Orbit 1
P3

Post Keplerian (PK) parameters

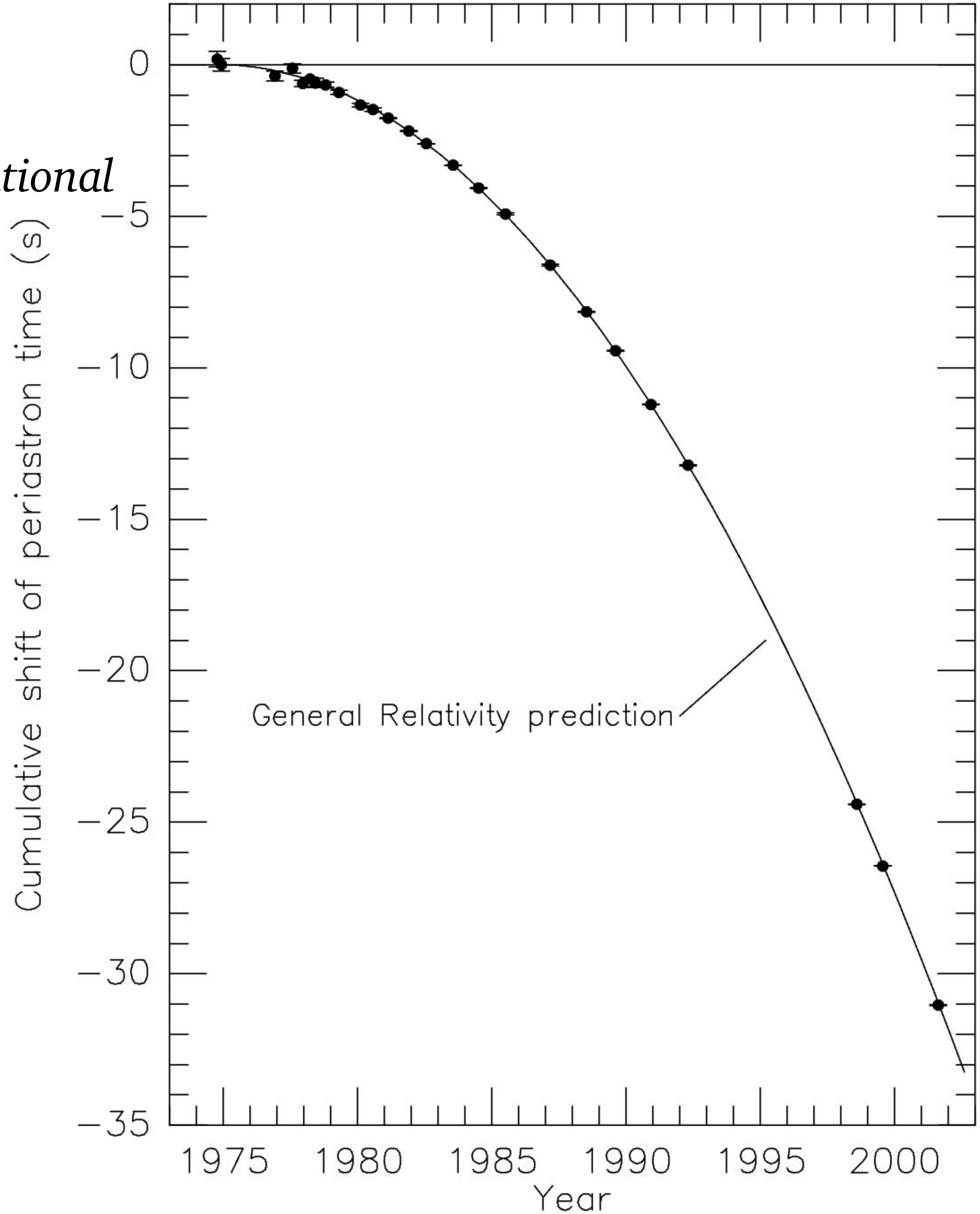






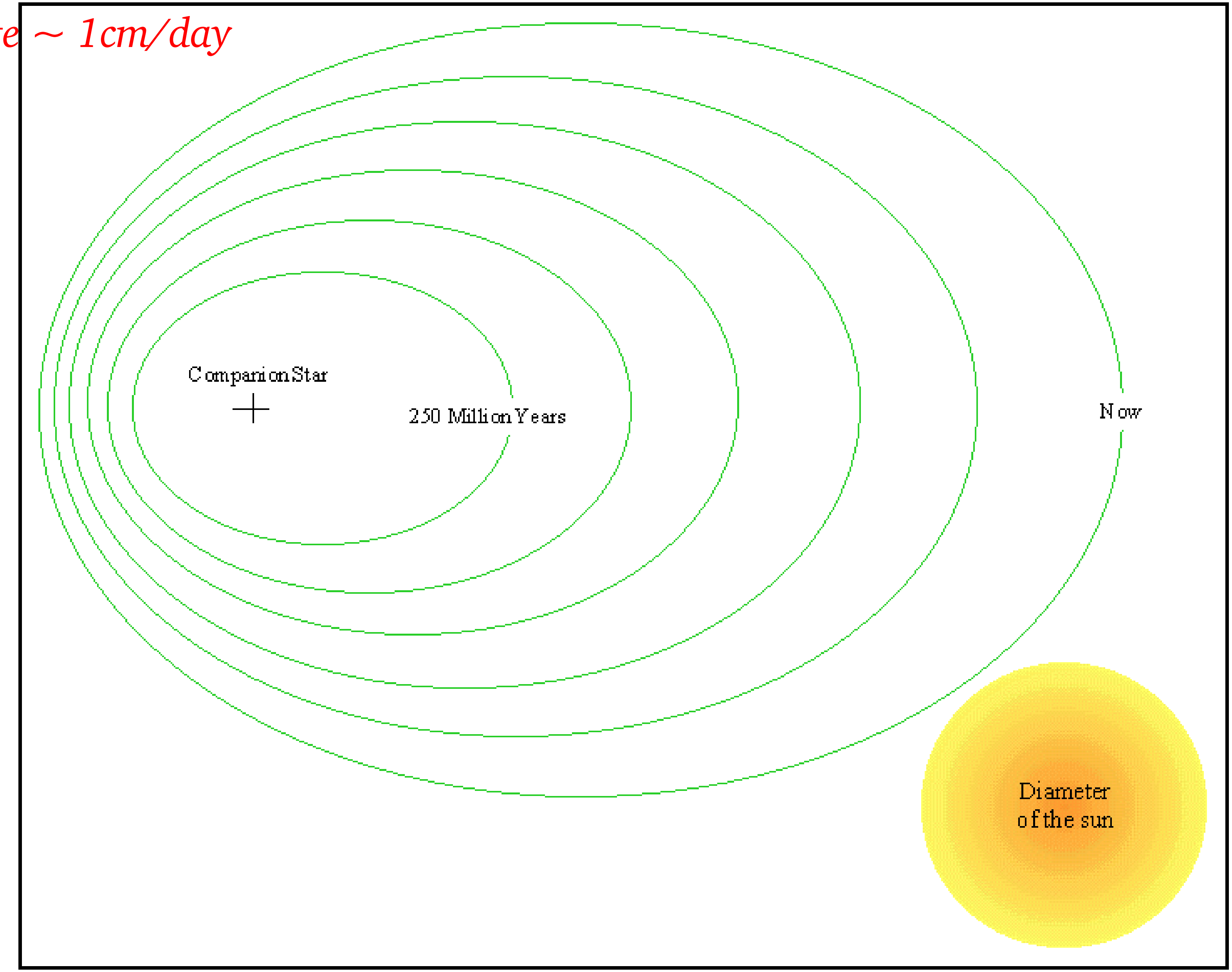
Hulse and Taylor:
orbital decay with release of gravitational waves:

orbital shrinkage ~ 1cm/day



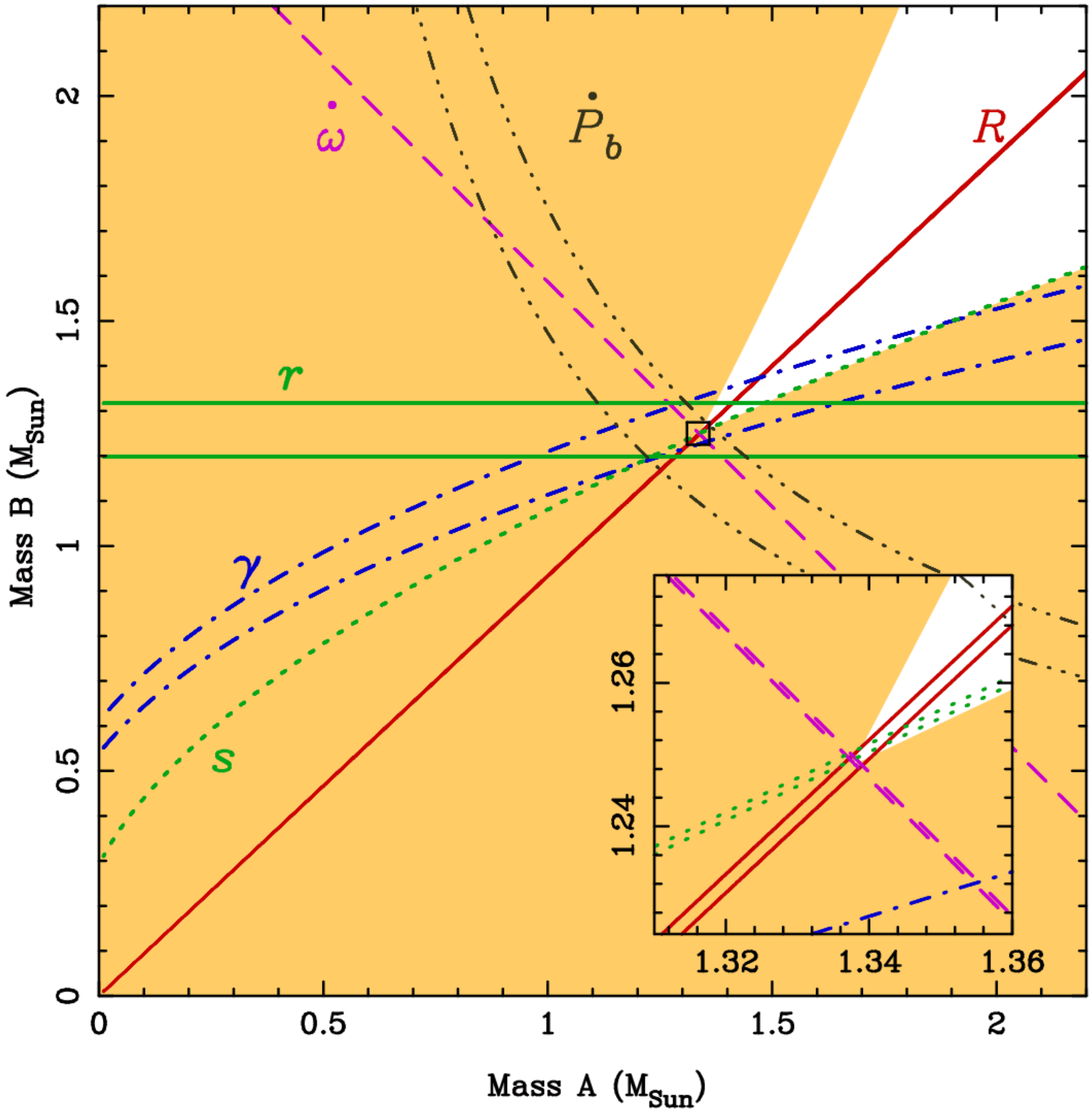
Hulse and Taylor: orbital decay with release of gravitationalwaves:

orbital shrinkage ~ 1cm/day

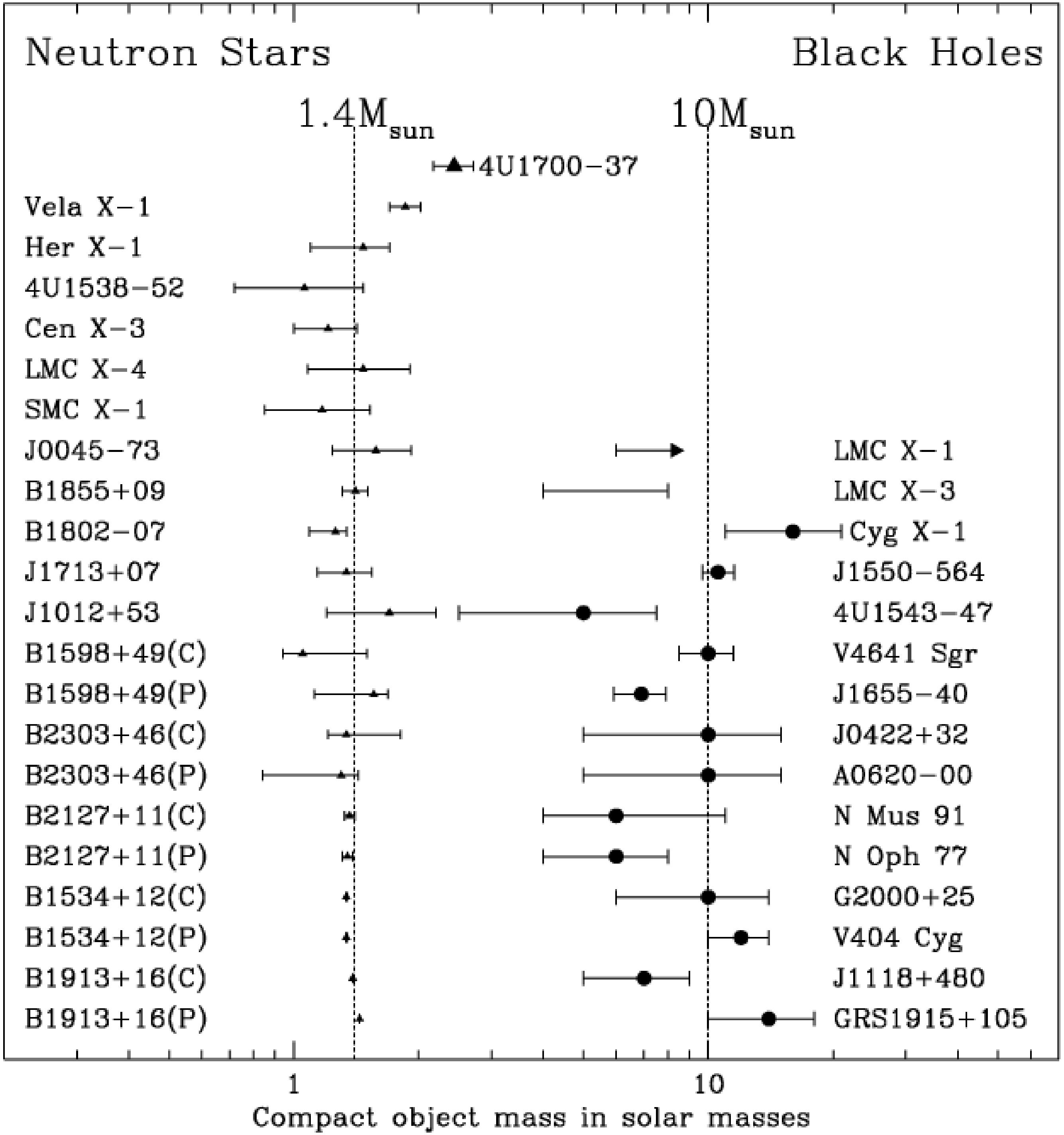


the double pulsar J0737-3039A (22.7 ms) & B (2.8 s) [Burgay + 2003]

M= 2.59 (



masses of the binary system J0737-3039A&B


Neutron Stars
Black Holes
1.4M sun
10M sun
4U1700−37
Vela X−1
Her X−1
4U1538−52
Cen X−3
LMC X−4
SMC X−1
J0045−73
B1855+09
B1802−07
J1713+07
J1012+53
B1598+49(C)
B1598+49(P)
B2303+46(C)
B2303+46(P)
B2127+11(C)
B2127+11(P)
B1534+12(C)
B1534+12(P)
B1913+16(C)
B1913+16(P)
LMC X−1
LMC X−3
Cyg X−1
J1550−564
4U1543−47
V4641 Sgr
J1655−40
J0422+32
A0620−00
N Mus 91
N Oph 77
G2000+25
V404 Cyg
J1118+480
GRS1915+105
1
10
Compact object mass in solar masses

mass determination

efficient in double systems with both "compact" stars (NS-NS)
no mass transfer, no tidal effects, predictions of general relativity can be tested

e.g. PSR B1913+16
1993: Nobel Prize in Physics: Russell Hulse & Joseph Taylor (Princeton University)
discovery of a pulsar (in 1974) in a binary system, in orbit with another star around a common center of mass

coherent emission

Post Keplerian (PK) parameters

mass determination

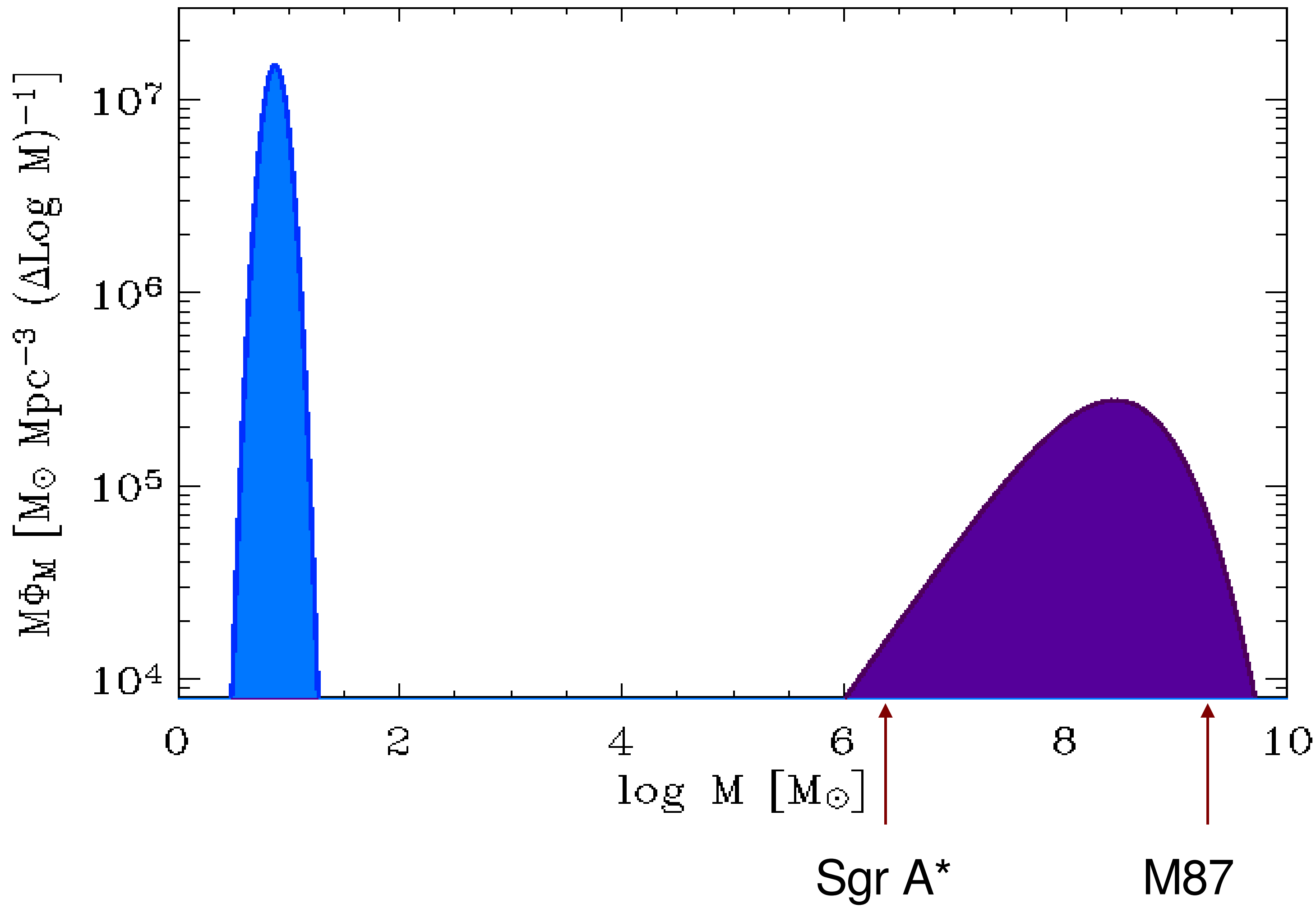
TOV limit

Two main classes: Stellar mass BHs $M_{BH} < 10^2 M_\odot$

Galactic size BHs $10^5 M_\odot < M_{BH} < 10^6 M_\odot$

Alternative division:

Primordial BHs	$M_{BH} < 1 M_\odot$	(early Universe BHs)
Stellar BHs	$2.5 M_\odot < M_{BH} < 50 M_\odot$	(either direct formation from massive star evolution or accretion onto NSs)
Intermediate mass BHs	$50 M_\odot < M_{BH} < 10^5 M_\odot$	(ULX sources ?)
High redshift BHs	$10^5 M_\odot < M_{BH} < 10^6 M_\odot$	(direct collapse of supermassive clouds in massive dark halos in early Universe [z~25])
Supermassive BHs	$10^6 M_\odot < M_{BH} < 10^{10} M_\odot$	(at the center of spheroidal galaxies, may be long lasting growth/accretion. Found at z=6 already!)



