



Thermal Bremsstrahlung

"Radiation due to the acceleration of a charge in the Coulomb field of another charge is called **bremsstrahlung** or **free-free** emission"

A full understanding of the process requires a quantum treatment, since photons of energies [maybe] are comparable to that of the emitting particle can be produced

However, a classical treatment is justified in some (most of) regimes, and the formulas so obtained have the correct functional dependence for most of the physical parameters. Therefore, we first give a classical treatment and then state the quantum results as corrections (Gaunt factors) to the classical formulas"

George B. Rybicki & Alan P. Lightman in "Radiative processes in astrophysics" p. 155



on textbooks

G. Ghisellini: "Radiative Processes in High Energy Astrophysics" § 2

M. Longair: "High Energy Astrophysics" § 6.3 – 6.6

G. B. Rybicki & A.P. Lightman: "Radiative Processes in Astrophysics" § 5

C. Fanti & R. Fanti: "Lezioni di radioastronomia" § 5

T. Padmanabhan: "Theoretical Astrophysics" § 6.9



Continuum from radiative processes from free-free transitions

Classical physics holds when $\lambda_{\text{dB}} = \frac{h \sqrt{1 - v^2/c^2}}{m v}$ smaller than d = the typical scale of the interaction

→ Quantum mechanics ($\Delta x \Delta p \geq h/2\pi$) can be avoided

For non relativistic velocities $\frac{h}{m v} \ll d$

h = Planck 's constant

m = mass of the (lightest) particle

v = velocity of the (lightest) particle

d = characteristic scale of the interaction

or λ of the electromagnetic wave if it is involved)

λ_{dB} = de Broglie wavelength

Implication → Particles act like points when $h v \ll E_{\text{particle}}$





Free electrons are accelerated (decelerated) in the Coulomb field of atomic (ionized) nuclei (free-free) and radiate energy

If Electrons and nuclei are in **thermal** equilibrium at the temperature T , their emission is termed "**thermal bremsstrahlung**"

Electrons move at velocity v wrt the ions and during "collisions" are deflected. The electrostatic interaction decelerates the electron which emits a photon bringing away part of its kinetic energy

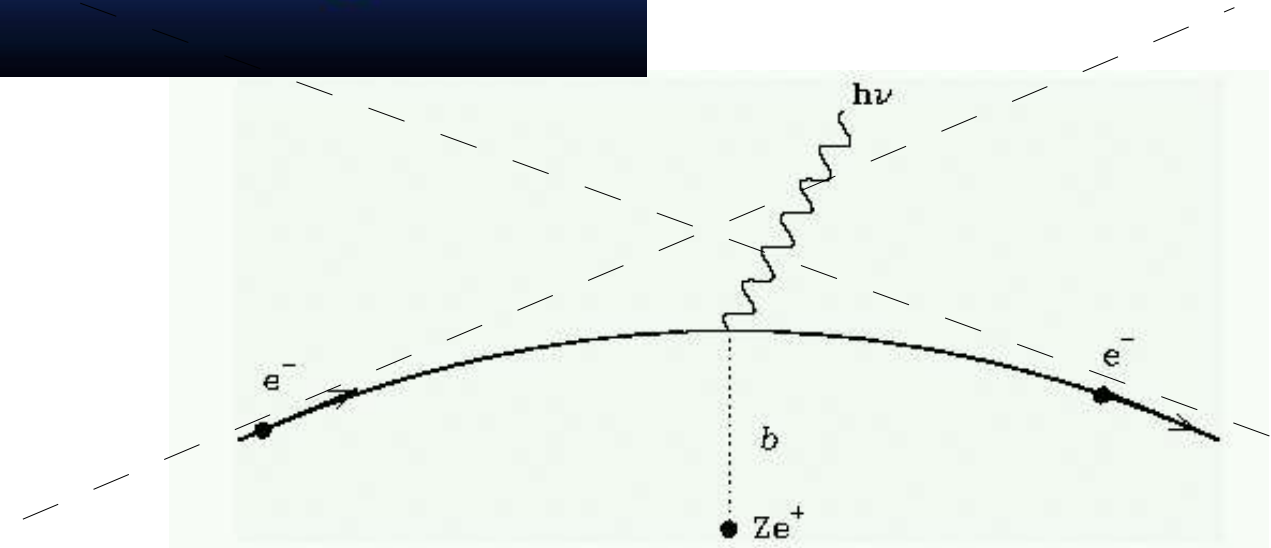
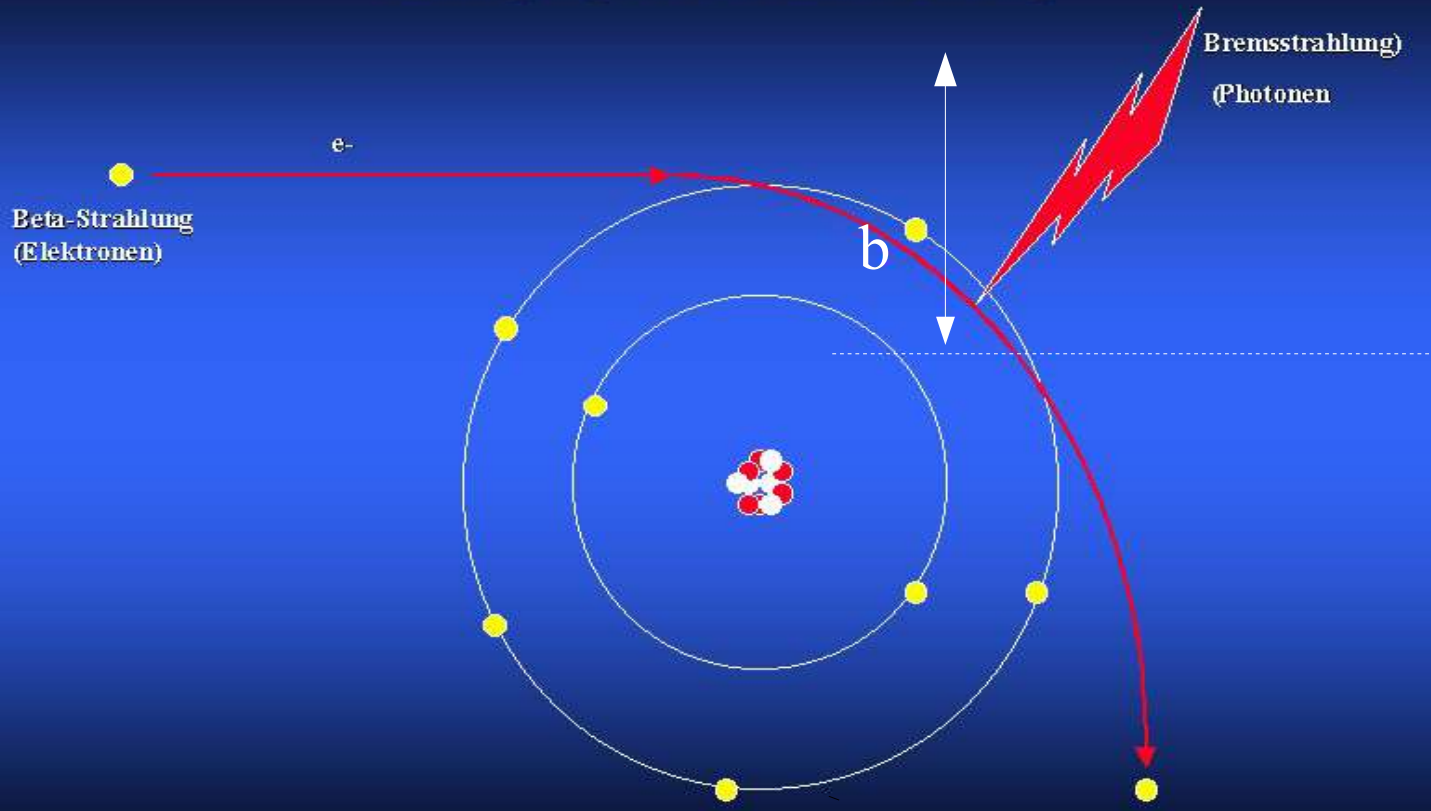
N.B. **e** and **p** have also homologous interactions (**e** with **e**; **p** with **p**) but they do not emit radiation since the electric dipole is zero.

Astrophysical examples:

1. HII regions (10^4 °K)
2. Galactic bulges & hot-coronae (10^7 °K)
3. Intergalactic gas in clusters/groups (10^{7-8} °K)

Bremsstrahlung is the main cooling process for high T ($> 10^7$ K) plasma

Erzeugung von Bremsstrahlung



Radiated power (single particle) is given by the Larmor's formula:

$$P = -\frac{dE}{dt} = \frac{2e^2}{3c^3} a^2$$

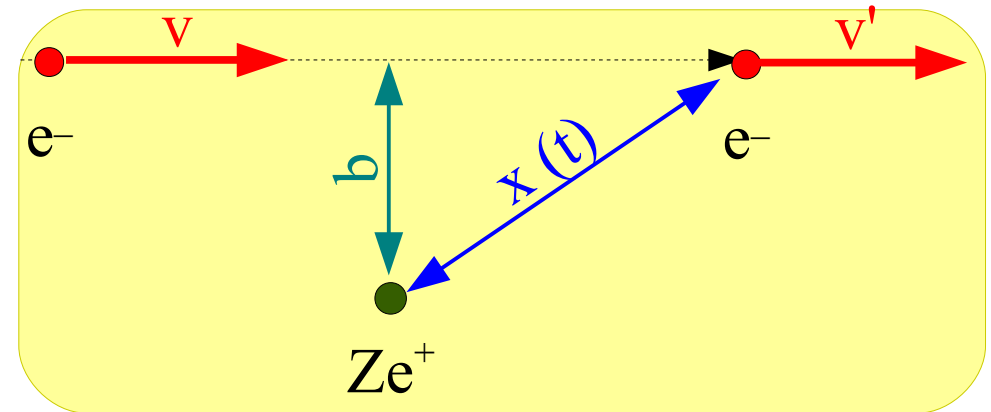
acceleration is from Coulomb's law

$$F = ma \simeq -\frac{Ze^2}{x^2} \quad \rightarrow \quad a = \frac{Ze^2}{mx^2}$$

where x = actual distance between electron (e^-) and ion (Ze^+)

more appropriately $x \sim x(t)$ then

$$P(t) = \frac{2e^2}{3c^3} a^2(t) = \frac{2Z^2e^6}{3c^3} \frac{1}{m^2} \frac{1}{[x(t)]^4}$$

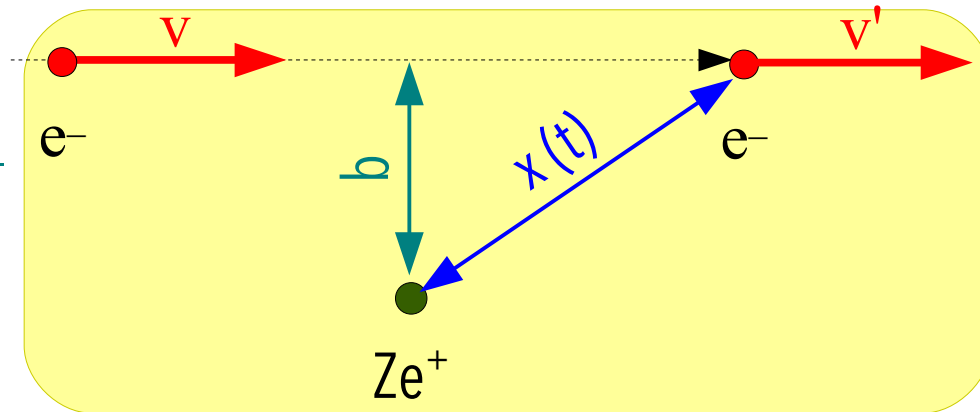


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Remarks:

