



"Particles accelerated by a magnetic field will radiate. For non – relativistic velocities the complete nature of the radiation is rather simple and is called <u>cyclotron radiation</u>. The frequency of emission is simply the frequency of gyration in the magnetic field.

However, for extreme relativistic particles the frequency spectrum is much more complex and can extend to many times the gyration frequency. This radiation is known as <u>synchrotron radiation</u>"

George B. Rybicky & Alan P. Lightman in "Radiative processes in astrophysics" p. 167



Synchrotron radiation (cyclotron)

A moving charge *q* is deflected by a uniform magnetic field *B* according to:





In the direction perpendicular to the B field we have a circular motion and in the non-relativistic case we have the well known cyclotron relations:





 $h/p \ll r_{rel}$

 $(\mathbf{r}_{\mathbf{I}})$

The relativistic case is dealt with by considering the appropriate expression of the particle mass:

$$m = \frac{m_0}{\sqrt{1 - \beta^2}} = m_0 \gamma$$
$$r_{rel} = \frac{\gamma m_0 c}{qH} v_\perp = \gamma r_L$$
$$T_{rel} = \frac{2\pi r_L}{v_\perp} = \frac{2\pi m_0 c \gamma}{qH} = \gamma T_L$$
$$\omega_{rel} = \frac{2\pi}{T_r el} = \frac{qH}{\gamma m c} = \frac{\omega_L}{\gamma}$$

if we consider **v~c**, we have:

$$r_{rel} = \frac{\gamma m_0 c^2}{qH} = \frac{\epsilon}{qH}$$

N.B. "classical" physics is ok, since for typical B fields strengths,



Non uniform static magnetic field

Let's consider the RF in which the curvature radius has uniform motion ($v_{||} = const$): the particle "feels" an electric field which is indeed the varying magnetic field, which modifies $v_{||}$

$$\vec{\nabla} \times E' = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$$

for each projected orbit the variation of energy can be written as

$$\Delta\left(\frac{1}{2}m\,v_{\perp}^{2}\right) = \oint q\,\vec{E}\,'\cdot d\,\vec{l} = q\int_{S}\vec{\nabla}\times\vec{E}\,'\cdot d\,\vec{S} = -\frac{q}{c}\int_{S}$$

if the magnetic field changes smoothly within a single orbit

$$\begin{split} \frac{\partial H}{\partial t} &\approx \frac{\Delta H}{T_L} \\ \Delta \epsilon_{\perp} &= \Delta \left(\frac{1}{2}mv_{\perp}^2\right) = \frac{q}{c}\frac{\Delta H}{T_L}(\pi r_L^2) \approx \epsilon_{\perp}\frac{\Delta H}{H} \\ \frac{\Delta \epsilon_{\perp}}{\epsilon_{\perp}} &\approx \frac{\Delta H}{H} \qquad \longrightarrow \frac{\epsilon_{\perp}}{H} = \text{const} \end{split}$$



In the observer's frame

the motion takes place within a static non uniform field;
the Lorentz force does not make work and the total kinetic energy of the particle does not change with time.

If V_{\perp} increases (decreases) going into higher H regions, then V_{\parallel} must decrease (increase). If we consider the energy (scalar), if \mathcal{E}_{\perp} increases (decreases) then \mathcal{E}_{\parallel} must decrease (increase) of the same amount



Magnetic mirrors



$$\frac{\Delta \epsilon_{\perp}}{\epsilon_{\perp}} \approx \frac{\Delta H}{H} \qquad \longrightarrow \frac{\epsilon_{\perp}}{H} = \text{const}$$

is equivalent to:
$$\frac{\sin^2 \theta}{H} = \text{const}$$

if H_{0} and θ_{0} (pitch angle) are known at a given point on the particle trajectory, then:

$$\sin\theta = \sin\theta_o \sqrt{\frac{H}{H_o}}$$

if H increases, then also the pitch angle increases, up to reach the maximum value of 90° and cannot penetrate further regions where H is stronger. It is then induced to move in the reverse direction

Charged particles may be trapped into regions where the magnetic field is strong enough (example, terrestrial magnetic field)



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Example:

Earth Magnetic Field and charged particles



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If a particle is initially very energetic, the magnetic bottle will not be able to confine the particle and it will escape. In exactly the same way, the aurora borealis / australis (Northern and Southern Lights) occurs when charged particles escape from the Van Allen radiation belt. These interact with the atmosphere through optical excitations of the gaseous atoms.