$M\Phi_M$ [$M_\odot$ Mpc^{-3} (ΔLog M)$^{-1}$]
10^7
10^6
10^5
10^4
0
2
4
6
8
10
log M [$M_\odot$]
Sgr A*
M87

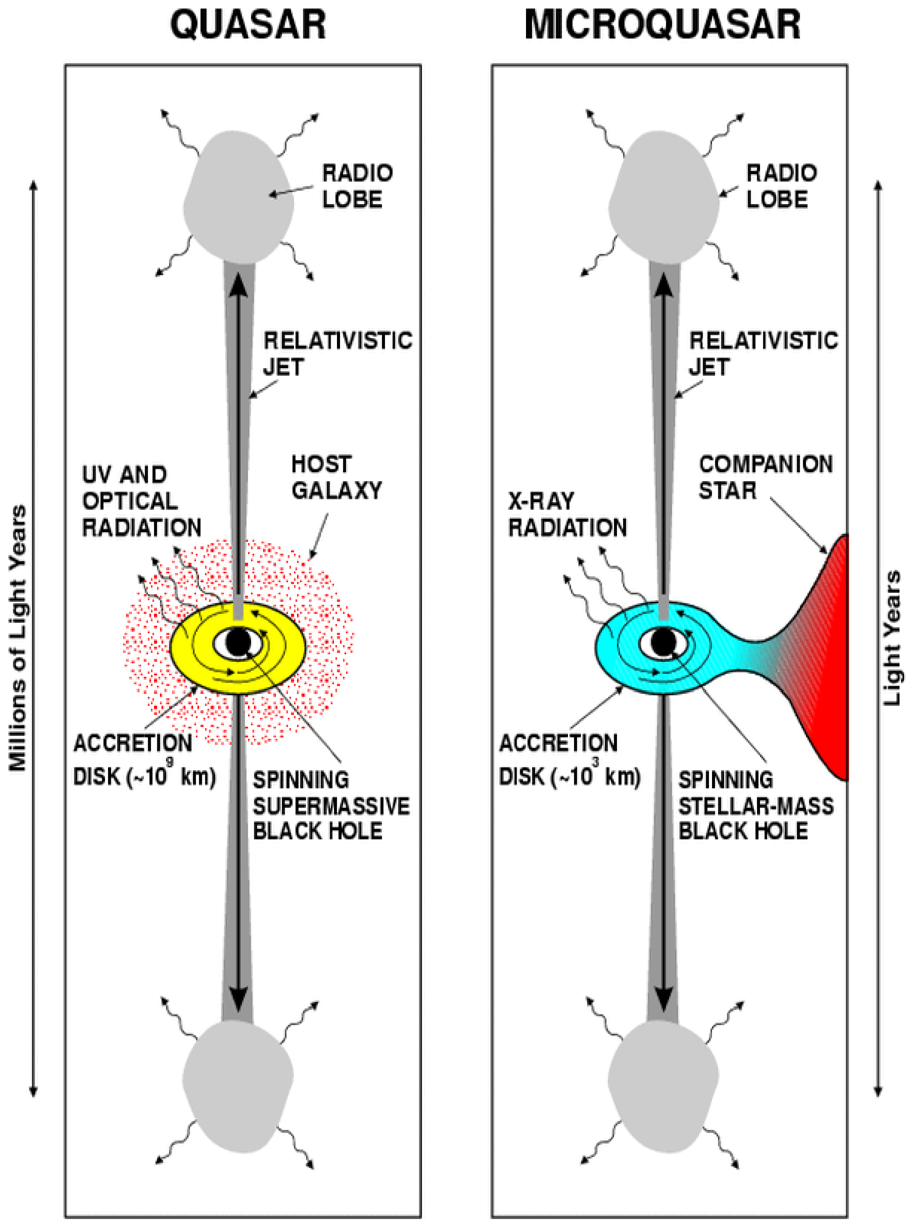
Standard method in X-ray binaries:

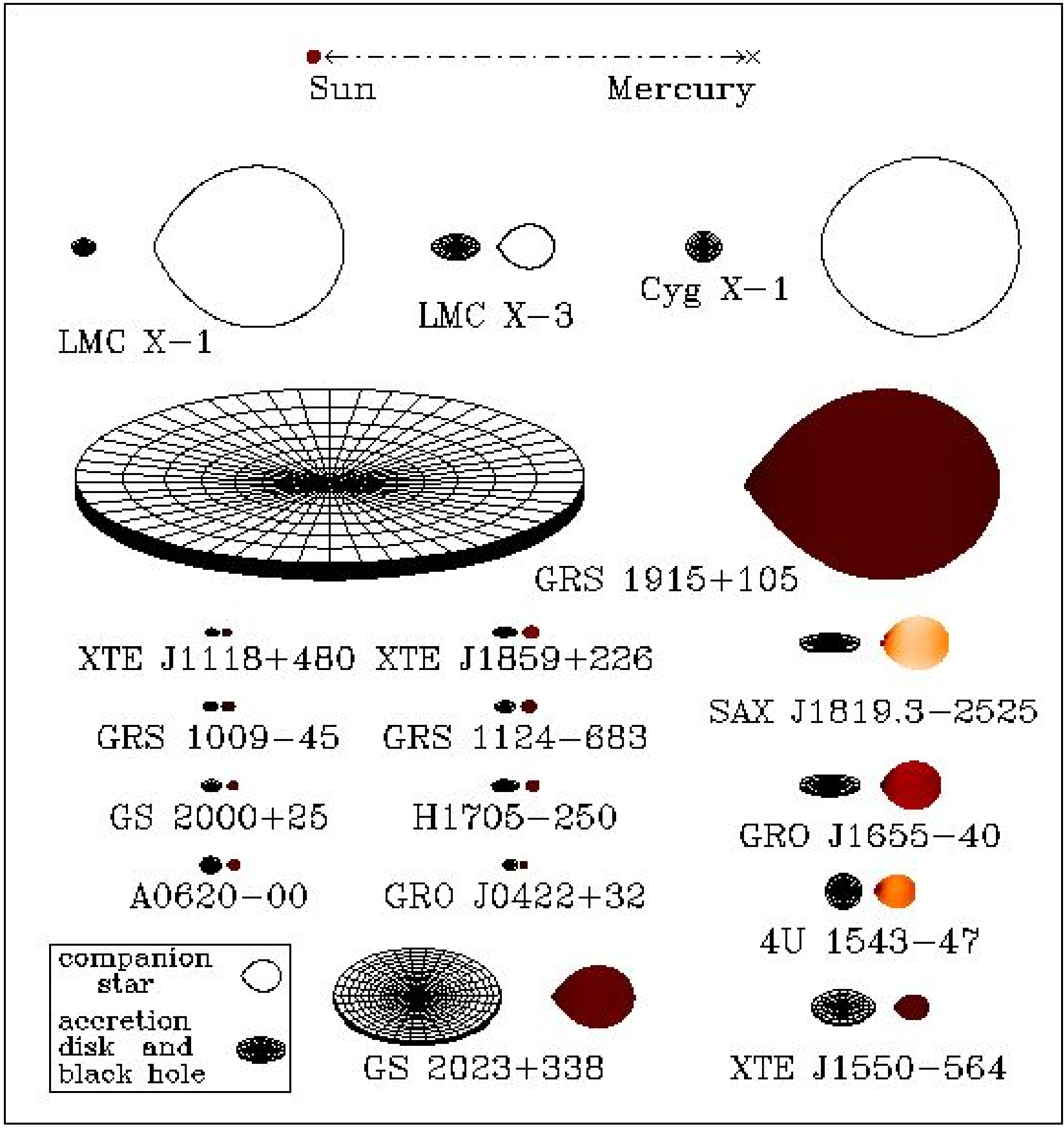
$$f(M_{BH}, i) = \frac{K_c^3 P_b}{2\pi G} = \frac{M_{BH}^3 \sin^3 i}{(M_{BH} + M_c)^2}$$

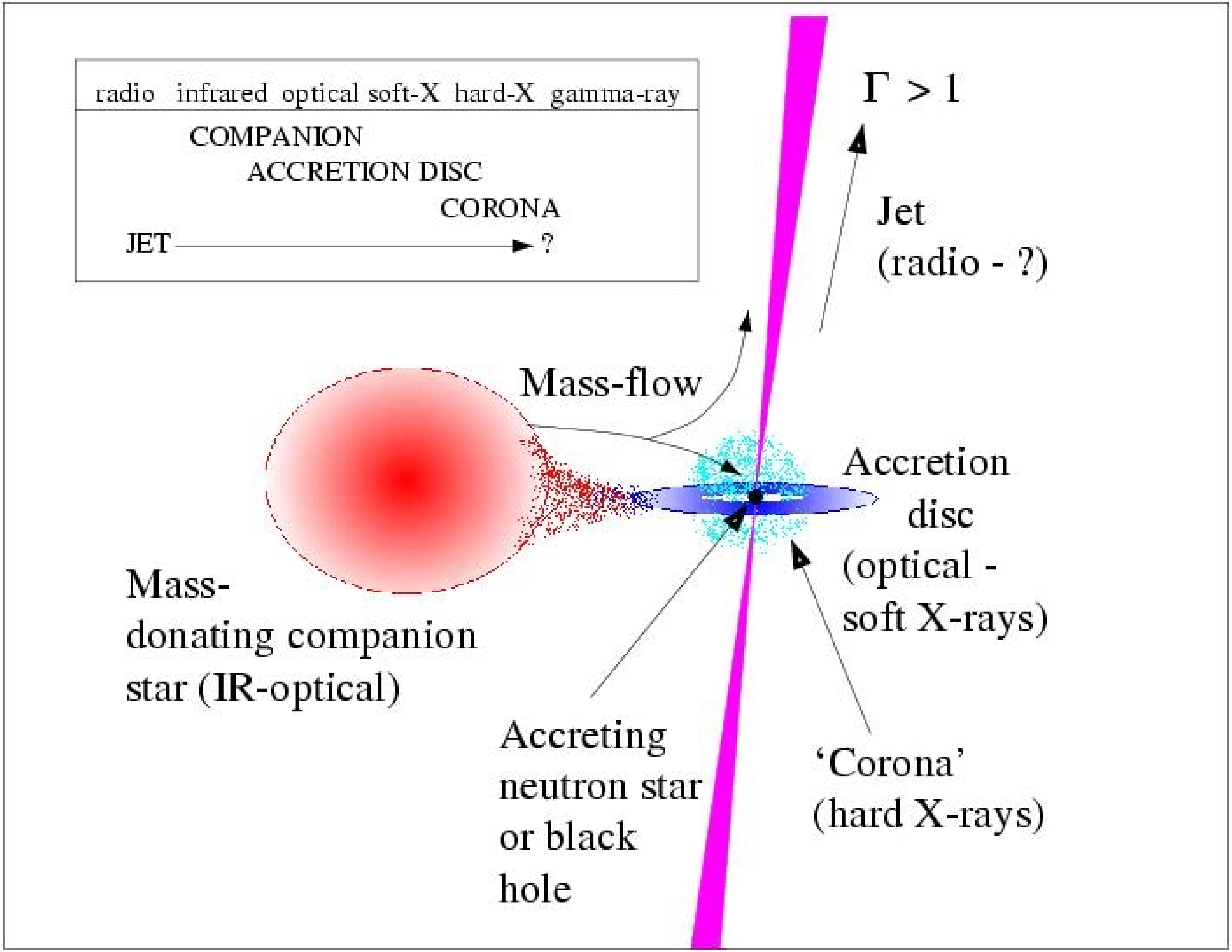
where P_b *is the orbital period and* K_c *is the maximum LoS Doppler velocity of the companion star. They can be both generally measured. Other parameters like the mass of the companion star* M_c *and the inclination of the orbit* i *need to be estimated.*