1. $a \sim m^{-1} \rightarrow P \sim m^{-2}$
2. $a \sim x^{-2} \rightarrow P \sim x^{-4}$



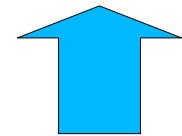
Radiated power is relevant

1. for electrons (positrons) only i.e. light particles
2. when the two charges are at the closest distance: $(x \sim b)$

The interaction lasts very shortly: $\Delta t \sim \frac{2b}{v}$ where $v =$ velocity

The emitted energy in a single "collision" (consider $x \sim b$)

$$P \Delta t = \frac{2}{3} \frac{e^2}{c^3} \left(\frac{Ze^2}{mx^2} \right)^2 \frac{2b}{v} = \frac{4}{3} \frac{Z^2 e^6}{c^3 m^2} \frac{1}{b^3 v} \quad x \simeq b$$



Radiation comes in pulses Δt long

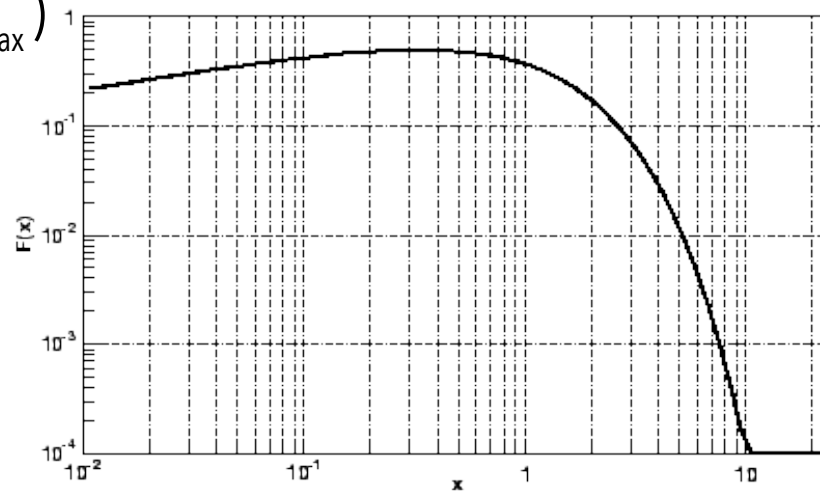
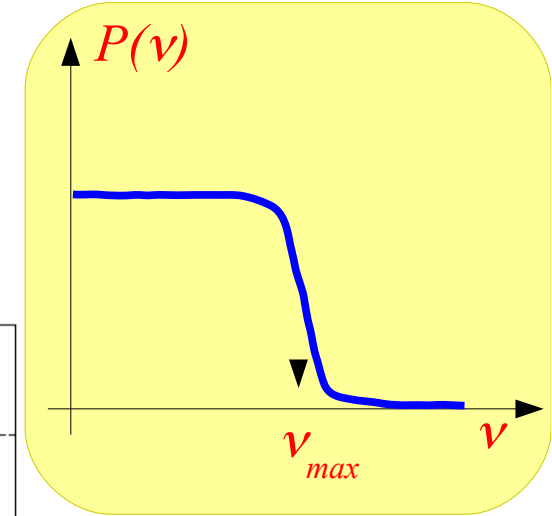
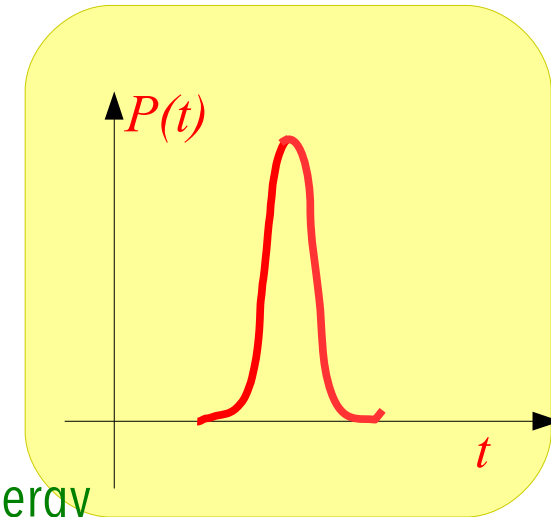
Asymmetric profile (decrease of the e^- velocity)

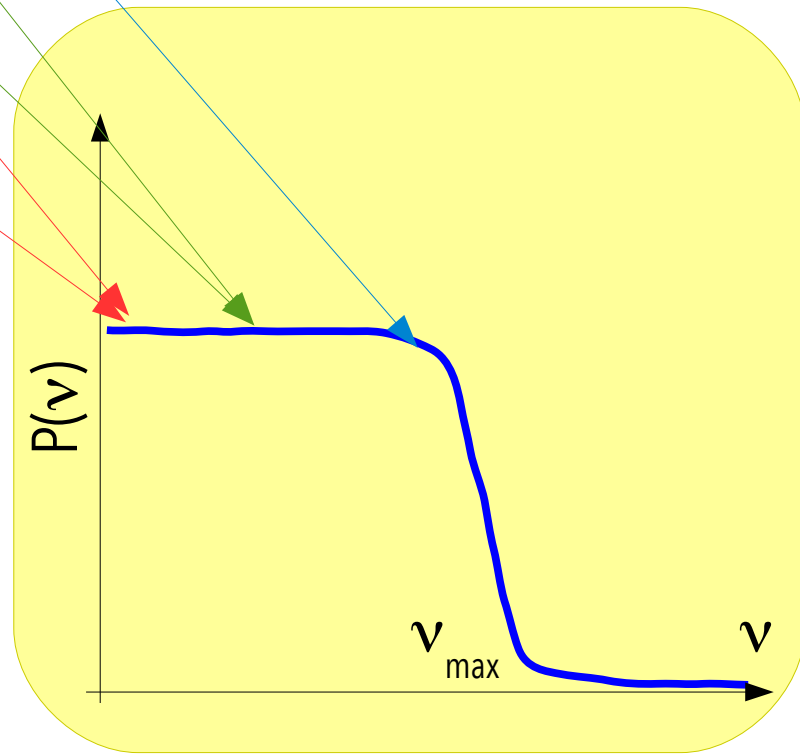
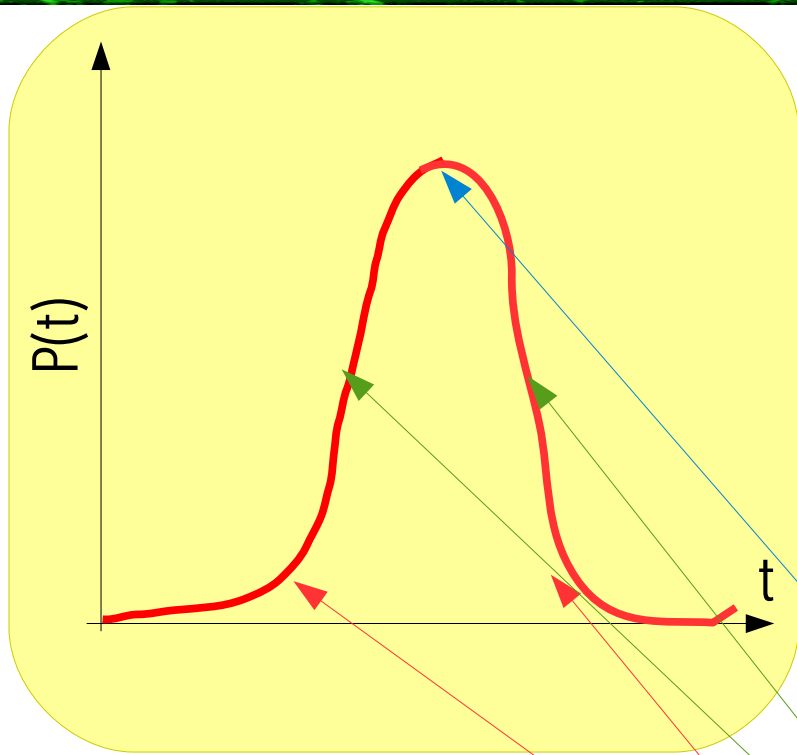
Fourier analysis of the pulse \rightarrow Spectral distribution of the radiated energy

Flat up to a frequency $\nu_{\max} \simeq \frac{1}{2\Delta t} \approx \frac{v}{4b}$

Related to the kinetic energy of the electrons

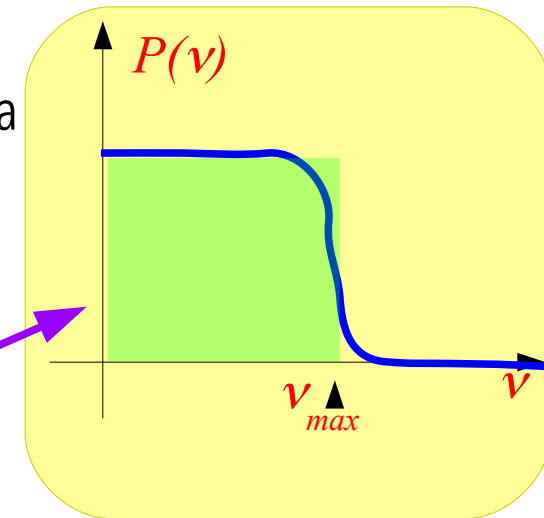
(exponential decay beyond ν_{\max})





The radiated energy per unit frequency (assuming flat spectrum) from a single event can be written as

$$\frac{P \Delta t}{\Delta \nu} \approx \frac{P \Delta t}{\nu_{\max}} = 2P(\Delta t)^2 \approx \frac{16 Z^2 e^6}{3 c^3 m_e^2} \frac{1}{b^2 \nu^2} \approx \frac{1}{b^2 \nu^2}$$



Exercise:

Determine the range of distances (order of mag) in which the released energy is significant

$$e = 4.8 \cdot 10^{-10} \text{ statC} \quad m_e = 9 \cdot 10^{-28} \text{ g} \quad v = 100 \text{ km/s}$$

$$\text{Remember.... Bohr's radius} \quad a_0 \sim 5 \cdot 10^{-13} \text{ cm}$$

$$\text{Typical ISM densities} \quad n_{\text{ISM}} \sim 1 \text{ cm}^{-3}$$

Unphysical case: a cloud with ion number density n_Z and free electron number density n_e

In a unit time, each electron experiences a number of collisions with an impact parameter between b and $b + db$ is

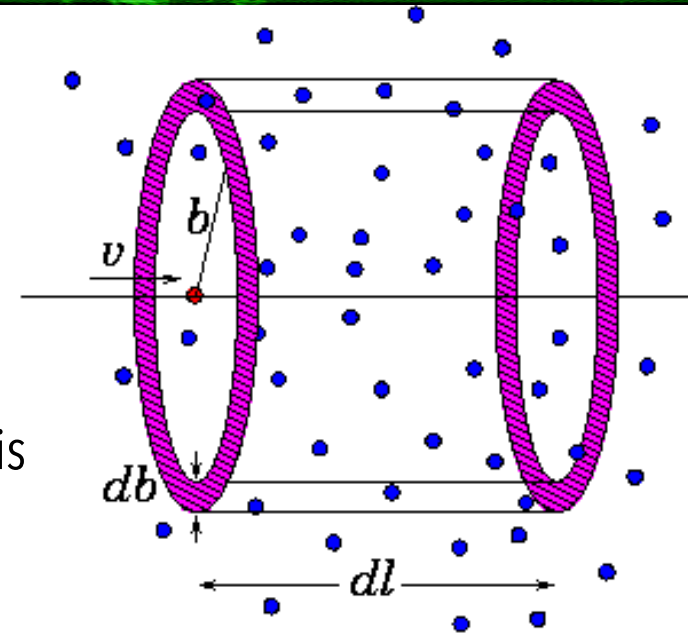
$$2\pi n_Z v b db$$

The total number of collisions is $2\pi n_e n_Z v b db$

Total emissivity from a cloud emitting via bremsstrahlung (single velocity!)