Fast charges within a spiral galaxy: do they escape?

Compute the Larmor radius of a relativistic proton:

 $m_p = 1.67e-24 \ g$ $\gamma = ?$ $B \sim 1 \ \mu G$







Cyclotron radiation

Radiating energy... " a charge moving in a magnetic field is accelerated and then radiates" [CYCLOTRON]

- dipole in the plane of the circular orbit (dipole angular distribution)
- linear polarization or circular or elliptical polarization





Cyclotron radiation

Relativistic charge ($\gamma > 1$) moving in a uniform magnetic field:

Must write the Larmor's formula in an invariant form: scalars are not affected, let's write

$$d\tau = dt/\gamma$$
$$p_i = [\mathbf{p}, (i/c)W]$$
$$W^2 = p^2c^2 + m^2c^4$$

$$P = \frac{dW}{dt} = \frac{2q^2}{3m^2c^3} \left[\frac{dp_i}{d\tau}\frac{dp_i}{d\tau}\right]$$
$$\left[\frac{dp_i}{d\tau}\frac{dp_i}{d\tau}\right] = \left(\frac{dp}{d\tau}\right)^2 - \frac{1}{c^2}\left(\frac{dW}{d\tau}\right)^2 = \left(\frac{dp}{d\tau}\right)^2 - \beta^2\left(\frac{dp}{d\tau}\right)^2$$

$$P_{rel} = \frac{dW}{dt} = \frac{2q^2}{3m^2c^3}\gamma^2 \left[\left(\frac{d\mathbf{p}}{dt}\right)^2 - \beta^2 \left(\frac{dp}{dt}\right)^2 \right]$$

Classical, non relativistic, Larmor case

linear acceleration (no $a_c)~|d {\bf p}/d \tau| \simeq d p/d \tau$

$$P = \frac{dW}{dt} = \frac{2q^2}{3m^2c^3}\frac{1}{\gamma^2}\left(\frac{dp}{d\tau}\right)^2 = \frac{2q^2}{3m^2c^3}\left(\frac{dp}{dt}\right)^2$$

centripetal acceleration $|d\mathbf{p}/d\tau| >> \beta(dp/d\tau) = \frac{1}{c}(dW/d\tau)$

$$P = \frac{dW}{dt} = \frac{2q^2}{3 m^2 c^3} \left(\frac{d\mathbf{p}}{d\tau}\right)^2 = \frac{2q^2}{3 m^2 c^3} \gamma^2 \left(\frac{d\mathbf{p}}{dt}\right)^2$$

Relevant in cyclotron, relativistic cyclotron and synchrotron emission!



$$\frac{d\boldsymbol{p}}{dt} = \frac{q}{c}\boldsymbol{v} \times \boldsymbol{H}$$
$$|\frac{d\boldsymbol{p}}{dt}| = q\beta H \sin\theta$$

 $a_c = v_\perp / mc$ H



If we consider an electron, its gyration frequency is: $v[MHz] \approx 2.5 H[G]$ in general the magnetic fields are much lower, except in very particular stars, and collapsed bodies. Example: 34keV abs feature in Her-X1 (see Longair)



The energy is radiated in various harmonics of the gyration frequency:

$$v_{k} = k v_{rel} \left(1 - \frac{v_{\parallel} \cos \theta}{c} \right) \qquad k = 1, 2, 3, 4, \dots n$$
Doppler shift along the LOS

The energy in the various harmonics follows:





frequency of emitted radiation

 $pprox \beta^2$

observer

l l + 1as a result of a relativistic effect on electron motion and its emitted radiation, fields on the approaching side are amplified, while they are attenuated on the receding side. The effective field can therefore be represented as the superposition





It is generated by ultra-relativistic electrons, for which curvature of the trajectory plays the role in determining the emitted power

$$-\frac{dW}{dt} = \frac{2}{3}\frac{q^4}{m^2c^3}\beta^2\gamma^2H^2\sin^2\theta$$

if $\beta \simeq 1$ and $\epsilon = m_o c^2 \gamma$ then it can be rewritten as

$$-\frac{dW}{dt} = \frac{2}{3} \frac{q^4}{m_o^4 c^7} \varepsilon^2 H^2 \sin^2 \theta = 2c \sigma_T \gamma^2 \frac{H^2}{8\pi} \sin^2 \theta \simeq 1.62 \cdot 10^{-15} \gamma^2 H^2 \sin^2 \theta \ erg \, s^{-1}$$

and σ_{τ} is known as electron Thomson cross section defined as

$$\sigma_{\tau} = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = \frac{8\pi}{3} r_o^2 = 6.6524586 \dots \cdot 10^{-25} cm^{-2}$$



pulse duration (1)