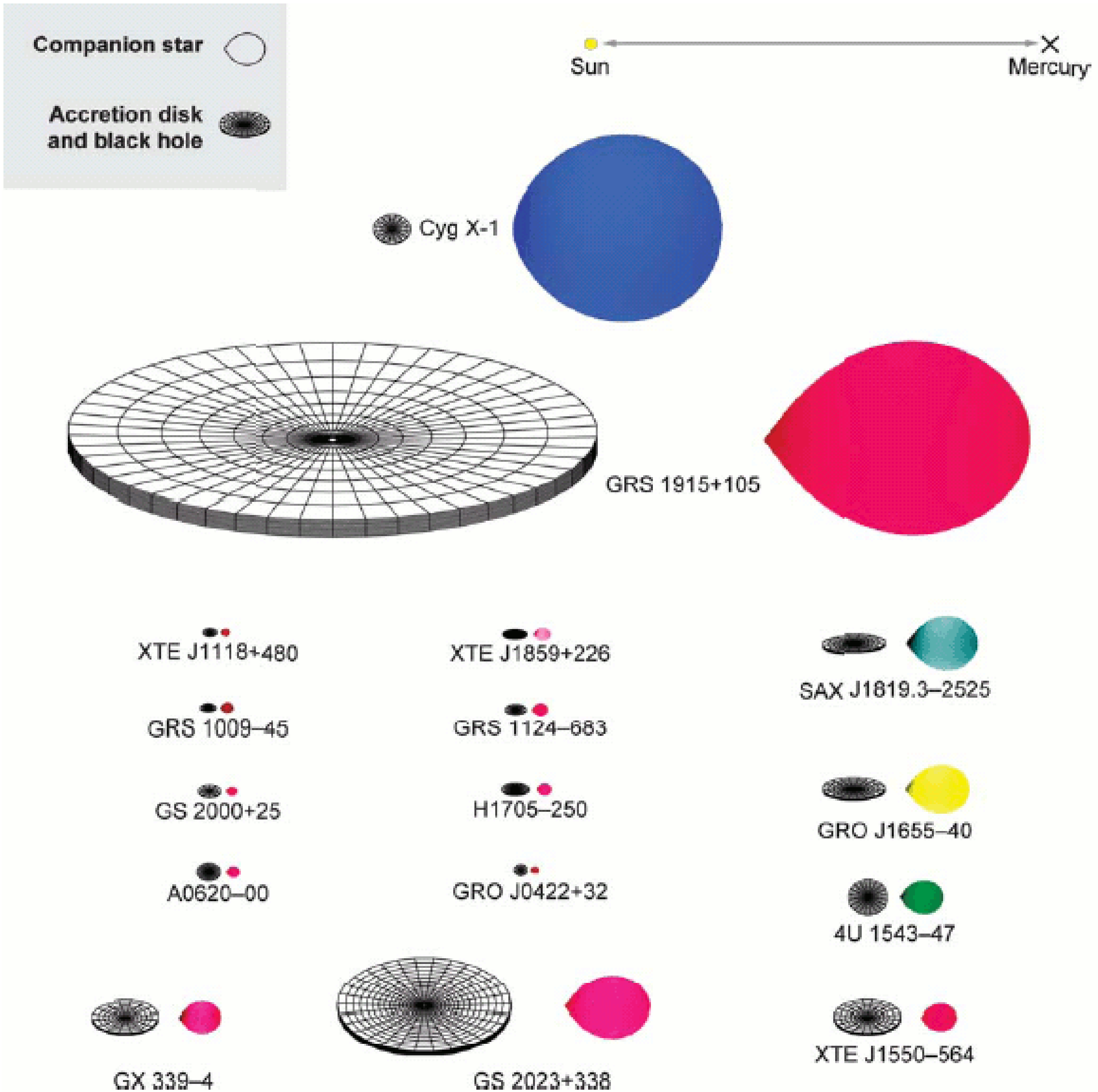
about 20-30 XRB have the mass of the compact object consistent with a stellar BH.

About $10^8 - 10^9$ *BH in binary systems are believed to exist in our own galaxy (Remillard & McClintock 2006).*

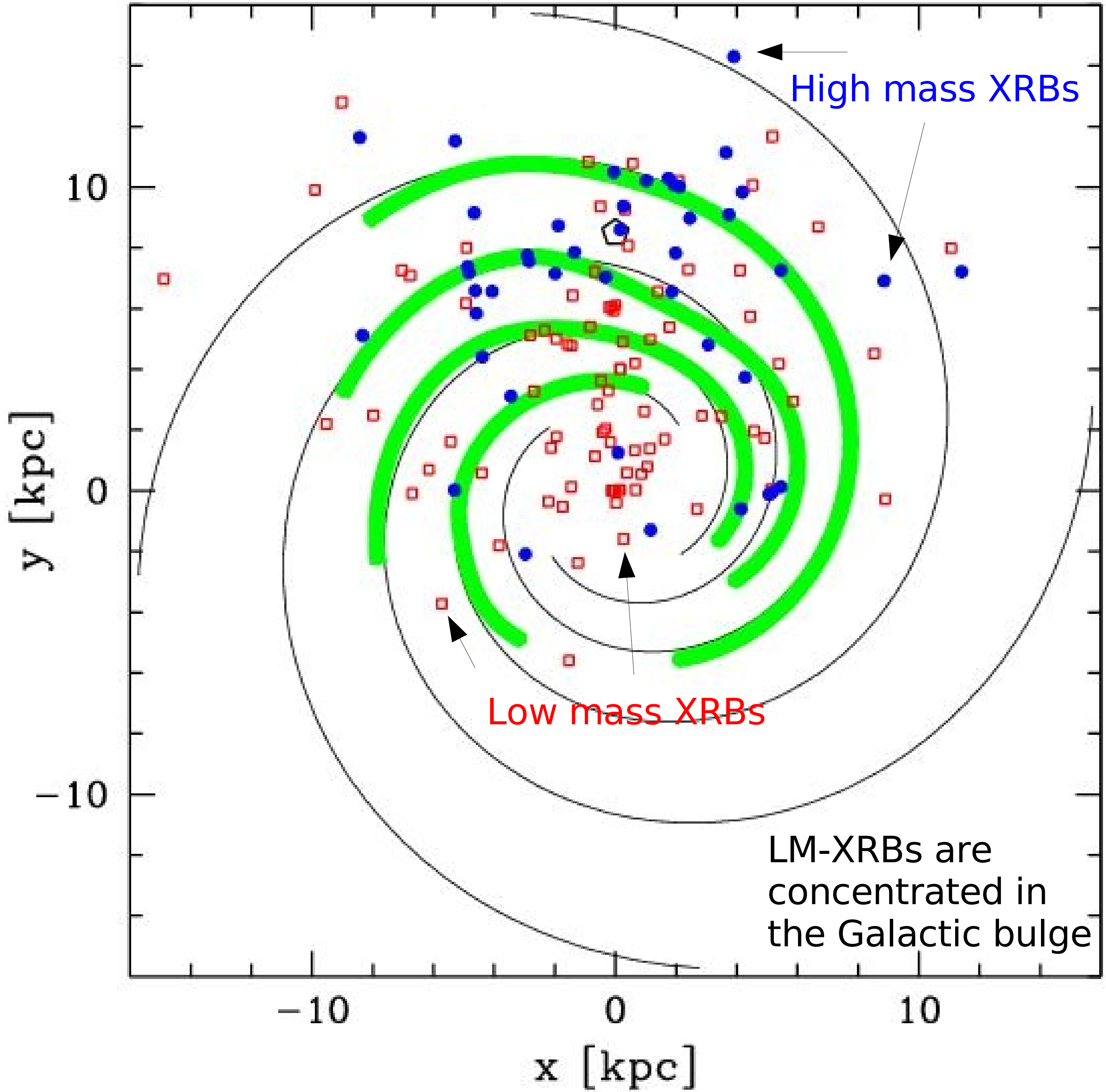

QUASAR
MICROQUASAR
RADIO LOBE
RELATIVISTIC JET
UV AND OPTICAL RADIATION
HOST GALAXY
ACCRETION DISK (~10^9 km)
SPINNING SUPERMASSIVE BLACK HOLE
Millions of Light Years
X-RAY RADIATION
COMPANION STAR
ACCRETION DISK (~10^3 km)
SPINNING STELLAR-MASS BLACK HOLE
Light Years


Sun
Mercury
LMC X−1
LMC X−3
Cyg X−1
GRS 1915+105
XTE J1118+480
XTE J1859+226
SAX J1819.3−2525
GRS 1009−45
GRS 1124−683
GS 2000+25
H1705−250
GRO J1655−40
A0620−00
GRO J0422+32
4U 1543−47
companion star
accretion disk and black hole
GS 2023+338
XTE J1550−564


radio infrared optical soft-X hard-X gamma-ray
COMPANION
ACCRETION DISC
CORONA
JET ?
Γ > 1
Jet
(radio - ?)
Mass-flow
Accretion
disc
(optical -
soft X-rays)
Mass-
donating companion
star (IR-optical)
Accreting
neutron star
or black
hole
'Corona'
(hard X-rays)



Companion star
Accretion disk and black hole
Sun
Mercury
Cyg X-1
GRS 1915+105
XTE J1118+480
XTE J1859+226
SAX J1819.3–2525
GRS 1009–45
GRS 1124–683
GS 2000+25
H1705–250
GRO J1655–40
A0620–00
GRO J0422+32
4U 1543–47
GX 339–4
GS 2023+338
XTE J1550–564



High mass XRBs
Low mass XRBs
LM-XRBs are concentrated in the Galactic bulge
10
0
−10
y [kpc]
x [kpc]

TABLE 1. Galactic binary sources showing relativistic jets ($v > 0.1c$)

Source	X-ray[1]	Radio[1]	Accretor	Donor	D (kpc)	V_{app}[2]	V_{int}[3]	Θ[4]
GRS 1915+105	t	t	black hole?		10–12	1.2c-1.7c	0.92c-0.98c	66°-70°
GRO J1655-40 (V1033 Sco)	t	t	black hole	F3-6 IV	3.2	1.1c	0.92c	72°-85°
XTE J1748-288	t	t	black hole?		8–10	0.9c-1.5c	>0.93c	
XTE J1819–254 (V4641 Sgr)[5]	t	t			0.5?	~0.8c		
SS 433 (V1343 Aql)	t	t	neutron star	OB?	5.5	0.26c	0.26c	79°
Cygnus X-3 (V1521 Cyg)	t	t	neutron star?	WR?	8–10	~0.3c	~0.3c	>70°
CI Cam (XTE J0421+560)	t	t	neutron star?	Be?	~1	~0.15c	~0.15c	>70°
Circinus X-1 (BR Cir)	p	p	neutron star	MS	8.5	≥0.1c	≥0.1c	>70°
1E1740.7-2942	p	p	black hole?		8–10			
GRS 1758-258	p	p	black hole?		8–10			

(1) t ≡ transient, p ≡ persistent

(2) V_{app} is the apparent speed of the highest velocity component of the ejecta.