$$J_{br}(v, \nu) = \frac{dE}{d\nu dV dt} = 2\pi n_e n_Z v \int_{b_{min}}^{b_{max}} \frac{16 Z^2 e^6}{3 c^3 m_e^2} \frac{1}{b^2 v^2} b db =$$

$$= \frac{32 \pi e^6}{3 c^3 m_e^2} n_e n_Z Z^2 \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{32 \pi e^6}{3 c^3 m_e^2} \frac{1}{v} n_e n_Z Z^2 \ln\left(\frac{b_{max}}{b_{min}}\right)$$



Evaluation of b_{\max} :

at a given ν , we must consider only those interactions where electrons have impact parameters corresponding to $v_{\max} > v$ and then

$$b_{\max} \leq \frac{v}{4\nu} \quad \text{is a function of frequency}$$

Evaluation of b_{\min} :

a) classical physics requires that $\Delta v \leq v$

$$\Delta v \simeq a \Delta t = \frac{Ze^2}{m_e b^2} \frac{2b}{v} \leq v \quad \rightarrow \quad b_{\min-c} \geq \frac{2Ze^2}{m_e v^2}$$

b) quantum physics and the Heisenberg principle $\Delta p \Delta x \geq h/2\pi$

$$\Delta p = m_e \Delta v \approx m_e v \geq \frac{h}{2\pi \Delta x} \approx \frac{h}{2\pi b_{\min}} \quad \rightarrow \quad b_{\min-q} \geq \frac{h}{2\pi m_e v}$$



we must consider which b_{\min} has to be used



A set of particles at thermal equilibrium follow Maxwell-Boltzmann distribution:

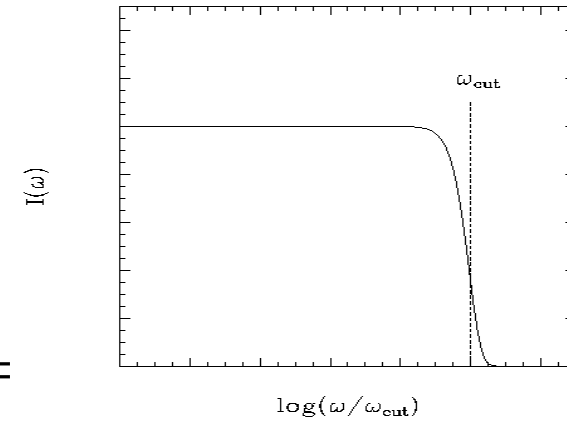
$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi kT} \right)^{3/2} e^{-m_e v^2 / 2kT} v^2 dv$$

Then $n_e(v) = n_e f(v)dv$ is the number density of electrons whose velocity is between v and $v+dv$ and $n_e(v)$ replaces n_e in $J_{br}(v)$

When the plasma is at equilibrium, the process is termed **thermal bremsstrahlung**

Total emissivity:

must integrate over velocity (energy) distribution ($0 \leq v < \infty$?)



$$J_{br}(v, T) = \frac{dE}{dt dV dv} = \int_{v_{min}}^{\infty} J_{br}(v, v) f(v) dv =$$

$$= 6.8 \cdot 10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_Z Z^2 g_{ff}(v, T) \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}$$

T ~ particle energy

high energy cut-off (T)

Plasma density (n_e^2)



$$b_{\max} \approx \frac{v}{4\nu}$$

frequency dependent!...velocity dependent!

e.g. if $v = 10^3 \text{ km/s}$ and $\nu = 10^{10} \text{ Hz}$ then $b_{\max} \approx \frac{10^6 \text{ m s}^{-1}}{4 \times 10^{10} \text{ s}^{-1}} \approx 2.5 \times 10^{-5} \text{ m}$

Thermal distribution \rightarrow range of velocities

Order of magnitude $\frac{3}{2}kT = \frac{1}{2}m\langle v^2 \rangle$

$$\langle v \rangle \approx \sqrt{3kT} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ } ^\circ\text{K}^{-1} T}{9.11 \times 10^{-31} \text{ kg}}} = 0.68 \times 10^4 \sqrt{T} \text{ m/s}$$

FYI,
Computations
never asked

For a minimum temperature of 10^4 K we get a typical velocity of 680 km/s .

At 10^{10} Hz we get: $b_{\max} \approx \frac{v}{4\nu} \approx \frac{6.8 \times 10^5 \text{ m/s}}{4 \times 10^{10} \text{ s}^{-1}} \approx 1.7 \times 10^{-5} \text{ m}$

$$b_{\min(q)} \geq \frac{h}{2\pi m_e v} \approx \frac{6.626 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}}{2\pi 9.11 \times 10^{-31} \text{ kg} \times 6.8 \times 10^5 \text{ m s}^{-1}} \approx 0.17 \times 10^{-10} \text{ m}$$

$$b_{\min(c)} \geq \frac{2Ze^2}{m_e v^2} \dots$$



which [classical/quantum] limit prevails:

>1 when $v \geq 0.01c$

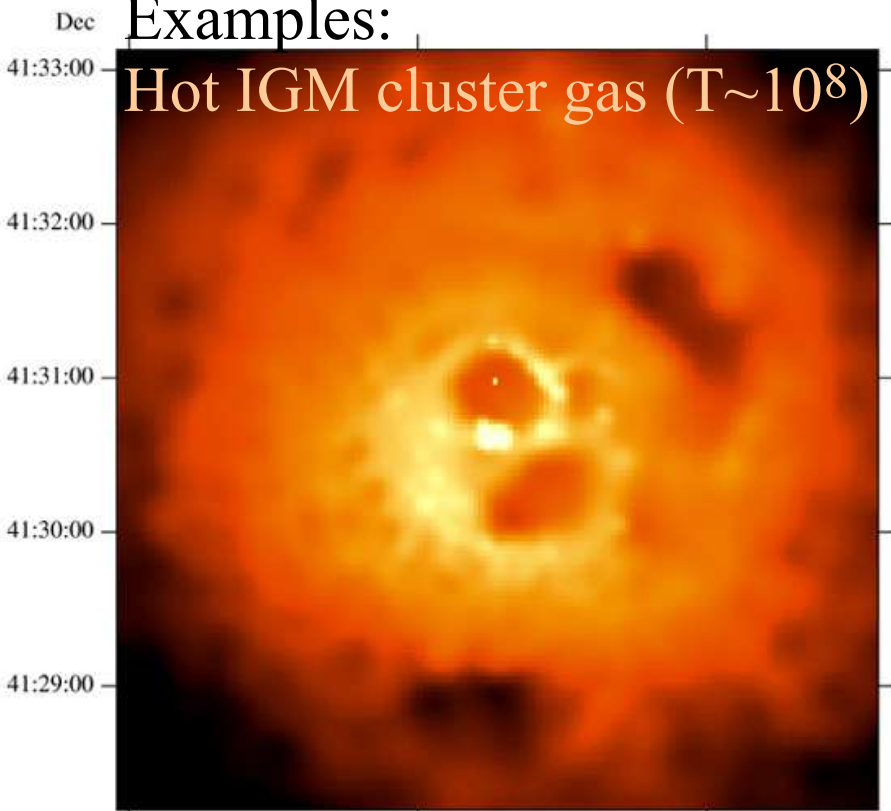
$$\frac{b_{\text{min-q}}}{b_{\text{min-c}}} \approx \frac{h/(2\pi m_e v)}{2Ze^2/(m_e v^2)} = \frac{h}{4\pi Ze^2} c \frac{v}{c} \approx \frac{137 v}{Z c}$$

but $v = \sqrt{3kT/m_e}$ then

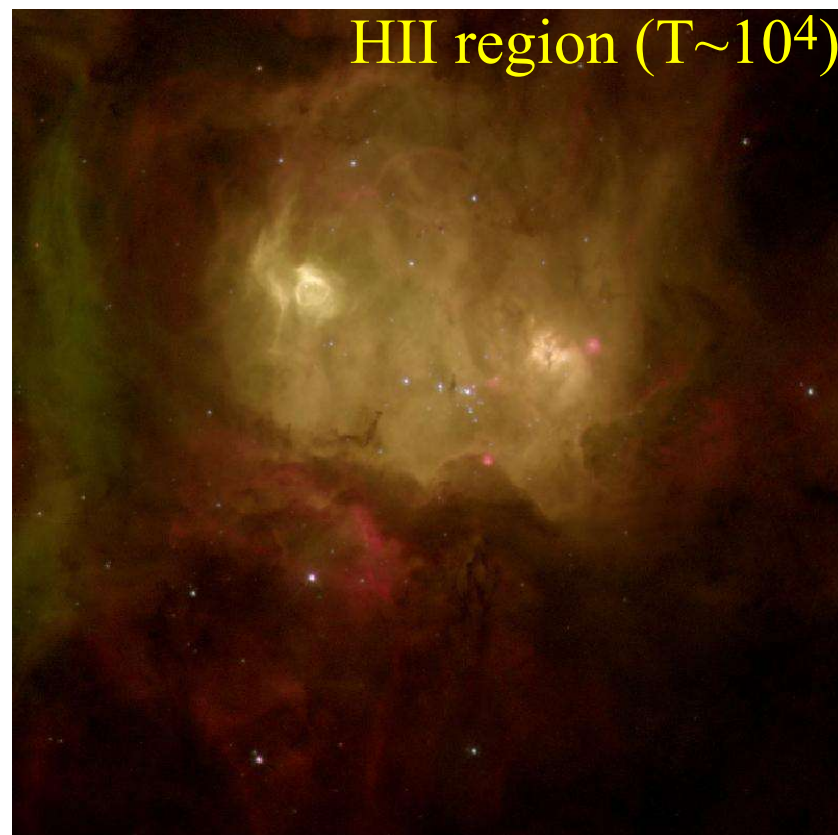
$$\frac{b_{\text{min-q}}}{b_{\text{min-c}}} \approx \frac{137}{Zc} \sqrt{\frac{3k}{m_e}} T^{1/2}$$

Examples:

Hot IGM cluster gas ($T \sim 10^8$)



HII region ($T \sim 10^4$)