 $\left(\Delta\theta = \frac{2}{v}\right)$

Pulse duration in the electron reference frame is

$$\Delta t = \frac{\Delta \theta}{\omega_{rel}} = \frac{m_e c \gamma}{e H} \frac{2}{\gamma} = \frac{2}{\omega_L}$$

In the observer frame, the duration of the pulse is shortened by propagation (Doppler) effects: the electron is closer to the observer in 2.





pulse duration (2)



algebra is very complex; a detailed discussion is in Rybicki-Lightman p.175

Fourier analysis of the pulse provides the spectrum of the radiated energy. The spectrum is the same as the relativistic cyclotron but with an infinite number of harmonics. The characteristic frequency is:

$$\frac{v_{s}}{4\pi} \simeq \frac{3}{4\pi} \frac{1}{\tau} = \frac{3}{4\pi} \gamma^{2} \omega_{L} = \frac{3}{4\pi} \gamma^{2} \frac{eH}{m_{e}c} = \frac{3}{4\pi} \frac{eH}{m_{e}c} \approx 6.24 \cdot 10^{18} \epsilon^{2} H \simeq 4.2 \cdot 10^{-9} \gamma^{2} H [\mu G] \quad \text{GHz}$$

example: an electron with $\gamma \sim 10^4$ and $H \sim 1 \ \mu G$ has $\sim 0.4 \ GHz$

The full expression of the emitted energy as a function of frequency is

$$\frac{dW(\nu)}{dt} = \frac{dW_{\parallel}}{dt} + \frac{dW_{\perp}}{dt} \approx \frac{\sqrt{3}e^3 H \sin\theta}{8\pi^2 cm_e} F(\frac{\nu}{\nu_s})$$

where

$$F(\frac{\nu}{\nu_s}) = \frac{\nu}{\nu_s} \int_{\nu/\nu_s}^{\infty} K_{5/3}(y) dy$$

and K is modified Bessel function





typical radio spectrum of a radio source





the full sky at 408 MHz

Synchrotron emission from relativistic electrons (positrons) within the Milky Way. In a field of 1 μ G, their energies correspond to $\gamma \sim 10^4$.







Synchrotron emission from relativistic electrons (positrons) in the crab nebula







Let's consider an ensemble of relativistic electrons with energies distributed according to **a <u>power – law</u>**:

$$N(\varepsilon)d\varepsilon = N_{o}\varepsilon^{-\delta}d\varepsilon$$

the specific emissivity of the whole population is

$$\begin{aligned}
J_{s}(v)d(v) &= \frac{dW_{s}(v,\varepsilon)}{dt}N(\varepsilon)d\varepsilon \\
&\approx N_{o}F\left(\frac{v}{v_{s}}\right)\varepsilon^{-\delta}d\varepsilon
\end{aligned}$$
(****)

there are various (approximate) ways to derive the total emissivity:

- 1. all the energy is radiated at the characteristic frequency
- 2. the energy is radiated at a constant rate over a small frequency range



1. all the energy is radiated at the characteristic frequency:

use

$$\nu \simeq \nu_s \approx \gamma^2 \nu_L = \left(\frac{\varepsilon}{m_e c^2}\right)^2 \nu_L \qquad \nu_L = \frac{eH}{2\pi m_e c}$$
$$\varepsilon = \gamma m_e c^2 = \left(\frac{\nu}{\nu_s}\right)^{1/2} m_e c^2$$

 $v_L e non v_s nella formula energia$

then

$$d\varepsilon = \frac{m_e c^2}{2\nu_L^{1/2}} \, \nu^{-1/2} \, d\nu$$

but remember!