(3) V_{int} is the intrinsic velocity of the ejecta.

(4) Θ is the angle between the direction of motion of the ejecta and the line of sight.

(5) This X-ray transient has initially been related to the optical variable GM Sgr. However, GM Sgr is a different var the optical counterpart of XTE J1819–254 was named V4641 Sgr [22].

Generally microquasars imply the presence of a stellar mass BH, which is a scaled down version of "conventional" quasars

High brightness temperature
Non-Thermal (power -law) spectra
Linear polarisation

Synchrotron radiation

$$\beta_{obs} \sim \left[\frac{\mu}{170\, mas/day}\right]\left[\frac{d}{kpc}\right] = \frac{\beta_{true}\sin\theta}{1 \mp \beta_{true}\cos\theta}$$

$$\beta_{true}\cos\theta = \frac{\mu_{appr} - \mu_{rec}}{\mu_{appr} + \mu_{rec}}$$

$$\tan\theta = \frac{2\mathrm{d}}{c} \cdot \frac{\mu_{appr}\mu_{rec}}{\mu_{appr} - \mu_{rec}}$$

$$\frac{S_{appr}}{S_{rec}} = \left(\frac{\delta_{app}}{\delta_{rec}}\right)^{k+\alpha} = \left(\frac{1+\beta_{true}\cos\theta}{1-\beta_{true}\cos\theta}\right)^{k+\alpha}$$

X-ray binaries: flux density variability

Flares: found in both transients and persistent (radio sources)

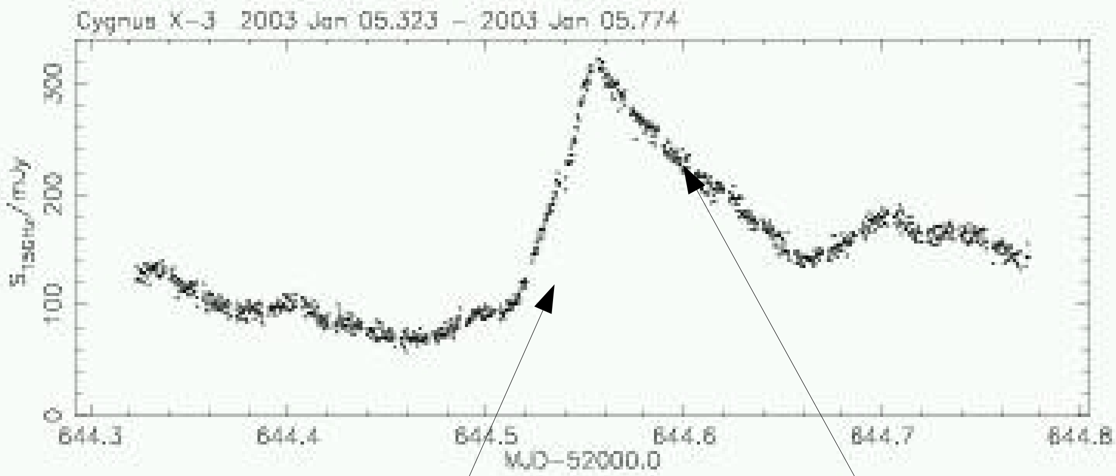


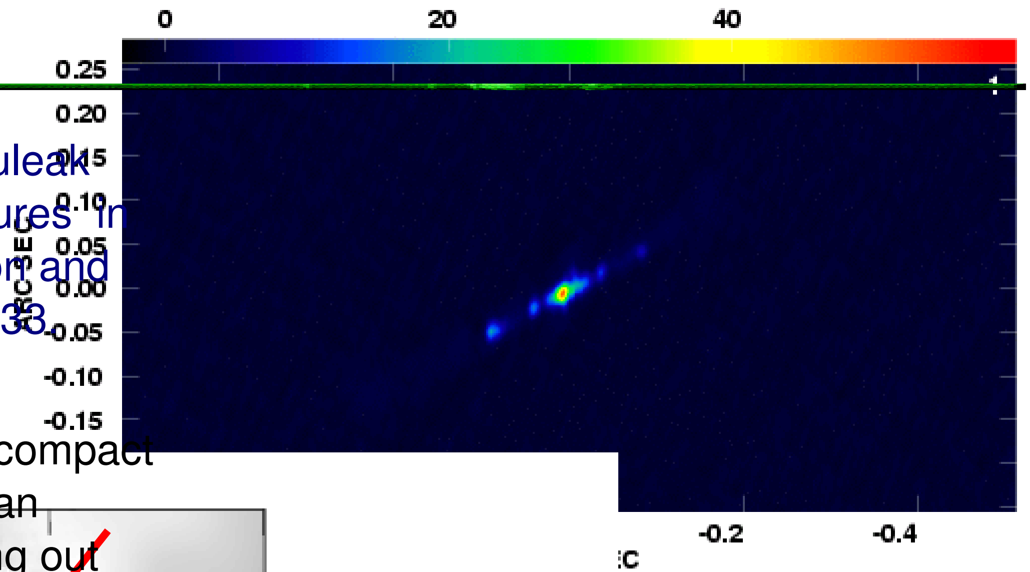
Fig. 9.3. Observation of a 'clean' radio flare event from the jet source Cyg X-3 at 15 GHz. The rise time of the event ~ 0.04 d, allows an estimation of the size of the region associated with the event, and thus the minimum energy. Observations from the Ryle Telescope (Guy Pooley, private communication).

Rise: *Synchrotron bubble?*
Injection/acceleration?

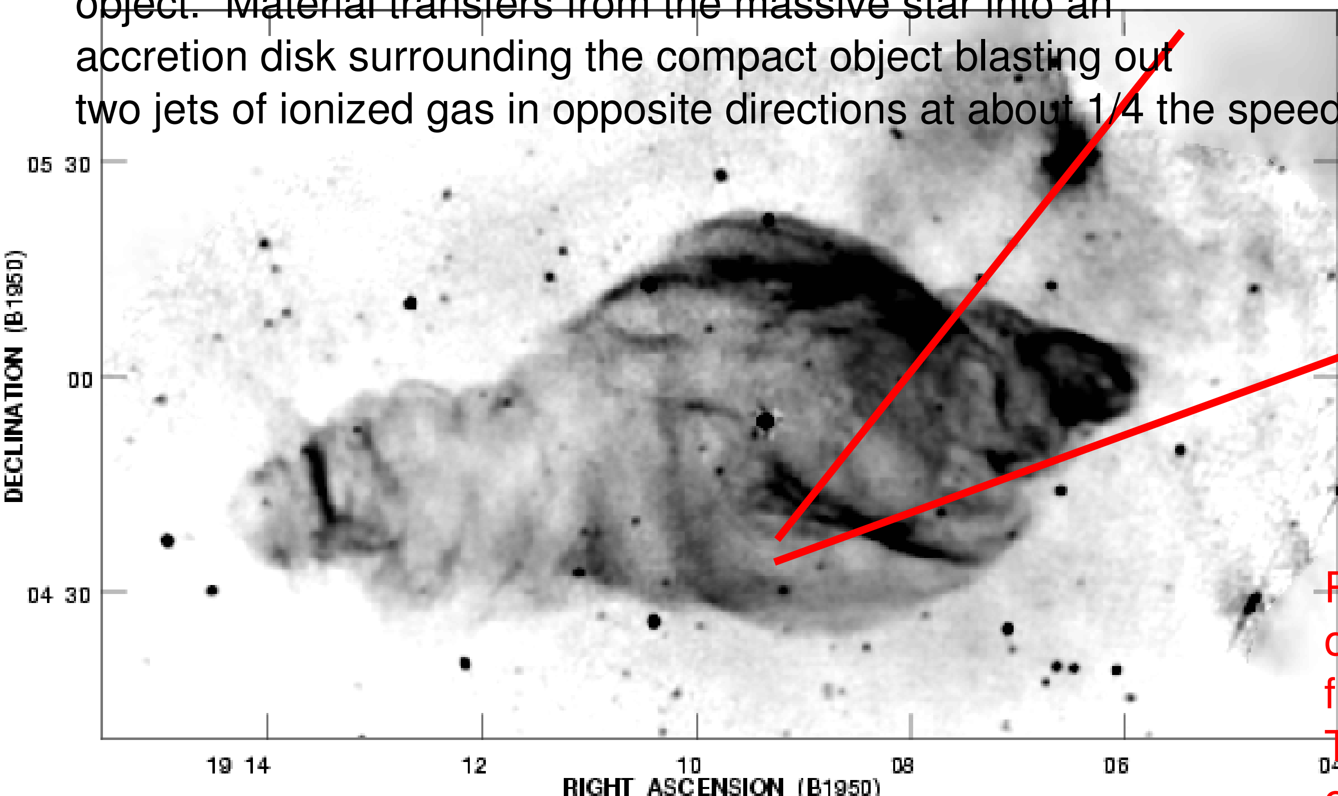
Decay: *Adiabatic expansion?*
Synchrotron + IC losses?

SS433/W50 (@4.7+/-0.3 kpc)

First noted in the '60s by Stephenson and Sanduleak (included in a catalog of stars with unusual features in their spectra). As the 433rd object in Stephenson and Sanduleak's catalog, it became known as SS 433.