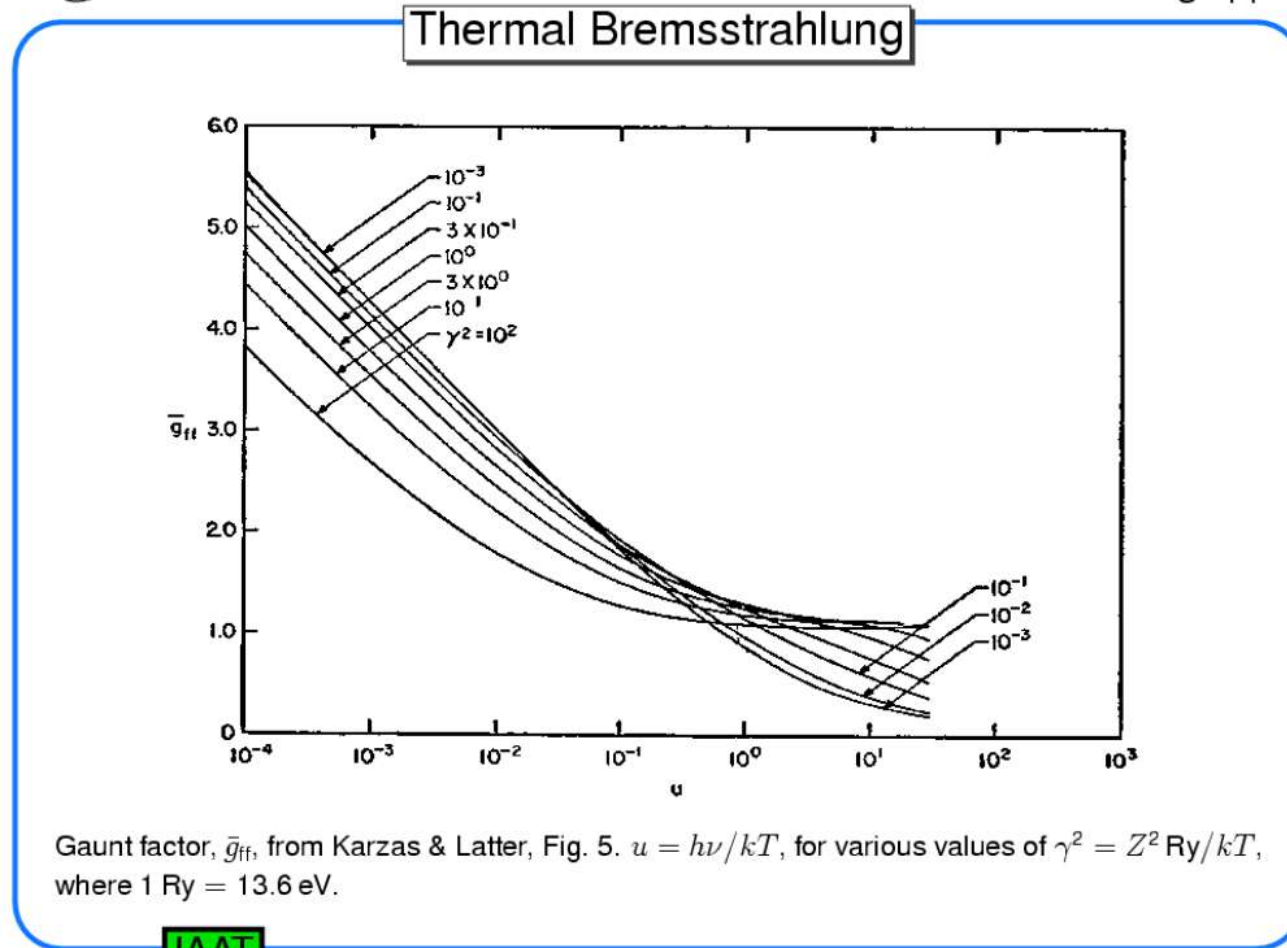


$$g_{ff}(\nu, T) = \frac{\sqrt{3}}{\pi} \ln \left(\frac{b_{\max}}{b_{\min}} \right)$$

The Gaunt Factor ranges from about 1 to about 10 in the radio domain



5-11



IAAT

Classical Treatment

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Values around 10-15 in the radio domain, slightly higher than 1 at higher energies



bremsstrahlung – interlude: photon frequency .vs. temperature

Consider an electron moving at a speed of $v = 1000 \text{ km s}^{-1}$.

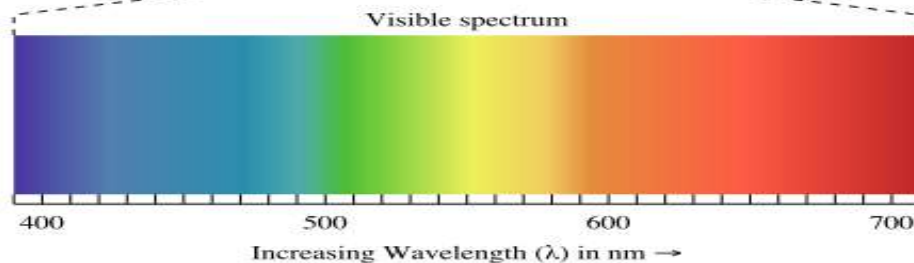
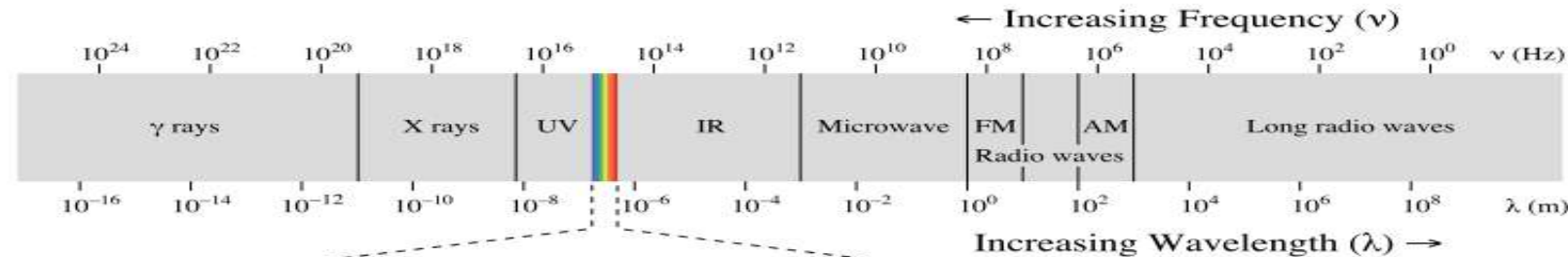
In case it would radiate all its kinetic energy in a single interaction

$$\Delta E = h\nu = \frac{1}{2}mv^2 = 0.5 \times 9.1 \times 10^{-31} \text{ kg} \times (10^6 \text{ m s}^{-1})^2 = 4.55 \times 10^{-19} \text{ J}$$

the frequency of the emitted radiation is $\nu = \frac{\Delta E}{h} = \frac{4.55 \times 10^{-19} \text{ kg m}^2 \text{ s}^{-2}}{6.626 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}} = 6.87 \times 10^{14} \text{ Hz}$

in case this electron belongs to a population of particles at thermal equilibrium for which $v = 1000 \text{ km s}^{-1}$ is the typical speed, determine which is the temperature of the plasma

$$T = \frac{\Delta E}{k} = \frac{4.55 \times 10^{-19} \text{ kg m}^2 \text{ s}^{-2}}{1.38 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1}} = 3.30 \times 10^4 \text{ K}$$



Planck's constant: $6.626 \times 10^{-27} \text{ erg s}^{-1} = 6.626 \times 10^{-34} \text{ J s}^{-1}$
Boltzmann's constant: $1.381 \times 10^{-16} \text{ erg K}^{-1} = 1.381 \times 10^{-23} \text{ J K}^{-1}$
 $m_e = 9.10938188 \times 10^{-31} \text{ kg}$

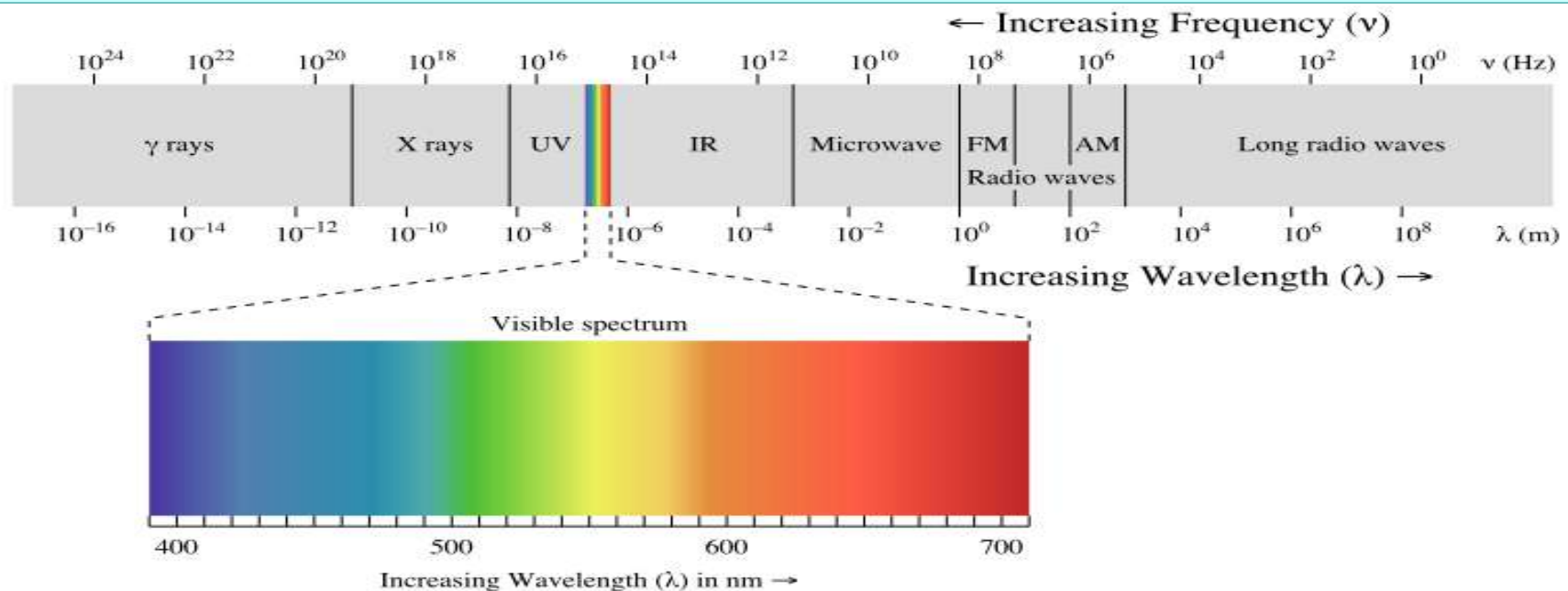
bremsstrahlung – interlude (2): the cut-off frequency

The cut-off frequency in the thermal bremsstrahlung spectrum depends on the temperature only, and is conventionally set when the exponential equals $1/e$.

$$\frac{h\nu}{kT} = 1 \quad \rightarrow \quad \nu = \frac{k}{h} T = \frac{1.38 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1}}{6.626 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}} \times T = 2.08 \times 10^{10} \times T \quad (\text{Hz})$$

It is interesting to determine the cut-off frequency for a warm plasma (HII region, $T \sim 10^4 \text{ K}$) and for a hot plasma (IGM in clusters of galaxies, $T \sim 10^8 \text{ K}$).

⇒ *In observations, the measure of the cut-off frequency is a way to determine the plasma temperature*





Total emissivity does not depend on frequency
(except the cut - off) and the spectrum is flat

$$J_{br}(\nu, T) = 6.8 \cdot 10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_z Z^2 g_{ff}(\nu, T) \quad \text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1}$$

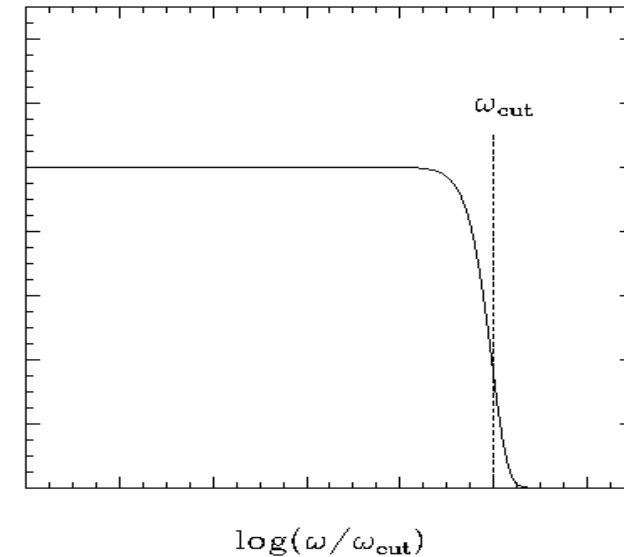
Total emitted energy per unit volume: must integrate over velocity (energy) distribution ($0 \leq v < \infty$?) at a given frequency ν , the velocity of the particle v must be as such

$$\frac{1}{2} m v^2 \geq h \nu \quad \rightarrow \quad v_{\min} \geq \sqrt{\frac{2 h \nu}{m_e}}$$

and integrating over the whole spectrum:

$$J_{br}(T) = \frac{dE}{dt dV} \simeq 2.4 \cdot 10^{-27} T^{1/2} n_e n_z Z^2 \bar{g}_{ff}(T) \quad \text{erg s}^{-1} \text{cm}^{-3}$$