$$(-)\frac{dW}{dt} = \frac{2}{3}\frac{q^4}{m^2c^3}\beta^2\gamma^2H^2\sin^2\theta$$

$$I_{s}(v) = \frac{dW_{s}(v,\varepsilon)}{dt} N(\varepsilon) \frac{d\varepsilon}{dv} \approx \text{ constant } N_{o}H^{(\delta+1)/2}v^{-(\delta-1)/2}$$
$$J_{s}(v) \sim N_{o}H^{(\delta+1)/2}v^{-\alpha}$$
where
$$\alpha = \frac{\delta - 1}{2}$$

is known as SPECTRAL INDEX of the synchrotron radiation

The total spectrum is interpreted as the superposition of many contributions from the various electrons



The total spectrum is interpreted as the superposition of many contributions from the various electrons each emitting at its own characteristic frequency



Synchrotron self-absorption (0): summary

"a photon gives back its energy to an electron"

NOT in thermal equilibrium, then Kirchoff's law does not apply.

Concept of temperature still holds, but now there is a limit: the "electron (kinetic) temperature" T_{e}

SSA applies when $T_{_B} \sim T_{_e}$ $T_{_B}$ can NEVER EXCEED $T_{_e}$

The absorption coefficient is effectively determined by making use of the Einstein's coefficients (see further lectures)

$$\mu_{s} \approx N_{o} \nu^{-(\delta+4)/2} H_{\perp}^{(\delta+2)/2}$$



Synchrotron self-absorption (1):

use a trick:

"the power law distribution is the superposition of many Maxwellian at different T : $\gamma m_e c^2 \sim kT$

a given electron emits at its own frequency:

$$v_{s} \simeq 4.2 \cdot 10^{-9} \gamma^{2} H[\mu G] GHz$$

then $kT \sim \gamma m_{e}c^{2} \sim m_{e}c^{2} \left(\frac{\nu}{\nu_{s}}\right)^{1/2} H^{-1/2}$

For an absorbed source the brightness temperature is defined from

 $I(v) \stackrel{\text{def}}{=} 2kT_B \frac{v^2}{c^2} \quad \text{which must be} = \text{to the kinetic temperature of electrons}$ $I(v) \sim m_e c^2 \left(\frac{v}{v_s}\right)^{1/2} H^{-1/2} \frac{v^2}{c^2} \sim \frac{v^{5/2}}{H^{1/2}}$



The brightness temperature of a region does NEVER exceed the electron temperature:

$$T_B \simeq \frac{\lambda^2}{2k} \frac{S_{\nu}}{\theta_1 \cdot \theta_2} \quad \text{but} \quad S(\nu) \approx \nu^{-\alpha}$$

 $3k T_e = \gamma m_e c^2$

$$T_{B,max} = 3.0 \, x \, 10^{10} \, \frac{\nu_{max}^{1/2}}{(GHz)} \frac{H^{-1/2}}{(G)} \quad [K]$$

Synchrotron self-absorption (2):

$$\mu_{s} \approx N_{o} \nu^{-(\delta+4)/2} H_{\perp}^{(\delta+2)/2}$$
$$J_{s}(\nu) \approx \nu^{-\frac{\delta-1}{2}} H_{\perp}^{\frac{\delta+1}{2}}$$

to be inserted in

$$B_{s} = \frac{J_{s}(v)}{4 \pi \mu_{s}(v)} (1 - e^{-\tau_{s}(v)})$$

which becomes

$\underline{\delta-1}$ $\underline{\delta+1}$	
$B_s(v) \approx v^2 H_{\perp}^2$	$\tau \ll 1$ optically thin regime
$B_s(v) \approx v^{5/2} H_{\perp}^{-1/2}$	$\tau \gg 1$ optically thick regime

the first case is that derived from the earlier description which provides a power-law distribution



Synchrotron self-absorption (4):

Let's derive the expression for J_s to get the peak values:

$$v_{p} \approx S_{p}^{2/5} \theta^{-4/5} H_{\perp}^{1/5} (1+z)^{1/5}$$

Important issue: from observations we observe: peak flux density, turnover frequency angular size then.....