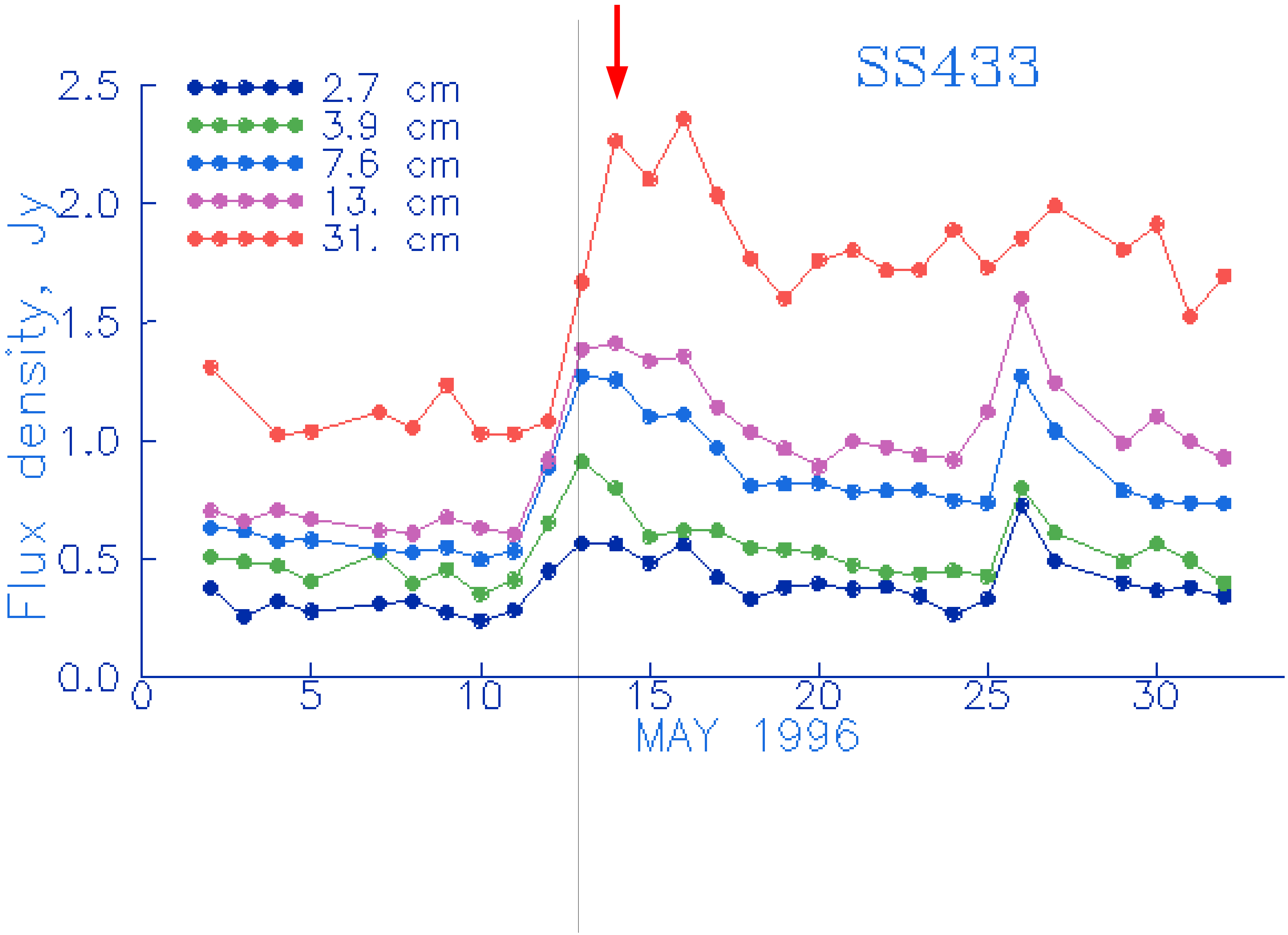
A massive, hot star is locked in a mutual orbit with a compact object. Material transfers from the massive star into an accretion disk surrounding the compact object blasting out two jets of ionized gas in opposite directions at about 1/4 the speed of light!



Radiation from the jet tilted toward the observer is blue-shifted, while radiation from the jet tilted away is red-shifted. The binary system itself completes an orbit in about 13 days while the jets precess (wobble like a top) with a period of about 164 days.

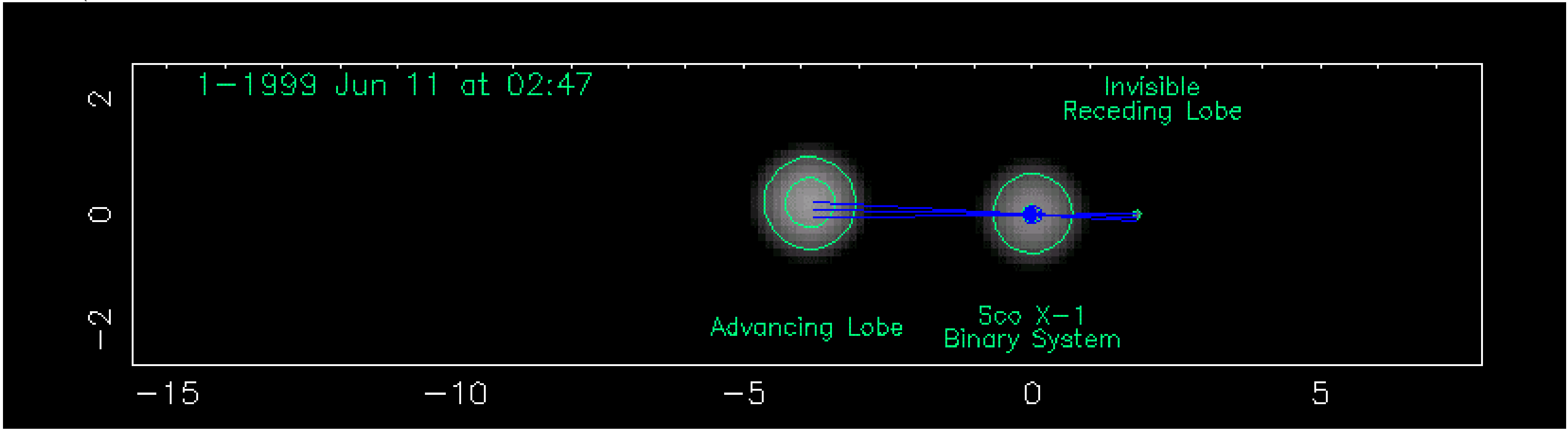
Radio / Optical SNR W50

Time variability vs frequency



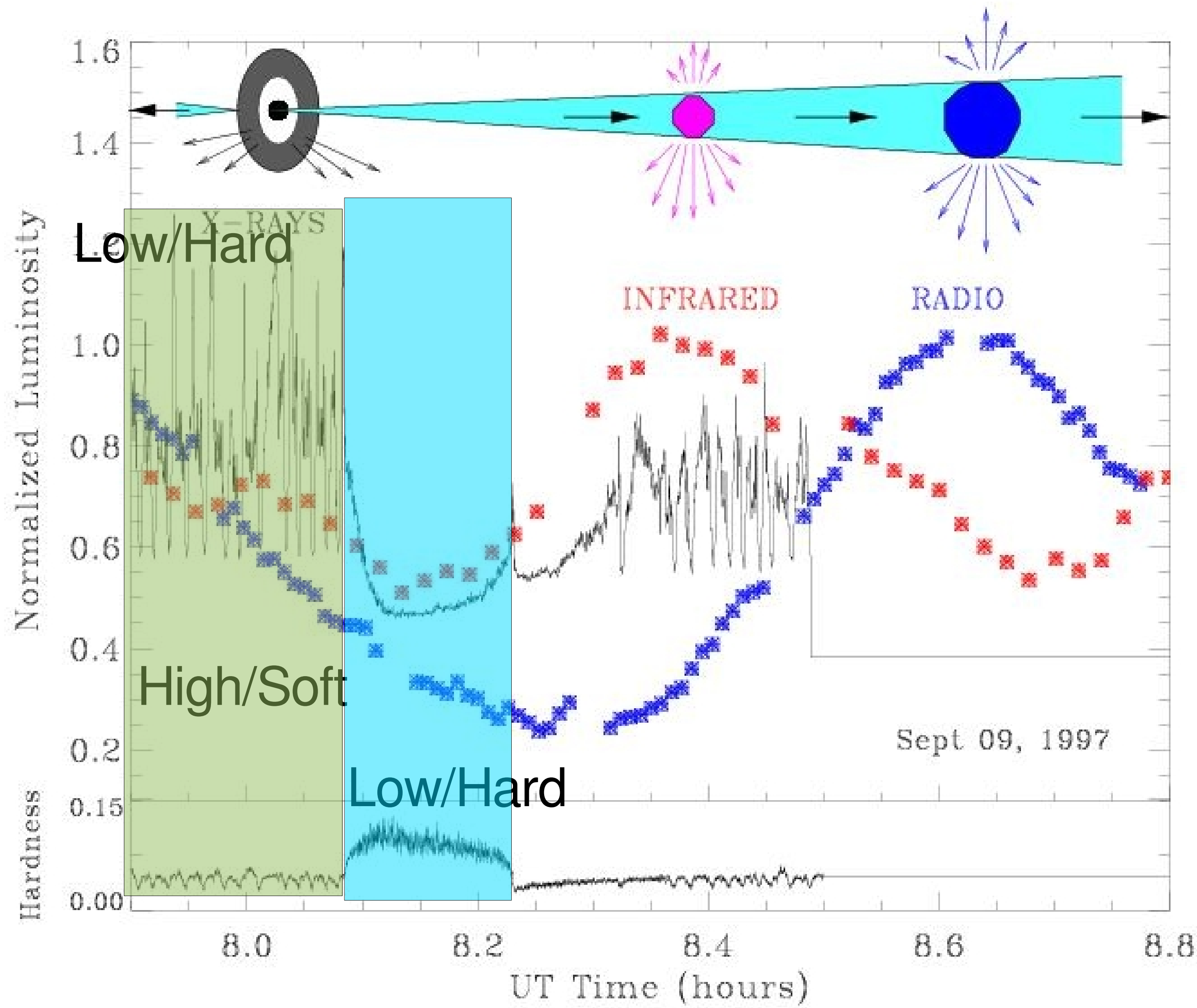
SCO X-1 monitored at radio wavelengths (VLBA)

Observed for 2.3 days and images every 50', interpolation every 15'