$\bar{g}_{ff}(T)$ average gaunt factor





Let's consider the total thermal energy of a plasma and the radiative losses:
the **cooling time** is

$$t_{\text{br}} = \frac{E_{\text{th}}^{\text{tot}}}{J_{\text{br}}(T)} = \frac{3/2(n_e + n_p)kT}{J_{\text{br}}(T)} \quad \text{we can assume} \quad n_e = n_p$$

$$\simeq \frac{3n_e kT}{2.4 \cdot 10^{-27} T^{1/2} n_e^2 \bar{g}_{\text{ff}}} = \frac{1.8 \cdot 10^{11}}{n_e \bar{g}_{\text{ff}}} T^{1/2} \text{ sec} \rightarrow = \frac{6 \cdot 10^3}{n_e \bar{g}_{\text{ff}}} T^{1/2} \text{ yr}$$

N.B. The "average" Gaunt Factor is about a factor of a few

Astrophysical examples:

HII regions: $n_e \sim 10^2 - 10^3 \text{ cm}^{-3}$; $T \sim 10^4 \text{ K}$

Galaxy clusters $n_e \sim 10^{-3} \text{ cm}^{-3}$; $T \sim 10^7 - 10^8 \text{ K}$

(thermal) bremsstrahlung is the main cooling process at temperatures above $T \sim 10^7 \text{ K}$

N.B.-2- all galaxy clusters have bremsstrahlung emission

A galactic HII region: **M42 (Orion nebula)**

Distance: 1.6 kly \sim 0.5 kpc angular size \sim 66' \rightarrow 1.1 $^\circ$

$T = 10^4$ K, $S_{1.4\text{GHz}} = 410$ Jy

Let's get some astrophysical information

$$\nu_{\text{cutoff}} \sim \frac{k}{h} T = 2.08 \cdot 10^{10} \left(\frac{T}{\text{K}} \right) \text{ Hz}$$

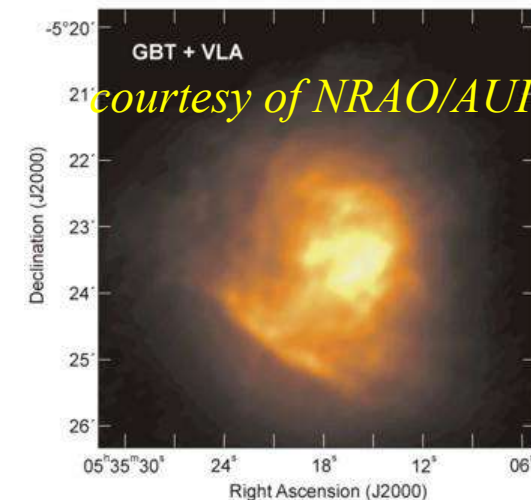
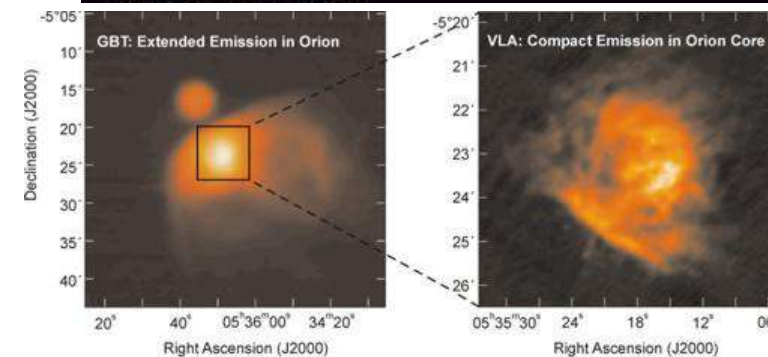
$$S = \frac{J_{\text{br}}(\nu, T) \cdot V}{4 \pi D^2}$$

$$t_{\text{br}} = \frac{6 \cdot 10^3}{n_e \bar{g}_{\text{ff}}} T^{1/2} \text{ yr}$$

$$j_{\text{br}}(T) = 2.4 \cdot 10^{-27} T^{1/2} n_e^2 \bar{g}_{\text{ff}} \text{ erg s}^{-1} \text{ cm}^{-3}$$

$$L_{\text{br}} = j_{\text{br}} \cdot V \text{ erg s}^{-1}$$

Pay attention to the UNITS

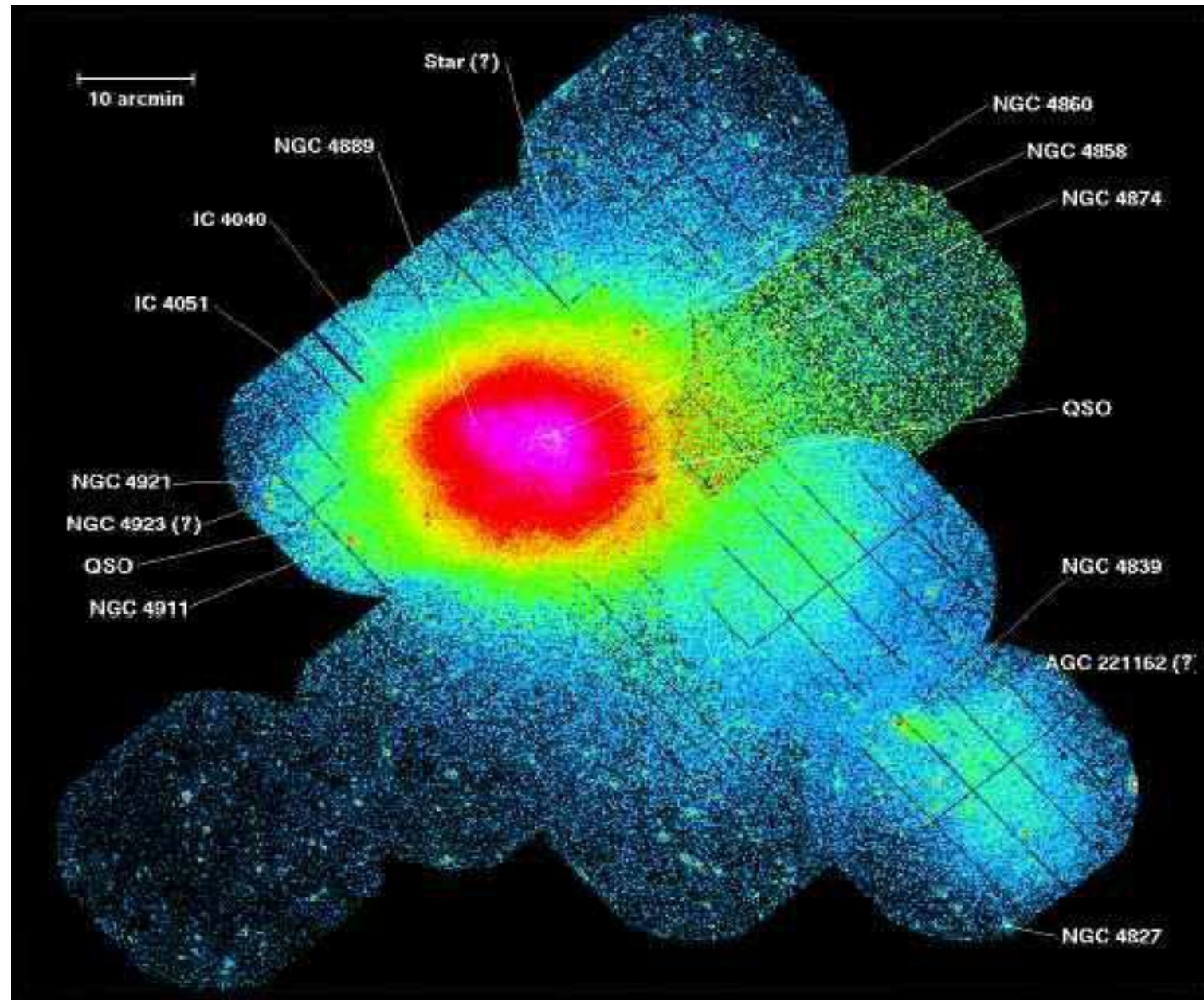


A cluster of galaxies:

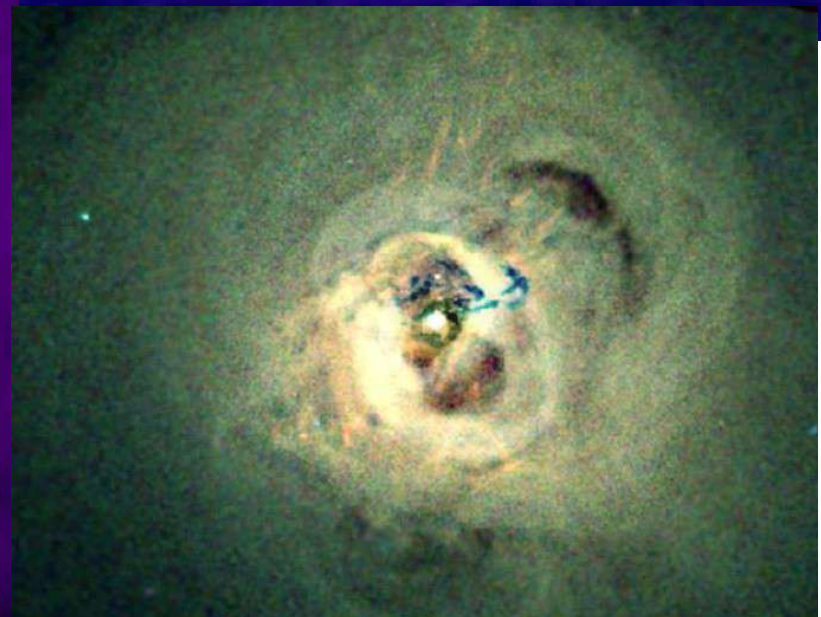
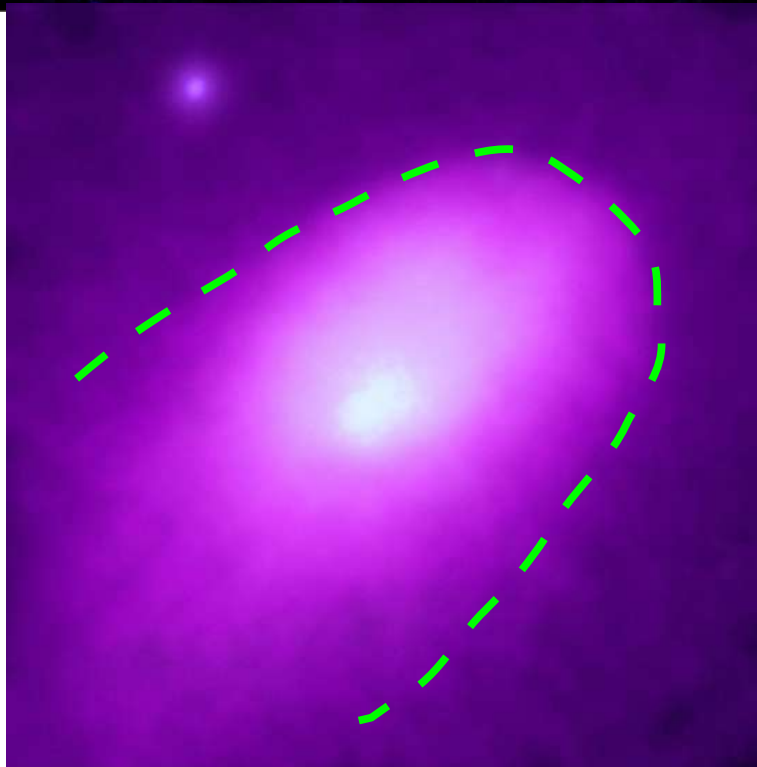
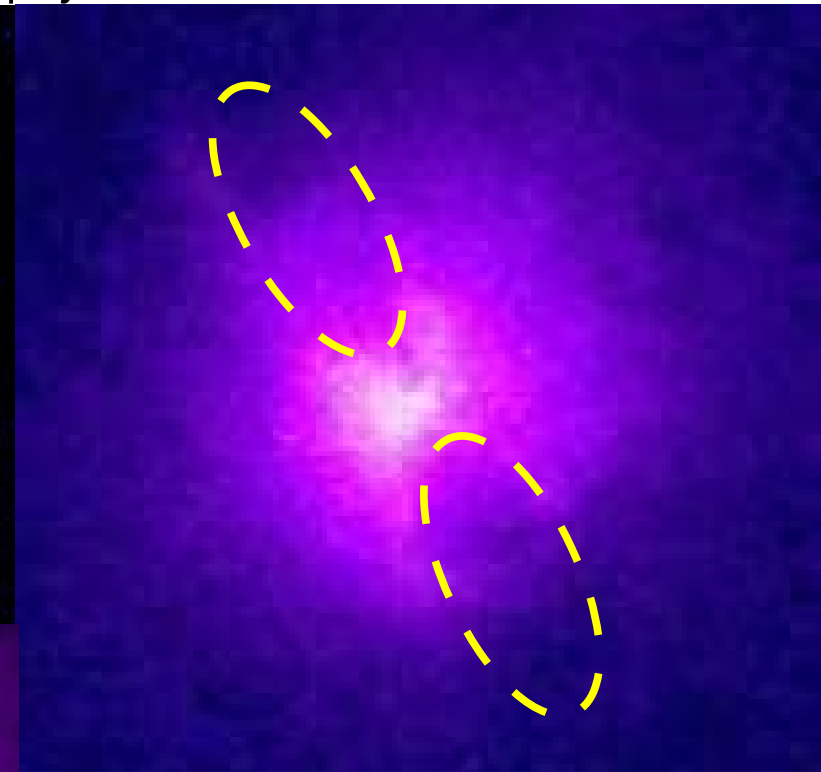
Coma ($z=0.0232$), size ~ 1 Mpc