Synchrotron self-absorption (5):





Energetics of a radio source

The total energy of a synchrotron emitting body must take into account both particles and the H field

$$U_{tot} = \varepsilon_{el} + \varepsilon_{pr} + U_H = (1+k)\varepsilon_{el} + U_H = U_p + U_H$$

If we consider relativistic electrons, their energy is given by

$$\varepsilon_{el} = V \int_{\varepsilon_{min}}^{\varepsilon_{max}} \varepsilon N(\varepsilon) d\varepsilon = \frac{N_o V}{2 - \delta} \left(\varepsilon_{max}^{2 - \delta} - \varepsilon_{min}^{2 - \delta} \right) \qquad \delta \neq 2$$

but we can get rid of $N_o V$ if we consider the source Lumonosity

$$L = 4\pi D^{2} \int_{\nu_{min}}^{\nu_{max}} S(\nu) d\nu = V \int_{\varepsilon_{min}}^{\varepsilon_{max}} N(\varepsilon) \frac{d\varepsilon}{dt} d\varepsilon$$
$$L \approx N_{o} V \int_{\varepsilon_{min}}^{\varepsilon_{max}} H^{2} \varepsilon^{2} \varepsilon^{-\delta} \sin^{2} \theta d\varepsilon = 2.4 \cdot 10^{-3} \frac{N_{o} V H^{2} \sin^{2} \theta}{3 - \delta} \left(\varepsilon_{max}^{3 - \delta} - \varepsilon_{min}^{3 - \delta}\right)$$
where $\varepsilon_{min/max} = \left(\frac{\nu_{min/max}}{6.24 \cdot 10^{18} H}\right)^{1/2}$



Energetics of a radio source (2)

$$\varepsilon_{el} = \frac{3-\delta}{2-\delta} \frac{\varepsilon_{max}^{2-\delta} - \varepsilon_{min}^{2-\delta}}{\varepsilon_{max}^{3-\delta} - \varepsilon_{min}^{3-\delta}} \frac{L}{H^2 \sin^2 \theta} \frac{1}{2.4 \times 10^{-3}} = \frac{3-\delta}{2-\delta} \frac{[\nu_{max}^{2-\delta} - \nu_{min}^{2-\delta}]H^{-(2-\delta)/2} \sin^{-(2-\delta)} \theta}{[\nu_{max}^{3-\delta} - \nu_{min}^{3-\delta}]H^{-(3-\delta)/2} \sin^{-(3-\delta)} \theta} \frac{L}{H^2 \sin^2 \theta} \frac{L}{2.4 \times 10^{-3}}$$

once chosen the minimum and maximum energy, considering an isotropic pitch angle distribution ($H^2sin^2\theta = 2/3 H^2$), the above becomes:

$$\varepsilon_{el} = C_{el} H^{-3/2} L$$

which represents the energy associated with the relativistic particles (electrons) radiating synchrotron emission





"Equipartition" Magnetic Field

$$H_{eq} = H(\varepsilon)_{min} = \left[\frac{3}{4}(1+k)\frac{C_{el}}{C_{H}}\right]^{2/7} \left(\frac{L}{V}\right)^{2/7}$$

such *equipartition field* provides the minimum total energy content in the radio source (related to the relativistic plasma), which amounts to

$$U_{tot,min} = \frac{7}{4} (1+k) \varepsilon_{el} = \frac{7}{4} (1+k) C_{el} H_{eq}^{-3/2} = 2 [(1+k) C_{el}]^{4/7} C_{H}^{3/7} L^{4/7} V^{3/7}$$

which can be written in a specific expression for radio emission at 1.4 GHz as

$$U_{tot,min} = 2 \cdot 10^{41} (1+k)^{4/7} \left(\frac{L_{1.4GHz}}{[Watt]}\right)^{4/7} \left(\frac{V}{[kpc]^3}\right)^{3/7} [erg]$$

and this can be used to define the energy density and the internal pressure as

$$u_{min} = \frac{U_{tot,min}}{V} = 6.8 \cdot 10^{-24} \left(\frac{L_{1.4GHz}}{[Watt]} \frac{[kpc]^3}{V} \right)^{4/7} [erg \ cm^{-3}]$$
$$P_{min} = P_H + P_{rel} = \frac{H^2}{8\pi} + (\Gamma - 1)u_{rel} = \frac{11}{21}u_{min}$$



Is equipartition Magnetic Field representative?