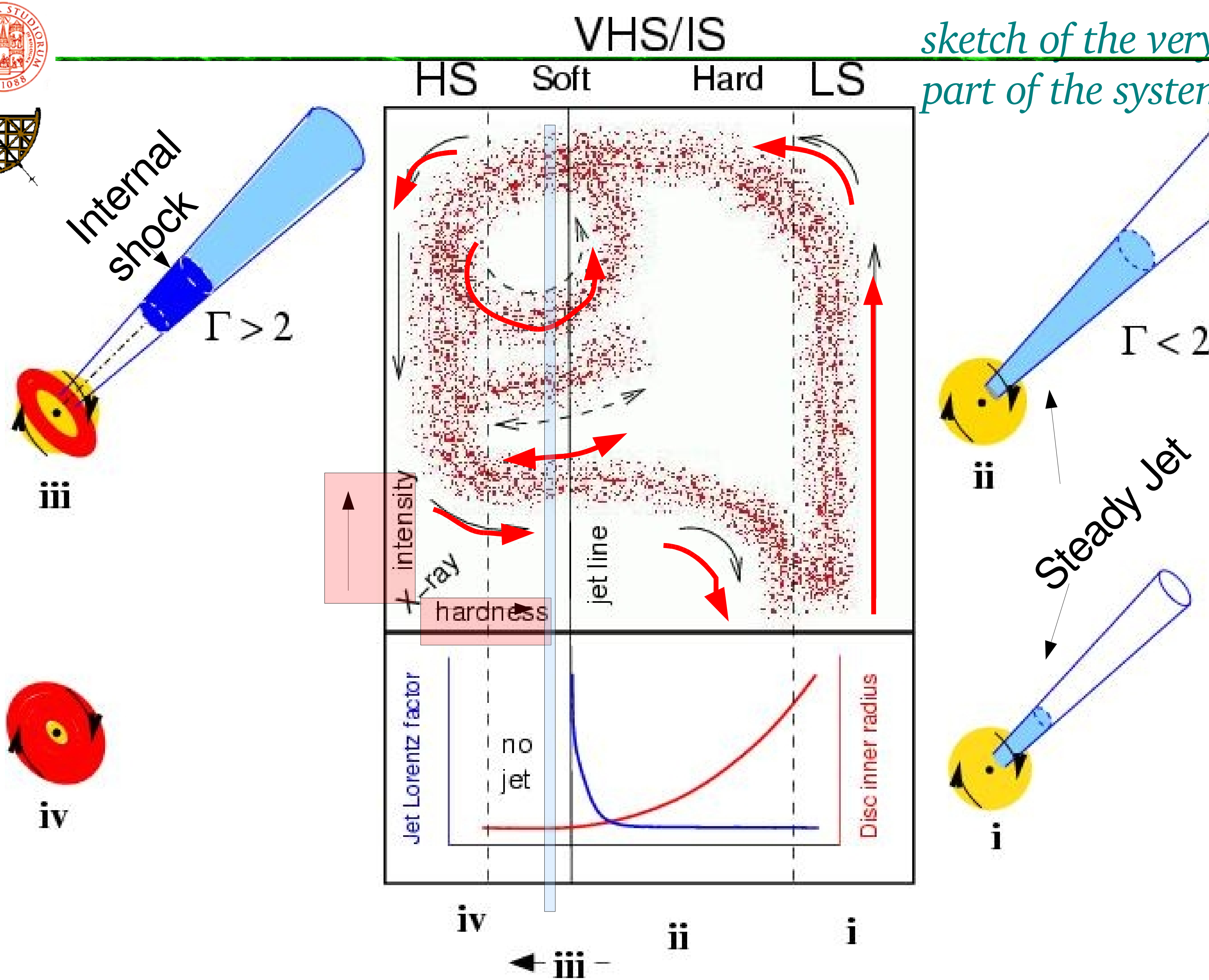
Discovered in 1962 on a rocket flight from White Sands Range (NM)
Neutron star + Sun-type secondary @ 9000 ly from Earth
Separation 0.001 AU
Period 18h53m
Maximum disk Temperature (~100 million K)
Magnetic Field perpendicular to the disk

GRS 1915+105



X-RAYS
Low/Hard
INFRARED
RADIO
High/Soft
Low/Hard
Sept 09, 1997
Normalized Luminosity
Hardness
UT Time (hours)
1.6
1.4
1.0
0.8
0.6
0.4
0.2
0.15
0.00
8.0
8.2
8.4
8.6
8.8

sketch of the very inner part of the system

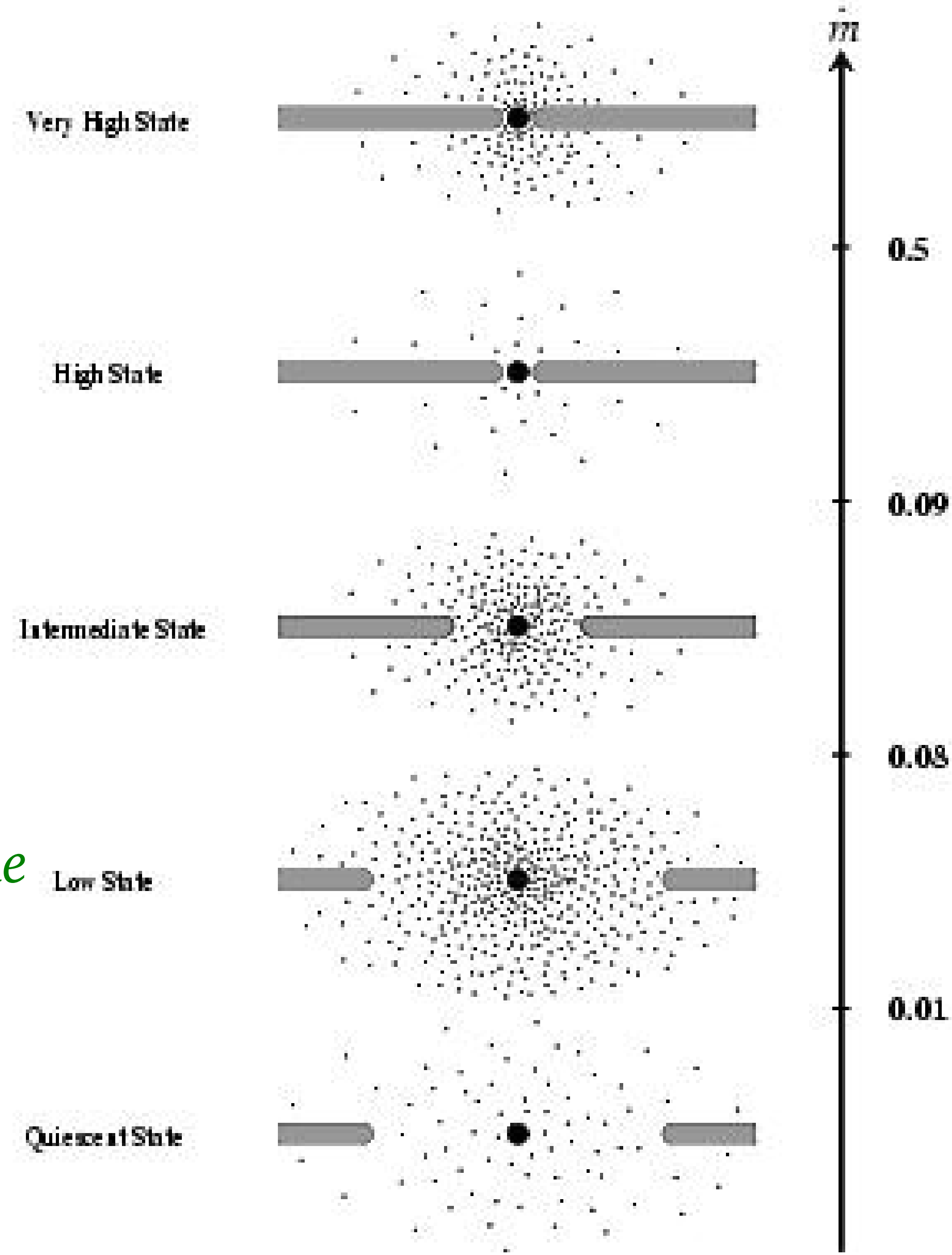


Disk status and structurein terms of accretion

high/low accretion rate

large/small disk

hot corona largely depends on the amount of high energy radiation



Relevant to define the properties of the matter involved.

At high densities, gas particles may be so close, that that interactions between them cannot be neglected.

What basic physical principle will become important as we increase the density and pressure of a highly ionised ideal gas ?

We are dealing with a FULLY IONIZED GAS

In normal matter the Pauli exclusion principle holds: the e^- in the gas must obey the law

"No more than two electrons (of opposite spin) can occupy the same quantum cell"

The quantum cell of an e^- is defined in phase space, and given by 6 values:

x, y, z, p_x, p_y, p_z

The volume of allowed phase space is given by

$$\Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z = h^3$$

The number of electrons in this cell must be at most 2 *between* p *and* $p+dp$ *there are* $4\pi p^2 dp dV/h^3$ *cells*

the maximum limit for f(p) is $8\pi p^2/h^3$

➔ *for a given* n_e *the M-B distribution conflicts the quantum limit to* $f(p)$ *for low* T

➔ *i.e. for a given* T*, this limit comes in for* n_e *exceeding a maximum value*

degenerate gas when for a given momentum $$f^{M-B}(p) > f^{Pauli}(p)$$

fully degenerate gas $$f^{M-B}(p) > f^{Pauli}(p) \quad \forall p$$

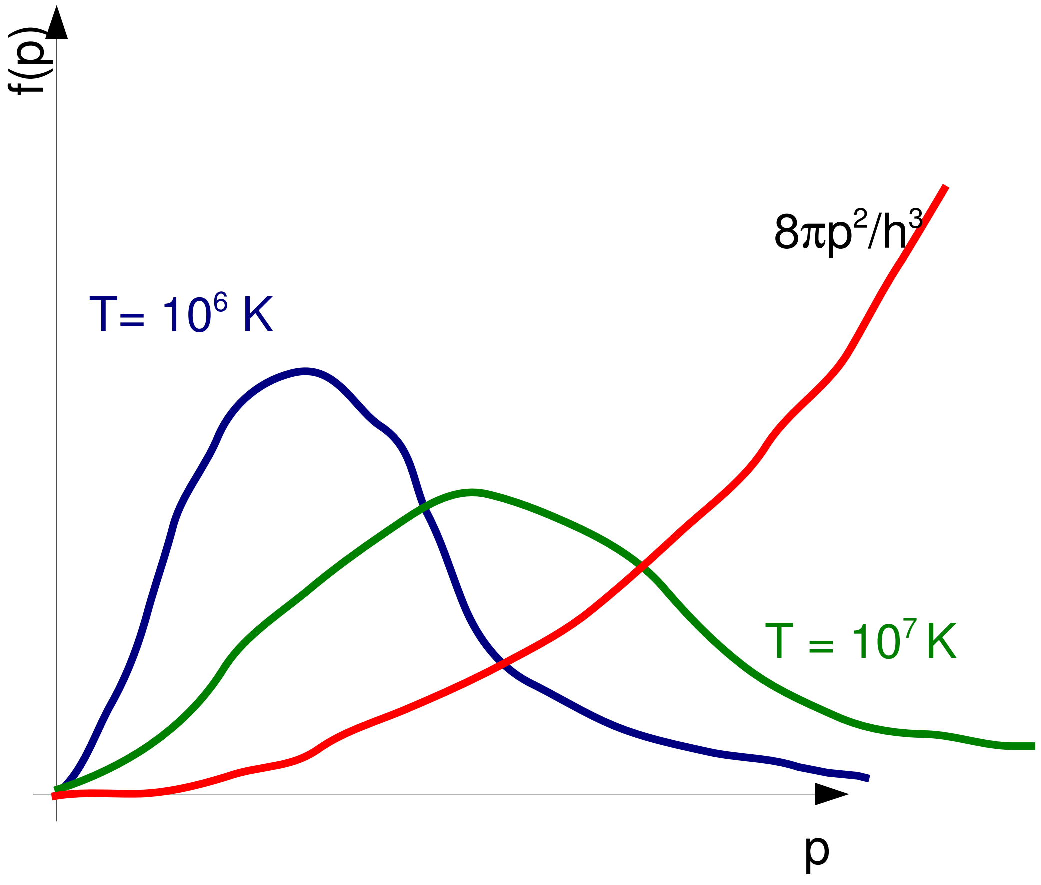
and this happens at $T = 0$*, where* $f(p)$ *is a delta function, and electrons may not all have* $p = 0$