Typical values of IC hot gas have radiative cooling time exceeding 10 Gyr

All galaxy clusters are X-ray emitters



Cluster of galaxies: a laboratory for plasma physics





Thermal free-free absorption: energy gained by a free moving electron.

For a Maxwellian velocity (energy) distribution, i.e. at thermal equilibrium, Kirchhoff's law holds

$$S(\nu) = \frac{j(\nu, T)}{\mu(\nu, T)} = B(\nu, T) = 2 \frac{h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$j(\nu, T) \equiv \mu(\nu, T) B(\nu, T) \quad [\text{Kirchhoff's law}]$$

For an isotropically emitting plasma cloud via thermal bremsstrahlung

$$\frac{dE}{dt dV d\nu} = 4\pi J_{br}(\nu, T)$$

Therefore we can get

$$\mu_{br}(\nu, T) = \frac{J_{br}(\nu, T)}{4\pi B(\nu, T)} = \frac{1}{4\pi} 6.8 \cdot 10^{-38} T^{-1/2} n_e n_z Z^2 \bar{g}_{ff}(\nu, T) \frac{c^2}{2h\nu^3} (1 - e^{-h\nu/kT})$$

$$\Rightarrow \mu_{br}(\nu, T) \approx T^{-1/2} n_e n_z \nu^{-3} (1 - e^{-h\nu/kT})$$

\Rightarrow strong dependency on frequency



- at high frequencies $\mu(\nu, T)$ is negligible (ν^{-3} and $e^{-h\nu/kT}$)

- at low frequencies ($h\nu \ll kT$)

$$\begin{aligned}\mu_{\text{br}}(\nu, T) &\approx T^{-1/2} n_e n_z \nu^{-3} (1 - e^{-h\nu/kT}) \\ &\approx T^{-1/2} n_e n_z \nu^{-3} \left(1 - 1 + \frac{h\nu}{kT}\right) = n_e n_z T^{-3/2} \nu^{-2}\end{aligned}$$

$$\mu_{\text{br}}(\nu, T) = \frac{4e^6}{3m_e h c} \left(\frac{2\pi}{3km_e}\right)^{1/2} n_e n_z Z^2 T^{-3/2} \nu^{-2} \bar{g}_{\text{ff}}(T)$$

$\bar{g}_{\text{ff}}(T) \approx 10$ therefore

$$\mu_{\text{br}}(\nu, T) \approx 0.018 T^{-3/2} \nu^{-2} n_e n_z Z^2 \bar{g}_{\text{ff}}(T) \simeq 0.2 T^{-3/2} \nu^{-2} n_e n_z Z^2$$



The total brightness from a free-free emitting cloud:

let's consider emission + absorption

$$B(\nu, T) = \frac{1}{4\pi} \frac{j_{br}(\nu, T)}{\mu_{br}(\nu, T)} (1 - e^{-\tau(\nu, T)}) = B_{BB}(\nu, T)(1 - e^{-\tau(\nu, T)})$$

according to radiative transfer

Therefore:

$$B(\nu, T) \approx \frac{T^{-1/2} n_e n_Z Z^2 \bar{g}_{ff}(\nu, T)}{T^{-3/2} \nu^{-2} n_e n_Z Z^2 \bar{g}_{ff}(\nu, T)} (1 - e^{-\tau(\nu, T)}) \approx T \nu^2 (1 - e^{-\tau(\nu, T)})$$

The opacity [$\tau(\nu, T) = \mu(\nu, T)l$; where l = mean free path] determines the spectral shape and the regime:

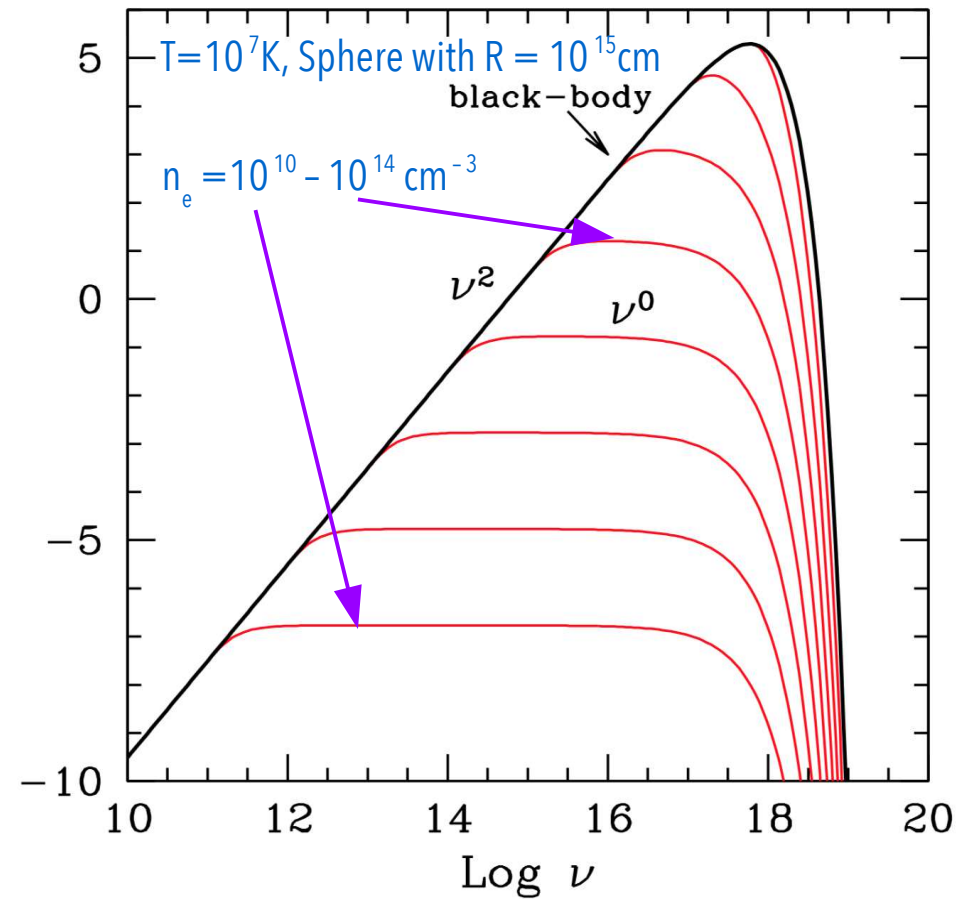
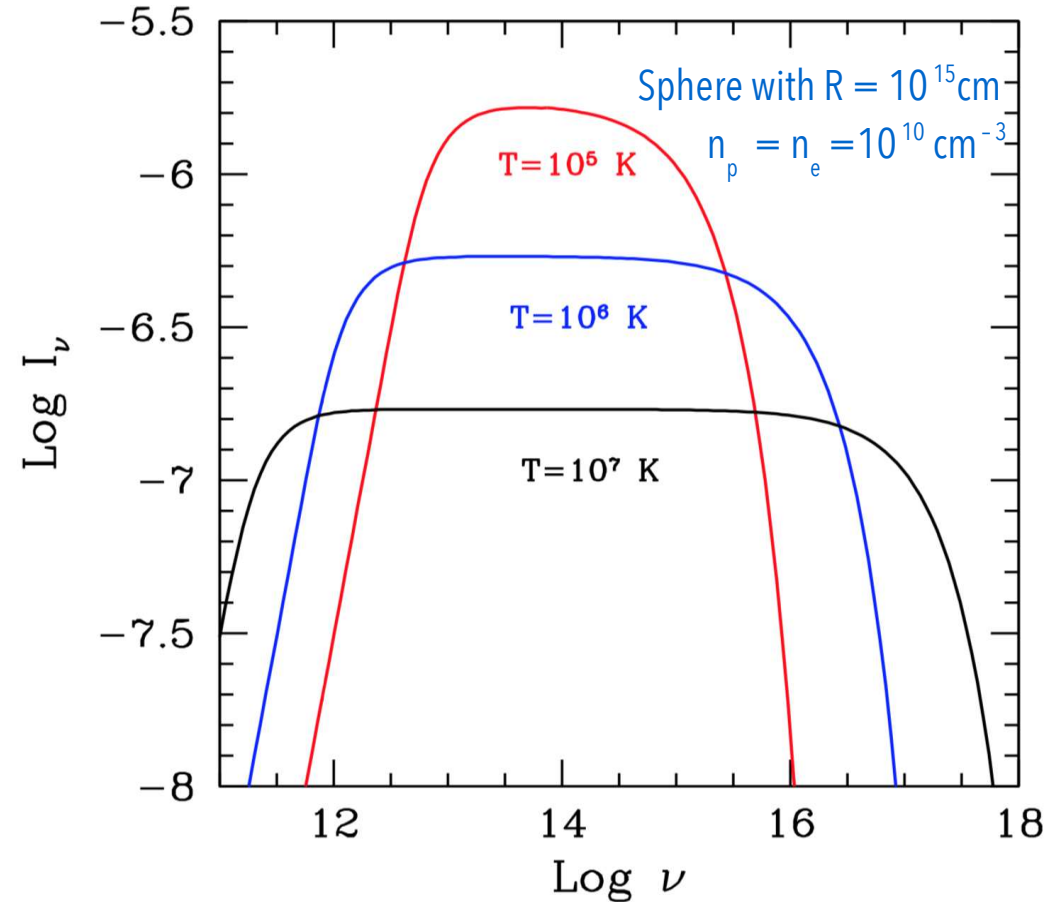
- 1. optically thick $\tau(\nu, T) \gg 1$
- 2. optically thin $\tau(\nu, T) \ll 1$