Energetics of a radio source

application to 3C219

etal

Physical quantities can be calculated either globally for the whole source, or locally, at various sites within the same radio source:

Pressure balance: *Pint* and *PISM / PICM* provide the ram pressure and then the espansion/growth velocity



$$u_{min} = \frac{U_{tot,min}}{V} = 6.8 \times 10^{-24} \left(\frac{L_{1.4 \text{ GHz}}}{[\text{Watt}]} \frac{1}{V \text{ [kpc^3]}}\right)^{4/7} \qquad [\text{erg cm}^{-3}]^{\text{EACE.}}$$
$$P_{min} = P_H + P_{rel} = \frac{H^2}{8\pi} + (\Gamma - 1)u_r = \frac{11}{21}u_{min}$$

<u>Time evolution</u> Particles radiate a

Particles radiate at expenses of their kinetic energy. Hence, the energy distribution of a synchrotron emitting region within a given volume V fully filled by magnetized relativistic plasma will change with time

Particle flow Energy losses Leakage Injection

$$\frac{\partial N(\varepsilon,t)}{\partial t} + \frac{\partial}{d\varepsilon} \left(\frac{d\varepsilon}{dt} N(\varepsilon,t) \right) + \frac{N(\varepsilon,t)}{T_{conf}} = Q(\varepsilon,t)$$

where the first term represents the particle flow, the second is for energy losses the third takes into account the leakage. On the right hand part a term representing a continuos injection/production of relativistic particles is added. In particular

 $N(\varepsilon, 0) = N_o \varepsilon^{-\delta}$ $Q(\varepsilon, t) = A \varepsilon^{-\delta}$

Radiative losses: $-\frac{d \varepsilon}{dt} = C_{sync} \varepsilon^{2} H^{2} \sin^{2} \theta \quad where \quad C_{sync} = \frac{2 e}{3 m_{e}^{4} c^{7}}$

it is possible to derive the energy of each particle as a function of time:

$$-\frac{d\varepsilon}{\varepsilon^{2}} = C_{sync}H^{2}\sin^{2}dt$$
$$\frac{1}{\varepsilon(t)} - \frac{1}{\varepsilon_{o}} = C_{sync}H^{2}\sin^{2}\theta t$$
$$\varepsilon(t) = \frac{\varepsilon_{o}}{1 + C_{sync}\varepsilon_{o}H^{2}\sin^{2}\theta t}$$

We can also define a characteristic time, the cooling time t^* , as the ratio between the total particle energy and its loss rate:

$$t^{*} = \frac{\varepsilon_{o}}{d \varepsilon / dt} = \frac{\varepsilon_{o}}{C_{sync} \varepsilon_{o}^{2} H^{2} \sin^{2} \theta} = \frac{1}{C_{sync} \varepsilon_{o} H^{2} \sin^{2} \theta}$$
$$\varepsilon(t) = \frac{\varepsilon_{o}}{1 + t / t^{*}}$$



Radiative losses:

High energy particles have shorter t* than low energy ones and the emission spectrum is modified accordingly due to the depletion of the high energy particles from the energy distribution:



A case study:



$$\frac{\partial \mathcal{N}(\varepsilon,t)}{\partial t} + \frac{\partial}{\partial \varepsilon} \Big(\frac{d\varepsilon}{dt} \mathcal{N}(\varepsilon,t) \Big) + \frac{\mathcal{N}(\varepsilon,t)}{T_{conf}} = Q(\varepsilon,t)$$

when the confinement time is extremely large and $Q(\varepsilon,t) = A \varepsilon^{-\delta}$ a balance between dying and refurbished particles is achieved at a particular energy, corresponding to a particular frequency, and, in turn, to a characteristic time; one solution of the equation is such that the synchrotron emissivity becomes:

$$J_s(\nu) \approx \nu^{-(\delta-1)/2} = \nu^{-\alpha} \qquad \nu << \nu^*$$
$$J_s(\nu) \approx \nu^{-\delta/2} = \nu^{-(\alpha+1/2)} \qquad \nu > \nu^*$$

Spectral "ageing"

The search for a break in the high frequency spectrum is a tool to estimate the radiative age of the dominant population of relativistic electrons

 $\log S_{v}$





Spectral "ageing" The age of electrons is different in various region of a radio

Info on the r–source formation and evolution

source



0.04

0.06

(JY/BEAM)