➔ *all the cells up to a given* p_F *are filled*

let's determine p_F*:*

$$n_e = \int_o^\infty f(p)\,dp = \int_o^{p_F} \frac{8\pi p^2}{h^3}\,dp = \frac{8\pi}{3h^3} p_F^3$$

$$p_F = \left(\frac{3h^3}{8\pi}\right)^{1/3} n_e^{1/3}$$


f(p)
8πp²/h³
T= 10⁶ K
T = 10⁷ K
p

and this produces the Fermi Energy:

$$E_F = \frac{p_F^2}{2m_e} \approx n_e^{2/3} \qquad \textit{(non-rel)}$$

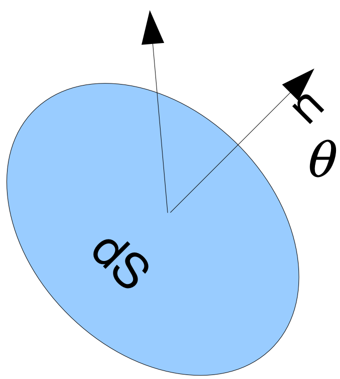
important consequence:
even at T=0 electrons must have $E \neq 0, \qquad 0 \leq E \leq E_F$

All this holds when the kinetic energy of the electrons is much larger than kT, i.e.

$$kT \ll \sqrt{p_F^2 c^2 + m_e^2 c^4} - m_e c^2$$

In case n_e *increases, also* E_F *increases and then electrons may become relativistic!*
namely $p = \gamma m_e v$

These electrons exert a pressure (=momentum flux through a unit surface per unit time)



$$dp = (2p)\times(n\,dS\,v\,dt)\times 1/2$$

$$P = \frac{n\,p^2}{\epsilon}$$ *is 1-D pressure*

$$P = \frac{n\,p^2}{3\epsilon}$$ *is 3-D pressure*

now we have to integrate over all the momenta of the electrons

$$P_e = \frac{8\pi}{3h^3}\int_o^{p_F} \frac{p^2}{\sqrt{p^2+m_e c^2}}\,p^2\,dp$$

$$P_e = \frac{8\pi m_e^4}{3h^3}\int_o^{p_F/m_e} \frac{u^4}{\sqrt{1+u^2}}\,du = \frac{\pi m_e^4}{3h^3}\left[u_o(2u_o^2 - 3)\sqrt{1+u_o^2}+3\sinh^{-1}u_o\right]$$

where $u_o = \frac{p_F}{m_e}$

The condition $u_o = \frac{p_F}{m_e} = 1$ *defines a borderline between the non relativistic and the relativistic* $u_o \gg 1$ *regimes. This corresponds to the following critical mass*

$$\rho_c = \frac{m_N \mu}{3\pi^2} \left(\frac{2\pi m_e c^2}{hc} \right)^3 = 0.97 \times 10^6 \mu \quad g/cm^3$$

$\rho \ll \rho_c$ *electrons are non-relativistic and if we consider the case* $u_o \to 0$ *we get the following expression for the pressure*

$$P_e = \frac{8\pi m_e^4 c^5}{15 h^3} x^5$$

$$\to \quad P_e(\rho) = \frac{h^2}{60\pi^4 m_e} \left(\frac{3\pi^2 \rho}{m_N \mu} \right)^{5/3} = 1.0036 \times 10^{13} \left(\frac{\rho}{\mu_e} \right)^{5/3} \text{cgs units}$$

$P = K\rho^\gamma$ *is a politropic form where* $K = \frac{h^2}{60\pi^4 m_e} \left(\frac{3\pi^2}{m_N} \right)^{5/3}$ *and* $\gamma = 5/3$

By considering the equations for hydrostatic equilibrium and star mass

$$\frac{dP}{dr} = -\frac{G\rho(r)M(r)}{r^2} \qquad \frac{dM(r)}{dr} = 4\pi r^2 \rho$$

integrating these equations outwards up to a given radius R where the pressure goes to 0

$$M_{\gamma=5/3} = \frac{2.72}{\mu^2}\left(\frac{\rho(r=0)}{\rho_c}\right)^{1/2} M_{sun} \qquad R = 2.0\times10^4 \frac{1}{\mu}\left(\frac{\rho(r=0)}{\rho_c}\right)^{-1/6} km$$

$\rho \gg \rho_c$ *electrons are relativistic and if we consider the case* $u_o \rightarrow \infty$ *we get the following expression for the pressure*

$$P_e = \frac{2\pi m_e^4 c^5}{3h^3} x^4$$

$$\rightarrow \quad P_e(\rho) = \frac{h}{24\pi^3}\left(\frac{3\pi^2 \rho}{m_N \mu}\right)^{4/3} = 1.2435\times 10^{15}\left(\frac{\rho}{\mu_e}\right)^{4/3} \text{(c.g.s. units)}$$

$P = K\rho^{\gamma}$ *is a politropic form where* $K = \frac{h}{24\pi^3}\left(\frac{3\pi^2}{m_N}\right)^{4/3}$ *and* $\gamma = 4/3$

integrating this equation along with those of the mass and hydrostatic equilibrium up to a given radius R where the pressure goes to 0

$$M_{\gamma=4/3} = 5.87\frac{1}{\mu^2} M_{sun} \qquad R_{\gamma=4/3} = 5.3\times 10^4 \frac{1}{\mu}\left(\frac{\rho(r=0)}{\rho_c}\right)^{-1/3} km$$

$\mu = 2$ *is expexted for* α *elements, with paired elements in nuclei for each e-*
The resulting limiting mass for WD is 1.4675 M_{sun}
In case of an hypothetic Fe nucleus ($\mu = 56/26 = 2.15$ *) then the limiting mass becomes 1.2653* M_{sun}

In a neutron star the equations are similar to those in WDs.
However, neutrons and residual protons play the role of the electrons
The equation for hydrostatic equilibrium for a neutron star is different, since special and general relativity effect must be taken into account

$$\frac{dP}{dr} = -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

where $\epsilon(r)$ *is the energy density which takes into account also interactions.*
This is known as the Tolman-Oppenheimer-Volkov (TOV) equation
The degeneracy pressure in a NS comes from the nuclear matter, whose density i

$$\rho = \frac{8\pi}{h^3}\int_o^{p_F} p^2\sqrt{p^2 + m_N^2 c^2}\,du =$$

$$= 3\rho_c\int_o^{p_F/m_N} u^2\sqrt{1+u^2}\,du = \frac{3\rho_c}{8}\left[u_o(2u_o^2 + 1)\sqrt{1+u_o^2} - \sin^{-1}u_o\right]$$

which provides the critical density $(\mu=1, m_e \rightarrow m_n)$

$$\rho_c = \frac{8\pi m_N^4 c^3}{3h^3} = 6.11\times10^{15}\ g/cm^3$$

the non relativistic and relativistic limits are

$$\rho \;\rightarrow\; \rho_c \left(\frac{p_F}{m_N}\right)^3 \quad \textit{non relativistic}$$

$$\rho \;\rightarrow\; \rho_c \left(\frac{p_F}{m_N}\right)^4 \quad \textit{relativistic}$$

likewise the pressure [----omissis-----] has the following limits

$$P \;\rightarrow\; \frac{1}{5}\;\rho_c \left(\frac{p_F}{m_N}\right)^5 \quad \textit{non relativistic}$$

$$\rho \;\rightarrow\; \frac{1}{4}\;\rho_c \left(\frac{p_F}{m_N}\right)^4 \quad \textit{relativistic}$$

[omissis] a few relations are still missing, TOV limit, considerations on real/theoretical EoS and consequences they will be on plase sooner or later…

The neutron drip line is a concept in particle and nuclear physics. An unstable atomic nucleus beyond the neutron drip line will leak free neutrons. In other words, the neutron drip line is the line on the Z, N plane (see table of nuclides) where the neutron separation energy is zero; the neutron drip line serves as a boundary for neutron-rich nuclear existence. This phenomenon may be contrasted with the proton drip line, as these concepts are similar, but occurring on opposite sides of nuclear stability.

We can see how this occurs by considering the energy levels in a nucleus. The energy of a neutron in a nucleus is its rest mass energy minus a binding energy. In addition to this, however, there is an energy due to degeneracy: for instance a neutron with energy E1 will be forced to a higher energy E2 if all the lower energy states are filled. This is because neutrons are fermions and obey Fermi-Dirac statistics. The work done in putting this neutron to a higher energy level results in a pressure which is the degeneracy pressure. So we can view the energy of a neutron in a nucleus as its rest mass energy minus an effective binding energy which decreases as we go to higher energy levels. Eventually this effective binding energy has become zero so that the highest occupied energy level, which is the Fermi energy, is equal to the rest mass of a neutron. At this point adding a neutron to the nucleus is not possible as the new neutron would have a negative effective binding energy — i.e it is more energetically favourable (system will have lowest overall energy) for the neutron to be created outside the nucleus. This is the neutron drip point.

In astrophysics, the neutron drip line is important in discussions of nucleosynthesis or neutron stars. In neutron stars, neutron heavy nuclei are found as relativistic electrons penetrate the nuclei and we get inverse beta decay, wherein the electron combines with a proton in the nucleus to make a neutron and an electron-neutrino:

$$p^+ + e^- \rightarrow n + \nu_e$$

As more and more neutrons are created in nuclei the energy levels for neutrons get filled up to an energy level equal to the rest mass of a neutron. At this point any electron penetrating a nucleus will create a neutron which will "drip" out of the nucleus. At this point we have:

$$E_n^F = m_n c^2$$

And from this point the equation

$$E_n^F = \sqrt{(p_n^F)^2 c^2 + m_n^2 c^4}$$

applies, where is the Fermi momentum of the neutron. As we go deeper into the neutron star the free neutron density increases, and as the Fermi momentum increases with increasing density, the Fermi energy increases, so that energy levels lower than the top level reach neutron drip and more and more neutrons drip out of nuclei so that we get nuclei in a neutron fluid. Eventually all the neutrons drip out of nuclei and we have reached the neutron fluid interior of the neutron star.