Bremsstrahlung

brightness/spectrum

(figures from Ghisellini)



$$B(\nu, T) \approx T \nu^2 (1 - e^{-\tau(\nu, T)})$$

$$\tau \gg 1 \rightarrow (1 - e^{-\tau}) = \left(1 - \frac{1}{e^\tau}\right) \approx 1$$

$$\rightarrow B(\nu, T) \approx T \nu^2$$

$$\tau \ll 1 \rightarrow (1 - e^{-\tau}) = (1 - 1 + \tau - o(\tau^2)) \approx \tau = \mu l_0$$

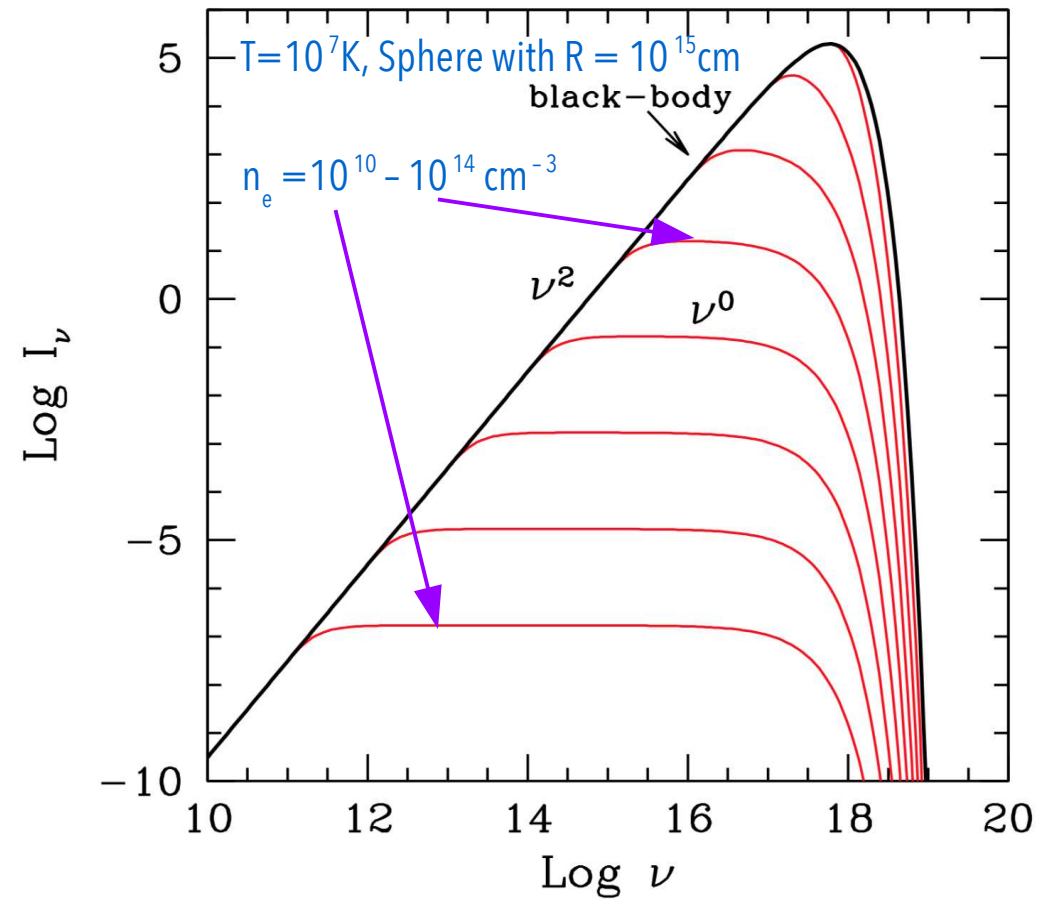
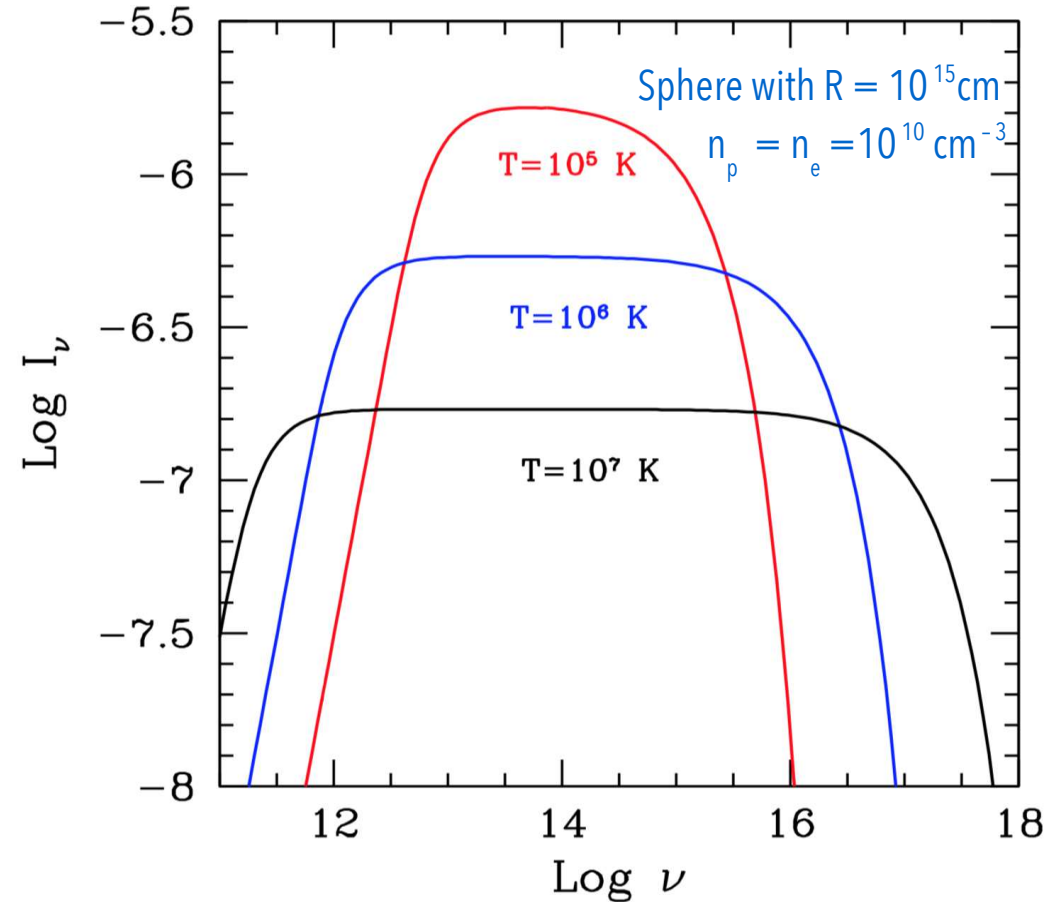
$$\text{but } \mu(\nu) \sim \nu^{-2} \rightarrow B(\nu, T) \approx T \nu^2 \tau \approx T \nu^0$$



Bremsstrahlung

brightness/spectrum

(figures from Ghisellini)



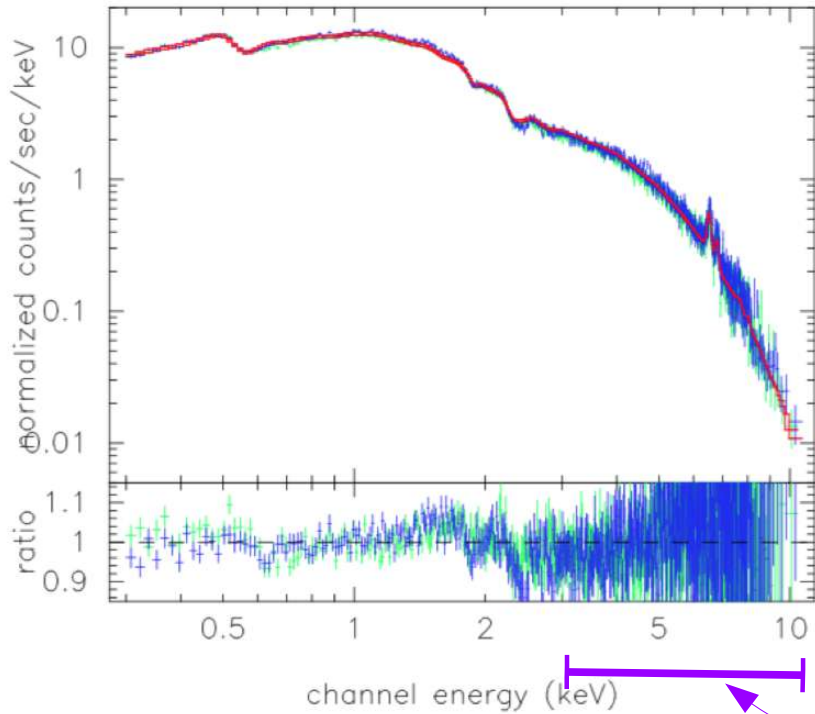
Fixed size and density ↑
 Emissivity decreases with temperature [$J_{br}(\nu, T) \approx T^{-1/2}$]
 The cut-off frequency increases linearly with T...and
 Self-absorption becomes less effective [$\mu(\nu, T) \approx T^{-3/2}$]

Fixed size and temperature ↑
 Emissivity increases with density [$J_{br}(\nu, T) \approx n_e^2$]
 The cut-off frequency stays constant
 Self-absorption reaches higher ν [$\mu(\nu, T) \approx n_e^2$]

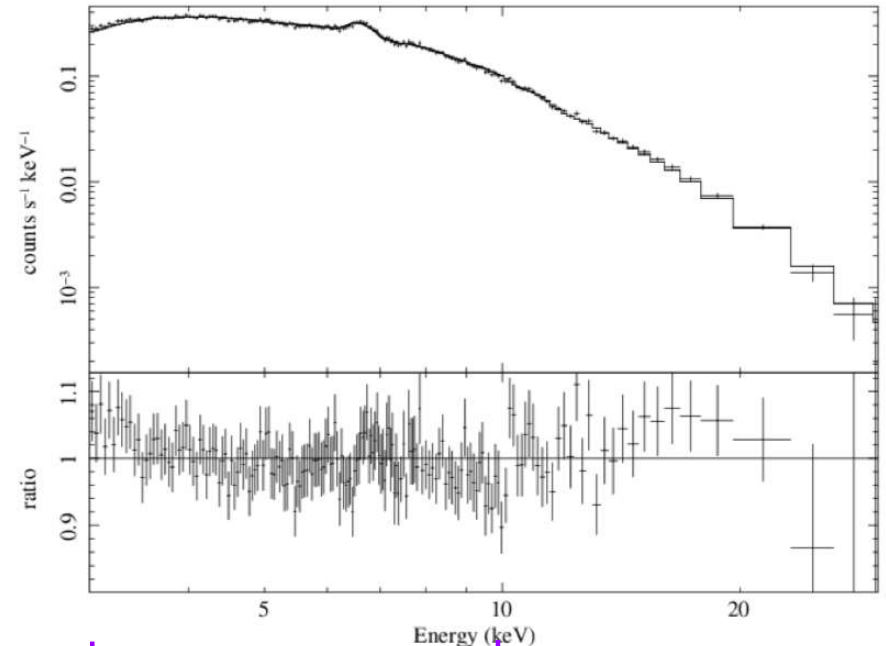


Bremsstrahlung : real X-ray spectra (3)

Coma cluster: Conlon + 2016



(a) XMM-Newton best fit flux model from Coma, within radius of ~ 300 kpc [18].



(b) NuSTAR spectrum and best-fit single temperature model for the central $12' \times 12'$ of the Coma cluster [19].