Core

280

300

320

0.08

8421.209MHz

Lobe

0.1

0.02

Hot

Spot

240

260

n

320

300

X 280

260

240

ę

e

0

g

Л

а

n

8

0147+400















Fresh electrons in an active nucleus // Old electrons in a dead AGN



3C285







Other energy losses

Ionization losses:

relativistic particles can interact with (neutral) matter with density n_o and let electrons free as a consequence of electrostatic force

$$-\left(\frac{d\,\varepsilon}{dt}\right)_{i} \approx n_{o}\ln(\varepsilon)$$

which has a characteristic time scale

$$t^* = 9.4 \cdot 10^{17} \frac{\varepsilon[erg]}{n_o} sec$$

Relativistic bremsstrahlung losses:

$$-\left(\frac{d \varepsilon}{dt}\right)_{rel-br} \approx n_o^2(\varepsilon)$$

The characteristic time of this phenomenon does not depend on the energy of the particles.

Both these processes are more relevant for low energy electrons since they scale with $\ln(\epsilon)$ and ϵ while synchrotron losses scale with ϵ^{2} .

Inverse Compton scattering losses:let's wait for a while.....

Adiabatic expansion:



Poisson's law:

$$T V^{\Gamma-1} = const \longrightarrow T r^{3(4/3-1)} = T r = const$$

$$\varepsilon \sim T \sim \frac{1}{r} \qquad \varepsilon r = \varepsilon_o r_o = const$$

$$\varepsilon(t) = \varepsilon_o \frac{r_o}{r} = \varepsilon_o \frac{r_o}{r_o + v_{exp}t}$$

$$\frac{d\varepsilon}{dt} = -\frac{v_{exp}}{r}\varepsilon$$

$$t_{ad}^* = \frac{r_o}{v_{exp}} = cost \qquad \varepsilon(t) = \frac{\varepsilon_o}{1 + t/t_{ad}^*}$$

magnetic field:

$$H(t) = H_o \left(\frac{r_o}{r}\right)^2$$

The shape of the spectrum does not change, but it is shifted at lower frequencies



Rybicki & Lightman, Fig. 6.7

Result of computations for both polarization directions:

$$\begin{pmatrix} P_{\parallel} \\ P_{\perp} \end{pmatrix} = \frac{\sqrt{3}}{2} \frac{e^3 B}{mc^2} \begin{pmatrix} F(\nu/\nu_c) - G(\nu/\nu_c) \\ F(\nu/\nu_c) + G(\nu/\nu_c) \end{pmatrix}$$
(6.31)

$$F(x) = x \int_x^\infty K_{5/3}(y) \, \mathrm{d}y$$

$$G(x) = x K_{2/3}(x)$$



Polarization (2)

The total emitted power for monoenergetic electrons is

$$P(\nu) = P_{\parallel}(\nu) + P_{\perp}(\nu) \propto F(\nu) \tag{6.34}$$

As before, the total emitted spectrum is found by integrating over the electron energy distribution. For a power-law:

$$\begin{pmatrix} P_{\parallel}(\nu) \\ P_{\perp}(\nu) \end{pmatrix} = \left(\frac{\sqrt{3}}{2}\right) n_0 \frac{e^3 B}{m_e c^2} \begin{pmatrix} J_F - J_G \\ J_F + J_G \end{pmatrix} \left(\frac{2\nu}{3\nu_{\rm L}}\right)^{-(p-1)/2}$$

$$(6.35)$$

where

$$J_F = \frac{2^{(p+1)/2}}{p+1} \Gamma\left(\frac{p}{4} + \frac{19}{12}\right) \Gamma\left(\frac{p}{4} - \frac{19}{12}\right)$$
(6.36)
$$J_G = 2^{(p-3)/2} \Gamma\left(\frac{p}{4} + \frac{7}{12}\right) \Gamma\left(\frac{p}{4} - \frac{1}{12}\right)$$
(6.37)



Polarization (3)

The degree of polarization is defined by

$$\frac{P_{\perp} - P_{\parallel}}{P_{\perp} + P_{\parallel}} = \frac{J_G}{J_F} = \frac{p+1}{p+7/3}$$

(6.38)

For p = 2.5 the degree of polarization is $\sim 70\%$. This is very large!!

Caveat: Faraday-rotation and B-field inhomogeneities can decrease the degree of polarization



polarization in M51



Example of polarized emission: M51 – vector length proportional to projected field strength – vector direction (B field shown here)







Example 2: polarized er



00 0

17 55

50

RIGHT ASCENSION (B1950)

45

09 18 00

add: exercise to compute the radiative age and the kinematic age