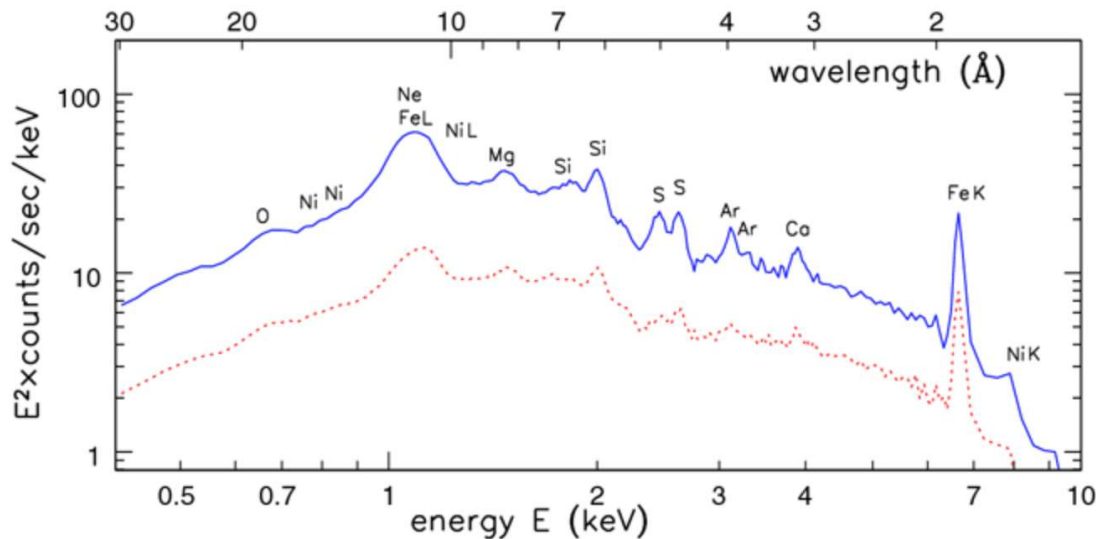


Fig 6.21 (K. Matsushita) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

The cut-off in these X-ray spectra is a measure of the plasma temperature

If distance is known, from the flux, we can derive the density (n_e)



Relativistic Bremsstrahlung

Minor remark: the Gaunt Factor is slightly different wrt the case of "thermal br."

The emissivity for relativistic bremsstrahlung as a function of ν for a given velocity v is:

$$j_{\text{rel,br}}(v, \nu) = \frac{32 \pi}{3} \frac{e^6}{m_e^2 c^3} \frac{1}{v} n_e n_z Z^2 \ln\left(\frac{183}{Z^{1/3}}\right)$$

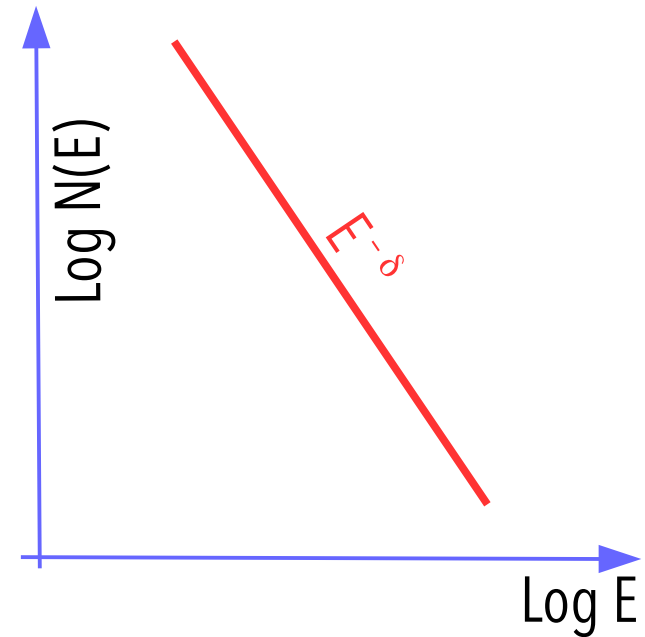
➔ N.B. T is not a relevant concept any more, v must be used

The typical velocities of the particles are now relativistic and the energy (velocity) distribution is described by a power law:

➔ $n_e(E) \approx n_{e,0} E^{-\delta}$

Integrating over velocities (energies)

$$j_{\text{rel,br}}(\nu) \approx \int_{h\nu}^{\infty} n_e(E) n_z Z^2 dE \approx \int_{h\nu}^{\infty} E^{-\delta} dE \simeq \frac{E^{-\delta+1}}{1-\delta} \approx \nu^{-\delta+1}$$



➔ The emitted spectrum is a power-law !



solar wind:

$$n_e \sim 4 - 7 \text{ cm}^{-3}$$

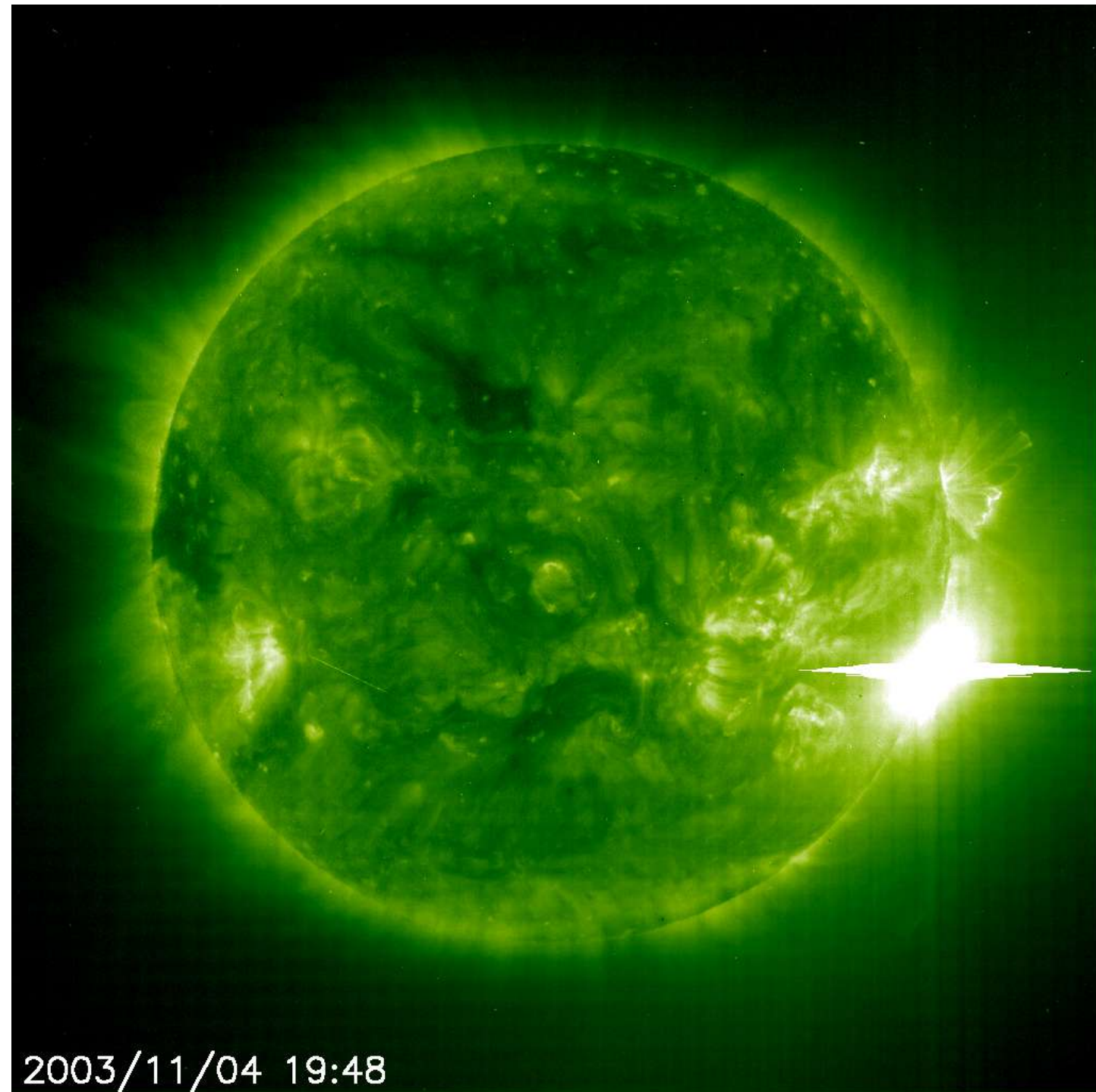
$$v_{\text{swind}} \sim 300 - 900 \text{ km s}^{-1}$$

$$T \sim 150\,000 \text{ K}$$

(not consistent with v !)

flares:

relativistic particles
with power-law energy
distribution



2003/11/04 19:48



Final remarks on bremsstrahlung emission:

a photon is emitted when a moving electron deeply penetrates the Coulomb field of positive ion and then is decelerated

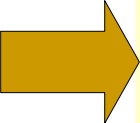
the energy of the photon can never exceed the kinetic energy of the electron

generally the "classical" description of the phenomenon is adequate, the quantum mechanics requires an appropriate Gaunt Factor only

the individual interaction produces polarized radiation (fixed direction of E and B fields of the e-m wave wrt the acceleration)

all the interactions produced in a hot plasma originate unpolarized emission (random orientation of the plane of interaction)

we have either "thermal" or "relativistic" bremsstrahlung, depending on the population (velocity/energy distribution) of the electrons.

 *Annoying question.... Given that only electrons radiate, they can cool, while positive ions do not... therefore, what happens to the thermal equilibrium?*

2. Una regione HII e' generata da una stella di tipo spettrale O. Si supponga che il volume sferico (raggio pari a 1 pc) sia omogeneamente occupato da un plasma di solo H, ad una temperatura uniforme pari a $T_{\text{HII}} = 1.6 \times 10^4 \text{ K}$ e con densita' numerica pari a $n_e = 10 \text{ cm}^{-3}$. Calcolare l'emissivita' specifica di bremsstrahlung, nonche' quella bolometrica (sempre per unita' di volume). Qualora venisse rimossa la stella O, dopo quanto tempo la regione non sarebbe piu' in grado di emettere? Se tale regione si trovasse ad una distanza di 250 pc, quale sarebbe il flusso osservato da un radiotelescopio a 10 GHz (si supponga che l'emissione sia gia' otticamente sottile a questa frequenza)

10 Nov 2017

Total emissivity (per unit volume)

$$J_{\text{br}}(\nu, T) = 6.8 \cdot 10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_Z Z^2 g_{\text{ff}}(\nu, T) \quad \text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1}$$

Total emitted energy per unit volume

$$J_{\text{br}}(T) = \frac{dW}{dt dV} \simeq 2.4 \cdot 10^{-27} T^{1/2} n_e n_Z Z^2 \bar{g}_{\text{ff}}(T) \quad \text{erg s}^{-1} \text{cm}^{-3} \quad \bar{g}_{\text{ff}}(T) \quad \text{average gaunt factor}$$

Cooling Time

$$t_{\text{br}} = \frac{1.8 \cdot 10^{11}}{n_e \bar{g}_{\text{ff}}} T^{1/2} \quad \text{sec} \quad \rightarrow \quad = \frac{6 \cdot 10^3}{n_e \bar{g}_{\text{ff}}} T^{1/2} \quad \text{yr}$$

4. Una regione HII, la “Cone Nebula” alias NGC2264 di cui sotto e' una immagine amatoriale, si trova ad una distanza di 800 pc ed ha un diametro apparente in cielo di 3 gradi. Assumendo che il gas (solo idrogeno per semplicita') si trovi a temperatura e densita' costanti, $T = 10^4$ K ed $n_e = 10^2 \text{ cm}^{-3}$, descrivere sinteticamente quali processi radiativi sono attivi e stimare il flusso a 30 GHz, in caso di emissione sia otticamente sottile a tale frequenza. 18 Sep 2018



Total emissivity (per unit volume)

$$J_{\text{br}}(\nu, T) = 6.8 \cdot 10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_z Z^2 g_{\text{ff}}(\nu, T) \quad \text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1}$$